EXERCISE

SHORT ANSWER TYPE QUESTIONS

Solve for *x*, the inequalities in Exercise 1 to 12.

 $\left[\because \quad \frac{1}{x} < \frac{1}{y} \quad \Rightarrow \quad x > y \right]$ $|x| - 3 \ge 2$ \Rightarrow + ∞ -5 +5 $|x| \geq 5$ \Rightarrow $x \le -5$ or $x \ge 5$ \Rightarrow $x \in (-\infty, -5] \cup [5, \infty)$ So, ...(*i*) Here $|x| - 3 \neq 0$ |x| - 3 < 0 or |x| - 3 > 0 \Rightarrow |x| < 3 or |x| > 3 \Rightarrow -3 < x < 3 or x < -3 or x > 3 \Rightarrow ...(*ii*) from in eq. (*i*) and (*ii*) we get $x \in (-\infty, -5] \cup (-3, 3) \cup [5, \infty)$ Q4. $|x-1| \le 5, |x| \ge 2$ **Sol.** $|x - 1| \le 5$ and $|x| \ge 2$ \Rightarrow $-5 \le x - 1 \le 5$ and $x \le -2$ or $x \ge 2$ \Rightarrow $-5+1 \le x \le 5+1$ \Rightarrow $-4 \le x \le 6$ and $x \le -2$ or $x \ge 2$ Hence $x < [-4, -2] \cup [2, 6]$ Q5. $-5 \le \frac{2-3x}{4} \le 9$ **Sol.** Given that $-5 \le \frac{2-3x}{4} \le 9$ $-20 \le 2 - 3x \le 36 \quad \Rightarrow \quad -20 - 2 \le -3x \le 36 - 2$ \Rightarrow $-22 \le -3x \le 34 \implies 22 \ge 3x \ge -34$ \Rightarrow $\frac{22}{3} \ge x \ge \frac{-34}{3}$ \Rightarrow $x \in \left[\frac{-34}{2}, \frac{22}{2}\right]$ Hence **Q6.** $4x + 3 \ge 2x + 17$, 3x - 5 < -2**Sol.** Given that $4x + 3 \ge 2x + 17$...(*i*) 3x - 5 < -2...(*ii*) from eq. (i) we get $4x + 3 \ge 2x + 17 \implies 4x - 2x \ge 17 - 3$ $2x \ge 14$ $\Rightarrow x \ge 7$ \Rightarrow From in eq. (*ii*) we get, 3x - 5 < -2 $3x < 5-2 \implies 3x < 3$ \Rightarrow *x* < 1 \Rightarrow

we set that the solution $x \ge 7$ and x < 1 is not possible. Hence there will be no solution of *x*.

- **Q7.** A company manufactures cassettes. Its cost and revenue functions are C(x) = 26,000 + 30x and R(x) = 43x, respectively, where *x* is the number of cassettes produced and sold in a week. How many cassettes must be sold by the company to realise some profit?
- **Sol.** Given that: Cost function, C(x) = 26,000 + 30xand revenue function R(x) = 43xNow for profit P(x), R(x) > C(x) $\Rightarrow 43x > 26000 + 30x \Rightarrow 26000 + 30x < 43x$ $\Rightarrow 30x - 43x < -26000 \Rightarrow -13x < -26000$ $\Rightarrow 13x > 26000 \Rightarrow x > 2000$ Hence, number of cassettes to be manufactured for some

profit must be more than 2000.

- **Q8.** The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 8·2 and 8·5. If the first two pH readings are 8·48 and 8·35, find the range of pH value for the third reading that will result in the acidity level being normal.
- **Sol.** Let the third pH value be *x*. Given that first pH value = 8.48 and second pH value = 8.35

...

$$8.48 + 8.35 + x$$

Average value of pH = $\frac{3}{3}$

But average value of pH lies between 8.2 and 8.5

 $\begin{array}{ll} \ddots & 8 \cdot 2 < \frac{8 \cdot 48 + 8 \cdot 35 + x}{3} < 8 \cdot 5 \\ \Rightarrow & 24 \cdot 6 < 16 \cdot 83 + x < 25 \cdot 5 \\ \Rightarrow & 24 \cdot 6 - 16 \cdot 83 < x < 25 \cdot 5 - 16 \cdot 83 \\ \Rightarrow & 7 \cdot 77 < x < 8 \cdot 67 \end{array}$

Hence, the third pH value lies between 7.77 and 8.67.

