EXERCISE 4.1

Choose the correct answer from the given four options in the following questions:

Q1. Which of the following is a quadratic equation?

(a) $x^2 + 2x + 1 = (4 - x)^2 + 3$ (b) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$

(c) $(k+1)x^2 + \frac{3}{2}x = 7$ (where $k = -1$)

(d) $x^3 - x^2 = (x - 1)^3$

Sol. (d): **Main concept used:** An equation of the form $ax^2 + bx + c = 0$ where a, b, c are real numbers and $a \neq 0$, is called a quadratic equation.

(a) $x^2 + 2x + 1 = (4 - x)^2 + 3$

$\Rightarrow$ $x^2 + 2x + 1 = (4)^2 + (x)^2 - 2(4)(x) + 3$

$\Rightarrow$ $2x + 1 = 16 - 8x + 3$

$\therefore$ Coefficient of x^2 is zero or $a = 0$. So, it is not a quadratic equation.

(b) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$

$\Rightarrow$ $-2x^2 = 10x - 2 - 2x^2 + \frac{2}{5}x$

$\Rightarrow$ $-2x^2 + 2x^2 = 10x - 2 + \frac{2}{5}x$

$\Rightarrow$ $0 = 10x - 2 + \frac{2}{5}x$

As the coefficient of x^2 in the above equation is zero or $a = 0$.
So, it is not a quadratic equation.

(c) $(k+1)x^2 + \frac{3}{2}x = 7$, where $k = -1$

$\Rightarrow$ $(-1+1)x^2 + \frac{3}{2}x = 7$

So, the coefficient of x^2 is zero or $a = 0$. Hence, the equation is not quadratic.

(d) $x^3 - x^2 = (x - 1)^3$

$\Rightarrow$ $x^3 - x^2 = (x)^3 - (1)^3 - 3(x)^2(1) + 3(x)(1)^2$

$\Rightarrow$ $x^3 - x^2 = x^3 - 1 - 3x^2 + 3x$

$\Rightarrow \quad -x^2 = -1 - 3x^2 + 3x$

$\Rightarrow \quad 2x^2 - 3x + 1 = 0$

As the coefficient of x^2 in the above equation is 3 or $a = 3$, so it is a quadratic equation.

Q2. Which of the following is not a quadratic equation?

(a) $2(x-1)^2 = 4x^2 - 2x + 1$ (b) $2x - x^2 = x^2 + 5$

(c) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ (d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

Sol. (c): **Main concept used:** An equation will not be a quadratic in which $a = 0$ in equation of the form $ax^2 + bx + c = 0$

(a) Given equation is $2(x-1)^2 = 4x^2 - 2x + 1$

$\Rightarrow \quad 2[(x)^2 + (1)^2 - 2(x)(1)] - 4x^2 + 2x - 1 = 0$

$\Rightarrow \quad 2x^2 + 2 - 4x - 4x^2 + 2x - 1 = 0$

$\Rightarrow \quad -2x^2 - 2x + 1 = 0$

$\therefore$ $a = -2$ so given equation is quadratic as, it is of the form $ax^2 + bx + c = 0$ and $a \neq 0$

(b) The given equation is $2x - x^2 = x^2 + 5$

$\Rightarrow \quad 2x - x^2 - x^2 - 5 = 0$

$\Rightarrow \quad -2x^2 + 2x - 5 = 0$

$\Rightarrow \quad 2x^2 - 2x + 5 = 0$

$\therefore$ $a = 2$ so the given so equation is quadratic, as it is of the form $ax^2 + bx + c = 0$ and $a \neq 0$.

(c) The given equation is $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$

$\Rightarrow \quad (\sqrt{2}x)^2 + (\sqrt{3})^2 + 2(\sqrt{2}x)(\sqrt{3}) + x^2 - 3x^2 + 5x = 0$

$\Rightarrow \quad 2x^2 + 3 + 2\sqrt{6}x + x^2 - 3x^2 + 5x = 0$

$\Rightarrow \quad 0 + (2\sqrt{6} + 5)x + 3 = 0$

As $a = 0$ so, the given equation is not quadratic.

(d) Given equation is $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

$\Rightarrow \quad (x^2)^2 + (2x)^2 + 2(x^2)(2x) - x^4 - 3 - 4x^3 = 0$

$\Rightarrow \quad x^4 + 4x^2 + 4x^3 - x^4 - 3 - 4x^3 = 0$

$\Rightarrow \quad 4x^2 - 3 = 0$

As $a = 2$, so the given equation is quadratic.

Q3. Which of the following equations has 2 as a root?

(a) $x^2 - 4x + 5 = 0$ (b) $x^2 + 3x - 12 = 0$

(c) $2x^2 - 7x + 6 = 0$ (d) $3x^2 - 6x - 2 = 0$

Sol. (c): **Main concept used:** Roots of equation must satisfy the given equation.

(a) Substituting $x = 2$ in the equation $x^2 - 4x + 5 = 0$, we get

$$(2)^2 - 4(2) + 5 = 0$$

$$\Rightarrow \quad 4 - 8 + 5 = 0$$
$$\Rightarrow \quad 9 - 8 = 0$$
$$\Rightarrow \quad 1 = 0, \text{ which is false}$$

As $x = 2$ does not satisfy the given equation so 2 is not the root of the given equation.

(*b*) Substituting $x = 2$ in the equation $x^2 + 3x - 12 = 0$, we get

$$(2)^2 + 3(2) - 12 = 0$$
$$\Rightarrow \quad 4 + 6 - 12 = 0$$
$$\Rightarrow \quad 10 - 12 = 0$$
$$\Rightarrow \quad -2 = 0, \text{ which is false}$$

As $x = 2$ does not satisfy the given equation so 2 is not the root of the given equation.

(*c*) Substituting $x = 2$ in the equation $2x^2 - 7x + 6 = 0$, we get

$$2(2)^2 - 7(2) + 6 = 0$$
$$\Rightarrow \quad 8 - 14 + 6 = 0$$
$$\Rightarrow \quad 14 - 14 = 0$$
$$\Rightarrow \quad 0 = 0, \text{ which is true}$$

As $x = 2$ satisfies the given equation so 2 is the root of the given equation.

(*d*) Substituting $x = 2$ in the equation $3x^2 - 6x - 2 = 0$, we get

$$3(2)^2 - 6(2) - 2 = 0$$
$$\Rightarrow \quad 12 - 12 - 2 = 0$$
$$\Rightarrow \quad -2 = 0, \text{ which is false}$$

As $x = 2$ does not satisfy the given equation so 2 is not the root of the given equation.

Q4. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

(*a*) 2 (*b*) -2 (*c*) $\frac{1}{4}$ (*d*) $\frac{1}{2}$

Sol. (*a*): As $\frac{1}{2}$ is the root of the given equation so $x = \frac{1}{2}$ must satisfy the given equation.

On substituting $x = \frac{1}{2}$ in the equation $x^2 + kx - \frac{5}{4} = 0$, we get

$$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$
$$\Rightarrow \quad \frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$
$$\Rightarrow \quad 1 + 2k - 5 = 0$$

$\Rightarrow \quad +2k = +4$

$\Rightarrow \quad k = +2$

Q5. Which of the following equations has the sum of its roots as 3?

(*a*) $2x^2 - 3x + 6 = 0$ (*b*) $-x^2 + 3x - 3 = 0$

(*c*) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ (*d*) $3x^2 - 3x + 3 = 0$

Sol. (*b*): **Main concept used:** Sum of roots (α, β) of quadratic equation $ax^2 + bx + c = 0$ is $\alpha + \beta = \frac{-b}{a}$

(*a*) Given equation is $2x^2 - 3x + 6 = 0$

Here, $\alpha + \beta = \frac{3}{2} \neq 3$

So, the given equation has not the sum of roots as 3.

(*b*) Given equation is $-x^2 + 3x - 3 = 0$

Here, $\alpha + \beta = \frac{-3}{-1} = 3$

$\therefore$ The given equation has sum of its roots as 3.

(*c*) Given equation is

$$\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$$

Here, $\alpha + \beta = \frac{-\left(\frac{-3}{\sqrt{2}}\right)}{\sqrt{2}} = \frac{+3}{\sqrt{2}\sqrt{2}} = \frac{3}{2} \neq 3$

So, the given equation has not the sum of roots as 3.