- **Q9.** A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 litres of the 9% solution, How many litres of 3% solution will have to be added?
- **Sol.** Let *x* litres of 3% solution be added to 460 litres of 9%.

:. Total amount of mixture = (460 + x) litres

Given that the acid contents in the resulting mixture is more than 5% but less than 7% acid.

$$\therefore 5\% \text{ of } (460 + x) < 460 \times \frac{9}{100} + \frac{3}{100} \times x < 7\% \text{ of } (460 + x)$$

$$\Rightarrow \frac{5}{100} (460 + x) < \frac{4140 + 3x}{100} < \frac{7}{100} (460 + x)$$

$$\Rightarrow 5(460 + x) < 4140 + 3x < 3220 + 7x$$

$$\Rightarrow 2300 + 5x < 4140 + 3x < 3220 + 7x$$

$$\Rightarrow 2300 + 5x < 4140 + 3x \text{ and } 4140 + 3x < 3220 + 7x$$

$$\Rightarrow 5x - 3x < 4140 - 2300 \text{ and } 3x - 7x < 3220 - 4140$$

$$\Rightarrow 2x < 1840 \text{ and } -4x < -920$$

$$\Rightarrow x < \frac{1840}{2} \text{ and } 4x > 920$$

$$\Rightarrow x < 920 \therefore x > \frac{920}{4}$$
and
$$x > 230$$

Hence the required amount of acid solution is more than 230 litres and less than 920 litres.

- Q10. A solution is to be kept between 40°C and 45°C. What is the range of temperature in degree Fahrenheit, if the conversion formula is $F = \frac{9}{5} \circ C + 32^{\circ}?$
- Sol. Given that the range of temperature of the solution is 40°C and 45°C

Formula of conversion is $F = \frac{9}{5} \circ C + 32^{\circ}$ a 5

=

$$\Rightarrow \qquad F - 32^\circ = \frac{7}{5} \circ C \quad \Rightarrow \quad \circ C = \frac{5}{9} (F - 32^\circ)$$

$$40^\circ < \circ C < 45^\circ$$

$$\Rightarrow \quad 40^\circ < \frac{5}{9} (F - 32^\circ) < 45^\circ \Rightarrow \quad 40^\circ \times \frac{9}{5} < F - 32^\circ < 45^\circ \times \frac{9}{5}$$

$$\Rightarrow \quad 72^\circ < F - 32^\circ < 81^\circ \quad \Rightarrow \quad 72^\circ + 32^\circ < F < 81^\circ + 32^\circ$$

$$\Rightarrow \quad 104^\circ < F < 113^\circ$$
Hence, the require range is 104°E to 113°E

Hence, the require range is 104°F to 113°F.

- Q11. The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm, then find the minimum length of the shortest side.
- **Sol.** Let the length of the shortest side be *x* cm
 - :. Length of the longest side = 2x cm

and the length of the third side = (x + 2) cm

Perimeter of the triangle = $x \operatorname{cm} + 2x \operatorname{cm} + (x+2) \operatorname{cm} = (4x+2) \operatorname{cm}$ As per the condition of the question,

Perimeter > 166 cm $4x + 2 > 166 \implies 4x > 166 - 2$ \Rightarrow \Rightarrow

 $4x > 164 \implies x > 41 \text{ cm}$

Hence, the minimum length of the shortest side of the triangle is 41 cm.

- Q12. In drilling world's deepest hole it was found that the temperature T is degree Celsius, x km below the earth's surface was given by $T = 30^\circ + 25^\circ (x - 3)$, $3 \le x \le 15$. At what depth will the temperature be between 155°C and 205°C?
- **Sol.** Given that, Temperature, $T = 30^\circ + 25^\circ (x 3)$, $3 \le x \le 15$ Range of the temperature is 155°C to 205°C

| itunge of the tem | peruture 15 100 C to 200 C |
|-------------------|--|
| | $155^{\circ} < T < 205^{\circ}$ |
| \Rightarrow | $155^{\circ} < 30^{\circ} + 25^{\circ}(x-3) < 205^{\circ}$ |
| \Rightarrow | $125^{\circ} < 25^{\circ}(x-3) < 175^{\circ}$ |
| \Rightarrow | $\frac{125}{25} < x - 3 < \frac{175}{25}$ |
| \Rightarrow | 5 < x - 3 < 7 |
| \Rightarrow | 8 < x < 10 |
| TT .1 . | 1 |

Hence the required temperature at the depth from 8 km to 10 km lies between 155°C and 205°C.

Q13. Solve the following system of inequalities

$$\frac{2x+1}{7x-1} > 5, \frac{x+7}{x-8} > 2$$

Sol. Let us first take the inequation

$$\frac{2x+1}{7x-1} > 5 \quad \Rightarrow \quad \frac{2x+1}{7x-1} - 5 > 0$$

$$\Rightarrow \qquad \frac{2x+1-5(7x-1)}{7x-1} > 0 \quad \Rightarrow \quad \frac{2x+1-35x+5}{7x-1} > 0$$

$$\Rightarrow \qquad \frac{-33x+6}{7x-1} > 0 \quad \Rightarrow \quad \frac{33x-6}{7x-1} < 0$$
So,
$$x \in \left(\frac{1}{7}, \frac{2}{11}\right) \qquad \dots(i)$$

Now from second inequality, we have

$$\Rightarrow \qquad \frac{x+7}{x-8} > 2 \quad \Rightarrow \quad \frac{x+7}{x-8} - 2 > 0$$
$$\Rightarrow \qquad \frac{x+7-2x+16}{x-8} > 0 \quad \Rightarrow \quad \frac{-x+23}{x-8} > 0$$

⇒ So

$$\frac{x-23}{x-8} < 0$$
 $x \in (8, 23)$...(*ii*)

Now nothing is common to the two values of *x i.e.* null set.

Hence, the given inequations has no solution.

- **Q14.** Find the linear inequalities for which the shaded region in the given figure is the solution set.
- **Sol.** Let us first consider the linear equation 3x + 2y = 48. We observe that the shaded region and the origin both on the same side of the graph of the line and



(0, 0) satisfy the constraint $3x + 2y \le 48$.

Similarly, we observe that the shaded region and the origin both on the same side of the graph of the line and (0, 0) satisfy the constraint $x + y \le 20$.

We also notice that the shaded region is in the first quadrant where x > and y > 0

Hence, the required linear inequalities are

 $3x + 2y \le 48$, $x + y \le 20$, $x \ge 0$, $y \ge 0$

- **Q15.** Find the linear inequalities for which the shaded region in the given figure is the solution set.
- **Sol.** Let us first consider the line x + y = 4

We observe that the shaded region and the origin (0, 0) are on the opposite side and (0, 0) satisfies the constraint $x + y \le 4$.

So, $x + y \ge 4$ is the linear



inequality. Now, take the linear equation x + y = 8.

In this case, the shaded region and the origin (0, 0) lie on the same side and (0, 0) also satisfies the constraint $x + y \le 8$.

So, $x + y \le 8$ is another linear inequality

Consider, x = 5. It is clear that the shaded area lies towards the origin. So $x \le 5$ is the constraints and similarly $y \le 5$ is also the constraints. We also observe that the shaded region lies in first quadrant, so $x \ge 0$, $y \ge 0$.

Hence, the required system of inequalities is

 $x + y \ge 4$, $x + y \le 8$, $x \le 5$, $y \le 5$, $x \ge 0$, $y \ge 0$.