(*d*) Given equation is $3x^2 - 3x + 3 = 0$

Here, $\alpha + \beta = \frac{-(-3)}{3} = 1 \neq 3$

So, the given equation has not the sum of roots as 3.

Q6. Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

(*a*) 0 only (*b*) 4 (*c*) 8 only (*d*) 0, 8

Sol.(*d*): **Main concept used:** The condition for equal roots of quadratic equation $ax^2 + bx + c = 0$ is $b^2 - 4ac = 0$.

Given equation is $2x^2 - kx + k = 0$

For equal roots, $b^2 - 4ac = 0$

$\Rightarrow \quad (-k)^2 - 4(2)(k) = 0 \qquad (a = 2, b = -k, c = +k)$

$\Rightarrow \quad k^2 - 8k = 0$

$\Rightarrow \quad k(k - 8) = 0$

$\Rightarrow \quad k = 0 \text{ or } k - 8 = 0$

$\Rightarrow \quad k = 0 \text{ or } k = 8$

So, the values of k are 0 and 8. Hence, the answer is (*d*).

Q7. Which constant must be added and subtracted to solve the quadratic equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square?

(*a*) $\frac{1}{8}$ (*b*) $\frac{1}{64}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{9}{64}$

Sol. (*b*): The given equation is $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$

So, to make the expression a complete square, we have to subtract $\left(\frac{1}{8}\right)^2$ or $\frac{1}{64}$.

$$\Rightarrow \quad (3x)^2 + \left(\frac{1}{8}\right)^2 + 2(3x)\left(\frac{1}{8}\right) - \sqrt{2} - \frac{1}{64} = 0$$

$$\Rightarrow \quad \left(3x + \frac{1}{8}\right)^2 = \sqrt{2} + \frac{1}{64}$$

Q8. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(*a*) two distinct real roots (*b*) two equal real roots

(*c*) no real roots (*d*) more than two real roots.

Sol. (*c*): **Main concept used:** After calculating $D = b^2 - 4ac$, check the following conditions:

(*i*) for no real roots $D < 0$ (*ii*) for two equal and real roots $D = 0$ (*iii*) for two distinct roots $D > 0$ and any quadratic equation must have only two roots.

Given equation is $2x^2 - \sqrt{5}x + 1 = 0$

$$D = b^2 - 4ac$$

$$= (-\sqrt{5})^2 - 4(2)(1) \qquad (a = 2, b = -\sqrt{5}, c = 1)$$

$$= 5 - 8$$

$$\Rightarrow \quad D = -3$$

As $D < 0$ so, the given equation has no real roots.

Q9. Which of the following equations has two distinct real roots?

(*a*) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ (*b*) $x^2 + x - 5 = 0$

(*c*) $x^2 + 3x + 2\sqrt{2} = 0$ (*d*) $5x^2 - 3x + 1 = 0$

Sol. (*b*): **Main concept used:** For real distinct roots $D > 0$

(*a*) Given equation is $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

$$D = b^2 - 4ac$$

$$= (-3\sqrt{2})^2 - 4(2)\left(\frac{9}{4}\right) \quad \left(a = 2, b = -3\sqrt{2}, c = \frac{9}{4}\right)$$

$$= 9 \times 2 - 18$$

$\Rightarrow \quad D = 0$

As D = 0, so the given equation has two real equal roots.

(*b*) $x^2 + x - 5 = 0$

$D = b^2 - 4ac$

$= (1)^2 - 4(1)(-5) \qquad (a = 1, b = 1, c = -5)$

$\Rightarrow \quad D = 1 + 20$

$\Rightarrow \quad D = 21$

As D > 0, so the given equation has two distinct real roots.

(*c*) $x^2 + 3x + 2\sqrt{2} = 0$

$D = b^2 - 4ac$

$= (3)^2 - 4(1)(2\sqrt{2}) \qquad (a = 1, b = 3, c = 2\sqrt{2})$

$\Rightarrow \quad D = 9 - 8\sqrt{2} = 9 - 8 \times 1.414 = 9 - 11.312$

$\Rightarrow \quad D = -2.312$

As D < 0, so the given equation has no real roots.

(*d*) $5x^2 - 3x + 1 = 0$

$D = b^2 - 4ac$

$= (-3)^2 - 4(5)(1) \qquad (a = 5, b = -3, c = 1)$

$= 9 - 20$

$\Rightarrow \quad D = -11$

As D < 0, so the given equation has no real roots.