Q16. Show that the following system of linear inequalities has no solution. $x + 2y \le 3$, $3x + 4y \ge 12$, $x \ge 0$, $y \ge 1$

Sol. Given that:
$$x + 2y \le 3$$
, $3x + 4y \ge 12$, $x \ge 0$, $y \ge 1$
Let $x + 2y = 3 \implies x = 3 - 2y$

Now putting (0, 0) in $x + 2y \le 3$

$$0 + 0 \le 3$$

 $0 \le 3$ True

Therefore, the shading will be towards (0, 0) Consider the in equation $3x + 4y \ge 12$ Let 3x + 4y = 12

| | x | 0 | 4 | 2 |
|--------------------------------------|---|---|---|-----|
| | у | 3 | 0 | 1.5 |
| 0 \cdot 2 \cdot 1 $>$ 10 | | | | |

Putting (0, 0) in $3x + 4y \ge 12$ 0+0 > 12

$$0 \ge 12$$
 False

Therefore, shading will be on the opposite side of the graph of line.

 $x \ge 0 \implies$ positive side of *y*-axis will be shaded. and $y \ge 1 \implies$ upper side of y = 1 will be shaded. Let us now draw the graph. It is clear from the graph that there is no common shaded region. Hence, the solution is null set.



...(*i*)

Q17. Solve the following system of linear inequalities

$$3x + 2y \ge 24$$
, $3x + y \le 15$, $x \ge 4$

Sol. Given that
$$3x + 2y \ge 24$$

Let $3x + 2y = 24$;

| <u> </u> | | | | |
|----------|----|---|---|--|
| x | 0 | 8 | 4 | |
| у | 12 | 0 | 6 | |

We observe that the graph of (*i*) inequality meets *x*-axis at (8, 0) and *y*-axis at (0, 12).

The origin (0, 0) does not satisfy the inequation (i)So, the shading will be on opposite side of the origin. From inequation (ii)

 $3x + y \le 15$ Let 3x + y = 15

| - | 0 | | | |
|---|---|----|---|---|
| | x | 0 | 5 | 3 |
| | y | 15 | 0 | 6 |

Put a point (0, 0) in inequation (*ii*) we get

 $\begin{array}{c} 0+0 \leq 15 \\ 0 \leq 15 \end{array} \text{ True} \end{array}$

So, the shading will be on the same side where the origin lies.

Now drawing $x \ge 4$, we observe a straight line parallel to *y*-axis passing through a point (4, 0) which does not satisfy the equation. So, the shading will be on right side of the line. Now it is clear from the graph that there is no region is common to all inequalities. Hence, the given system of inequalities has no solution.



Q18. Show that the solution set of the following system of linear inequalities is an unbounded region.

 $2x + y \ge 8$, $x + 2y \ge 10$, $x \ge 0$, $y \ge 0$

Sol. Let us consider the inequality $2x + y \ge 8$ as an equation

| 2x + y = 8 | | | | |
|------------|---|---|---|--|
| x | 0 | 4 | 2 | |
| у | 8 | 0 | 4 | |

The line 2x + y = 8 intersects *x*-axis at (4, 0) and *y*-axis at (0, 8) The origin (0, 0) does not satisfy the inequation $2x + y \ge 8$ Therefore shading will be done opposite side of the origin. Let us consider $x + 2y \ge 10$ as an equation x + 2y = 10.

| x | 0 | 10 | 4 |
|---|---|----|---|
| y | 5 | 0 | 3 |

The x + 2y = 10 meets the axes at (10, 0) and (0, 5)

The origin (0, 0) does not satisfy the in equation $x + 2y \ge 10$ Therefore, shading will be done on the opposite side of the origin



It is clear from the graph that the common shaded region is unbounded.

OBJECTIVE TYPE QUESTIONS

Q19. If *x* < 5, then

(a) -x < -5 (b) $-x \le -5$ (c) $-x \ge -5$ (d) $-x \ge -5$ Sol. If x < 5 then $-x \ge -5$

Here, the correct option is (*c*).