Q10. Which of the following equations has no real roots?

(*a*) $x^2 - 4x + 3\sqrt{2} = 0$ (*b*) $x^2 + 4x - 3\sqrt{2} = 0$

(*c*) $x^2 - 4x - 3\sqrt{2} = 0$ (*d*) $3x^2 + 4\sqrt{3}x + 4 = 0$

Sol. (*a*):

(*a*) Given equation is $x^2 - 4x + 3\sqrt{2} = 0$

$D = b^2 - 4ac$

$= (-4)^2 - 4(1)(3\sqrt{2}) \qquad (a = 1, b = -4, c = 3\sqrt{2})$

$= 16 - 12\sqrt{2} = 16 - 12 \times 1.414$

$= 16 - 16.968$

$\Rightarrow \quad D = -0.968$

As D < 0, so the given equation has no real roots.

(*b*) $x^2 + 4x - 3\sqrt{2} = 0$

$D = b^2 - 4ac$

$= (4)^2 - 4(1)(-3\sqrt{2}) \qquad (a = 1, b = 4, c = -3\sqrt{2})$

$\Rightarrow \quad D = 16 + 12\sqrt{2}$

$\therefore \quad D > 0$

Hence, the given equation has two distinct real roots.

(*c*) $x^2 - 4x - 3\sqrt{2} = 0$

$D = b^2 - 4ac$

$\Rightarrow \quad D = (-4)^2 - 4(1)(-3\sqrt{2}) \qquad (a = 1, b = -4, c = -3\sqrt{2})$

$\Rightarrow \quad D = 16 + 12\sqrt{2}$

$\therefore \quad D > 0$

So, the given equation has two real distinct roots.

(*d*) $3x^2 + 4\sqrt{3}x + 4$

$D = b^2 - 4ac$

$= (4\sqrt{3})^2 - 4(3)(4) \qquad (a = 3, b = 4\sqrt{3}, c = 4)$

$= 16 \times 3 - 48 = 48 - 48$

$\Rightarrow \quad D = 0$

So, the given equation has two real and equal roots.

Q11. $(x^2 + 1)^2 - x^2 = 0$ has

(*a*) four real roots (*b*) two real roots

(*c*) no real roots (*d*) one real root

Sol. (*c*): Given equation is $(x^2 + 1)^2 - x^2 = 0$

$\Rightarrow \quad (x^2)^2 + (1)^2 + 2(x^2)(1) - x^2 = 0$

$\Rightarrow \quad (x^2)^2 + 1x^2 + 1 = 0$

Let $x^2 = y$

$\Rightarrow \quad y^2 + 1y + 1 = 0$

Now, $D = b^2 - 4ac$

$= (1)^2 - 4(1)(1) = 1 - 4 \qquad (a = 1, b = 1, c = 1)$

$\Rightarrow \quad D = -3$

$\Rightarrow \quad D < 0$

So, the given equation $y^2 + y + 1 = 0$ has no values of y in equation $y^2 + 1y + 1 = 0$ or if y is not real then x^2 will not be real so no values of x are real or the given equation has no real roots.

EXERCISE 4.2

Q1. State whether the following quadratic equations have two distinct real roots. Justify your answer.

(*i*) $x^2 - 3x + 4 = 0$ (*ii*) $2x^2 + x - 1 = 0$

(*iii*) $2x^2 - 6x + \frac{9}{2} = 0$ (*iv*) $3x^2 - 4x + 1 = 0$

(*v*) $(x + 4)^2 - 8x = 0$ (*vi*) $(x - \sqrt{2})^2 - 2(x + 1) = 0$

(*vii*) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$ (*viii*) $x(1 - x) - 2 = 0$

(*ix*) $(x - 1)(x + 2) + 2 = 0$ (*x*) $(x + 1)(x - 2) + x = 0$

Sol. Main concept used: Quadratic equation $ax^2 + bx + c = 0$ will have two distinct real roots if D > 0 or $b^2 - 4ac > 0$.

(*i*) Given quadratic equation is $x^2 - 3x + 4 = 0$

Now, $D = b^2 - 4ac$

$= (-3)^2 - 4(1)(4)$ $(a = 1, b = -3, c = 4)$

$\Rightarrow \quad D = 9 - 16$

$\Rightarrow \quad D = -7 < 0$

$\therefore \quad D < 0$

So, the given equation has no real roots.

(*ii*) $2x^2 + x - 1 = 0$

Now, $\quad D = b^2 - 4ac$

$= (1)^2 - 4(2)(-1) \qquad (a = 2, b = 1, c = -1)$

$= 1 + 8$

$\Rightarrow \quad D = 9 > 0$

$\therefore \quad D > 0$

So, the given equation has two distinct real roots.

(*iii*) $2x^2 - 6x + \dfrac{9}{2} = 0$

Now, $\quad D = b^2 - 4ac$

$\Rightarrow \quad D = (-6)^2 - 4(2)\left(\dfrac{9}{2}\right) \qquad \left(a = 2, b = -6, c = \dfrac{9}{2}\right)$

$\Rightarrow \quad D = 36 - 36$

$\Rightarrow \quad D = 0$

So, the given equation has two real and equal roots.

(*iv*) $3x^2 - 4x + 1 = 0$

Now, $\quad D = b^2 - 4ac \qquad (a = 3, b = -4, c = 1)$

$= (-4)^2 - 4(3)(1) = 16 - 12$

$\Rightarrow \quad D = 4 > 0$

$\therefore \quad D > 0$

So, the given equation has two distinct real roots.

(*v*) $(x + 4)^2 - 8x = 0$

$\Rightarrow (x)^2 + (4)^2 + 2(x)(4) - 8x = 0$

$\Rightarrow \quad x^2 + 16 + 8x - 8x = 0$

$\Rightarrow \quad x^2 + 16 = 0$

$\Rightarrow \quad x^2 + 0x + 16 = 0$

Now, $\quad D = b^2 - 4ac$

$= (0)^2 - 4(1)(16) \qquad (a = 1, b = 0, c = 16)$

$\Rightarrow \quad D = -64 < 0$

As D < 0, so the given equation has no real roots.

(*vi*) $(x - \sqrt{2})^2 - 2(x + 1) = 0$

$\Rightarrow \quad x^2 - 2\sqrt{2}x + 2 - 2x - 2 = 0$

$\Rightarrow \quad x^2 - (2\sqrt{2} + 2)x = 0$

Now, $\quad D = b^2 - 4ac$

$= [-(2\sqrt{2} + 2)]^2 - 4 \times 1 \times 0$

$[\because a = 1, b = -(2\sqrt{2} + 2), c = 0]$

$= (2\sqrt{2}+2)^2 > 0$

As D > 0, so the given equation has real and unequal roots.

(*vii*) $\sqrt{2}x^2 - \dfrac{3}{\sqrt{2}}x + \dfrac{1}{\sqrt{2}} = 0$

$D = b^2 - 4ac$

$\Rightarrow \quad D = \left(\dfrac{-3}{\sqrt{2}}\right)^2 - 4(\sqrt{2})\left(\dfrac{1}{\sqrt{2}}\right) \quad \left(a = \sqrt{2}, b = \dfrac{-3}{\sqrt{2}}, c = +\dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{9}{2} - \dfrac{4}{1} = \dfrac{9-8}{2}$

$\Rightarrow \quad D = \dfrac{1}{2} > 0$

$\therefore \quad D > 0$

Hence, the given quadratic equation has two real and distinct roots.

(*viii*) $x(1-x) - 2 = 0$

$\Rightarrow \quad x - x^2 - 2 = 0$

$\Rightarrow \quad -x^2 + x - 2 = 0$

Now, $D = b^2 - 4ac$

$= (-1^2) - 4(-1)(-2)(a = -1, b = 1, c = -2)$

$= 1 - 8$

$\Rightarrow \quad D = -7 < 0$

So, the given equation has no real roots.

(*ix*) $(x-1)(x+2) + 2 = 0$

$\Rightarrow \quad x^2 + 2x - x - 2 + 2 = 0$

$\Rightarrow \quad x^2 + x = 0$

$\Rightarrow \quad x^2 + x + 0 = 0$

Now, $D = b^2 - 4ac \quad (a = 1, b = 1, c = 0)$

$= (1)^2 - 4(1)(0) = 1$

$\Rightarrow \quad D = 1 > 0$

So, the given equation has two distinct real roots.