Q20. Given that *x*, *y* and *b* are real numbers and x < y, b < 0 then

(a)
$$\frac{x}{b} < \frac{y}{b}$$
 (b) $\frac{x}{b} \le \frac{y}{b}$ (c) $\frac{x}{b} > \frac{y}{b}$ (d) $\frac{x}{b} \ge \frac{y}{b}$

Sol. Given that *x* < *y*, *b* < 0 $\Rightarrow \frac{x}{h} > \frac{y}{h}, b < 0$ Hence, the correct option is (*c*). **Q21.** If -3x + 17 < -13, then (a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$ (c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10)$ **Sol.** Given that − 3*x* + 17 < − 13 \Rightarrow -3x < -17 - 13 \Rightarrow -3x < -30 \Rightarrow 3x > 30 \Rightarrow x > 10 $x \in (10, \infty)$ \Rightarrow Hence, the correct option is (*a*). **Q22.** If *x* is a real number and |x| < 3 then (b) -3 < x < 3 (c) $x \le -3$ (d) $-3 \le x \le 3$ (a) $x \ge 3$ **Sol.** Given that |x| < 3 $[\because |x| < a \implies -a < x < a]$ \Rightarrow -3 < x < 3Hence, the correct option is (*b*). **Q23.** *x* and *b* are real numbers. If b > 0 and |x| > b then (a) $x \in (-b, \infty)$ (b) $x \in [-\infty, b]$ (d) $x \in (-\infty, -b) \cup (b, \infty)$ (c) $x \in (-b, b)$ **Sol.** Given that |x| > b, b > 0x < -b or x > b \Rightarrow $x \in (-\infty, -b) \cup (b, \infty)$ \Rightarrow Hence, the correct option is (*d*). **Q24.** If |x - 1| > 5, then (b) $x \in [-4, 6]$ (a) $x \in (-4, 6)$ (c) $x \in (-\infty, -4) \cup (6, \infty)$ (d) $x \in (-\infty, -4) \cup (6, \infty)$ **Sol.** Given that |x-1| > 5 \Rightarrow (x-1) < -5 or (x-1) > 5x < -5 + 1 or x > 5 + 1 \Rightarrow x < -4 or x > 6 \Rightarrow \Rightarrow $x \in (-\infty, -4) \cup (6, \infty)$ Hence, the correct option is (*c*). **Q25.** If $|x + 2| \le 9$, then (b) $x \in [-11, 7]$ (a) $x \in (-7, 11)$ (c) $x \in (-\infty, -7) \cup (11, \infty)(d)$ $x \in (-\infty, -7) \cup [11, \infty)$ **Sol.** Given that $|x + 2| \le 9$ $-9 \le x + 2 \le 9$ \Rightarrow $-9 - 2 \le x \le 9 - 2 \qquad [|x| \le a \implies -a \le x \le a]$ \Rightarrow $-11 \le x \le 7$ \Rightarrow



Solution of a linear inequality in variable *x* is represented on number line in exercises 27 to 30. Choose the correct answer from the given four options in each of the exercises (MCQ).

Q27. (a) $x \in (-\infty, 5)$ (b) $x \in (-\infty, 5]$ (c) $x \in [5, \infty)$ (d) $x \in (5, \infty)$ Sol. The gives graph represents all values of x greater than 5 including 5 on the real number line So, $x \in (5, \infty)$ Hence, the correct option is (d). Q28. (a) $x \in \left(\frac{9}{2}, \infty\right)$ (b) $x \in \left[\frac{9}{2}, \infty\right)$ (c) $x \in \left(-\infty, \frac{9}{2}\right)$ (d) $x \in \left(\infty, \frac{9}{2}\right)$ Sol. The given graph has all real values of x greater than and equal to $\frac{9}{2}$. So, $x \ge \frac{9}{2} \implies x \in \left[\frac{9}{2}, \infty\right)$ Hence, the correct option is (b).

Q29. (a)
$$x \in \left(-\infty, \frac{7}{2}\right)$$
 (b) $x \in \left(-\infty, \frac{7}{2}\right]$
(c) $x \in \left(\frac{7}{2}, -\infty\right)$ (d) $x \in \left(\frac{7}{2}, \infty\right)$

Sol. The given graph represents all the values of *x* less than $\frac{7}{2}$ on real number line.

So,
$$x \in \left(-\infty, \frac{7}{2}\right)$$

Hence, the correct option is (*a*).

- Q30. (a) $x \in (-\infty, -2)$ (b) $x \in (-\infty, -2]$ (c) $x \in (-2, -\infty)$ (d) $x \in (-2, -\infty]$ -2
- **Sol.** The given graph represents all real values of *x* less than and equal to -2So, $x \in (-\infty, -2]$ Hence, the correct option is (*b*).

True/False

Q31. State which of the following statements is True or False.

(*i*) If x < y and b < 0, then $\frac{x}{b} < \frac{y}{b}$ (*ii*) If xy > 0, then x > 0 and y < 0(*iii*) If xy > 0, then x < 0 and y < 0(*iv*) If xy < 0 then x < 0 and y < 0(*v*) If x < -5 and x < -2, then $x \in (-\infty, -5)$ (*vi*) If x < -5 and x > 2, then $x \in (-5, 2)$ (*vii*) If x > -2 and x < 9, then $x \in (-2, 9)$ (*viii*) If |x| > 5, then $x \in (-\infty, -5) \cup [5, \infty)$ (*ix*) If $|x| \le 4$, then $x \in [-4, 4]$ (*x*) Graph of x < 3 is



(*xi*) Graph of $x \ge 0$ is





Hence, statement (*i*) is False.

(*ii*) If *x*, *y* > 0 then *x* > 0, *y* > 0 or *x* < 0, *y* < 0
 Statement (*ii*) is False.



Hence, the statement (*xiii*) is False.

(*xiv*) Solution set of $x \ge 0$ and $y \le 1$ is



Hence, the statement (*xiv*) is False.

(*xv*) The given graph represents $x + y \ge 0$ Hence, the statement (*xv*) is True. **Q32.** Fill in the blanks of the following (*i*) If $-4x \ge 12$, then x = -3. (*ii*) If $-\frac{3}{4}x \le -3$, then x = 4. (*iii*) If $\frac{2}{x+2} > 0$, then x _____ - 2. (v) If x > y and z < 0 then -xz ______ -yz. (*vi*) If p > 0 and q < 0 then p - q _____ p. (*vii*) If |x+2| > 5, then $x \dots -7$ or $x \dots 3$. (*viii*) If $-2x + 1 \ge 9$ then x = -4. **Sol.** (i) $-4 \ge 12 \implies x \le -3$ Hence, the filler is (\leq) (ii) If $-\frac{3}{4}x \le -3 \implies x \ge 3x \times \frac{4}{3} \implies x \ge 4$ Hence, the filler is (>)(iii) If $\frac{2}{x+2} > 0 \implies x > -2$ Hence, the filler is (>) (iv) If $x > -5 \implies 4x > -20$ Hence, the filler is (>) (v) If x > y and z < 0, then $xz < yz \implies -xz > -yz$ Hence, the filler is (>) (*vi*) If p > 0 and q < 0then p - q > pHence, the filler is (>) (vii) If |x+2| > 5 then x + 2 < -5 or x + 2 > 5 $\Rightarrow x < -5 - 2$ or x > 5 - 2 \Rightarrow x < -7 or x > 3So $x \in (-\infty, -7) \cup (3, \infty)$ Hence, the filler is (<) or (>) (*viii*) If $-2x + 1 \ge 9$ then $-2x \ge 9 - 1 \implies -2x \ge 8 \implies 2x \le -8 \implies x \le -4$ Hence, the filler is (\leq) .