(*x*) $(x+1)(x-2) + x = 0$

$\Rightarrow \quad x^2 - 2x + x - 2 + x = 0$

$\Rightarrow \quad x^2 - 2 = 0$

$\Rightarrow \quad x^2 + 0x - 2 = 0$

Now, $D = b^2 - 4ac$

$= (0)^2 - 4(1)(-2) \quad (a = 1, b = 0, c = -2)$

$\Rightarrow \quad D = 8 > 0$

So, the given equation has two distinct real roots.

Q2. Write whether the following statements are true or false. Justify your answer.

(*i*) Every quadratic equation has exactly one root.

(*ii*) Every quadratic equation has atleast one real root.
(*iii*) Every quadratic equation has at least two roots.
(*iv*) Every quadratic equation has atmost two roots.
(*v*) If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.
(*vi*) If the coefficient of x^2 and the constant term have the same sign and if the coefficient of x term is zero, then the quadratic equation has no real roots.

Sol. (*i*) **False:** Consider a quadratic equation $x^2 - 4 = 0$ which has two distinct roots – 2 and 2. So, the given statement is false.

(*ii*) **False:** Consider a quadratic equation $x^2 + 1 = 0$ which has no real root. So, the given statement is false.

(*iii*) **False:** Consider the quadratic equation $x^2 - 4x + 4 = 0$ which has only 2 as root. So, the given statement is false.

(*iv*) **True:** Consider the quadratic equation $x^2 - 5x + 6 = 0$. Put 2 and 3 in x and the quadratic expression $x^2 - 5x + 6$ becomes equal to 0. So, 2 and 3 are the roots of the quadratic equation $x^2 - 5x + 6 = 0$. So, any quadratic equation can have atmost two roots i.e., one or two roots, but not more than two.

(*v*) **True:** In quadratic equation $ax^2 + bx + c = 0$, if a and c have opposite signs, then $ac < 0$.
Therefore, $b^2 - 4ac > 0$. So, the quadratic equation has real roots.
Hence, the given statement is true.

(*vi*) **True:** In quadratic equation $ax^2 + bx + c = 0$, if a and c have same sign and $b = 0$, then $b^2 - 4ac = (0)^2 - 4ac = -4ac < 0$.
So, the quadratic equation has no real roots.
Hence, the given statement is true.

Q3. A quadratic equation with integral coefficient has integral roots. Justify your answer.

Sol. No, a quadratic equation with integral coefficients (0, ± 1, ± 2, ± 3…) can have its roots in fraction, i.e., non integral.
For example, $5x^2 + 3x - 8 = 0$ has integral coefficients (coefficients 5, 3, – 8 are integers).

$$
\begin{aligned}
\text{Now,} \quad 5x^2 + 3x - 8 &= 0 \\
\Rightarrow \quad 5x^2 + 8x - 5x - 8 &= 0 \\
\Rightarrow \quad x(5x + 8) - 1(5x + 8) &= 0 \\
\Rightarrow \quad (5x + 8)(x - 1) &= 0 \\
\Rightarrow \quad 5x + 8 &= 0 \quad \text{or} \quad (x - 1) = 0
\end{aligned}
$$

Therefore, the roots are given by $x = \frac{-8}{5}$ and $x = 1$

So, the given statement is false.

Q4. Does there exist a quadratic equation, whose coefficients are rational but both of its roots are irrational? Justify your answer.

Sol. Yes, a quadratic equation having coefficients as rational number, has irrational roots.

For example, $2x^2 - 3x - 15 = 0$ has rational coefficients.

$$D = b^2 - 4ac \qquad (a = 2, b = -3, c = -15)$$

$$= (-3)^2 - 4(2)(-15) = 9 + 120$$

$$\Rightarrow \qquad D = 129$$

$\therefore$ Roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \qquad x = \frac{-(-3) \pm \sqrt{129}}{2 \times 2}$$

$$\Rightarrow \qquad x = \frac{3 \pm \sqrt{129}}{4}$$

The roots are irrational as $\sqrt{129}$ is irrational.

Q5. Does there exist a quadratic equation whose coefficients are all distinct irrationals but both the roots are rationals? Why?

Sol. Yes, there may be a quadratic equation whose coefficients are all distinct irrationals, but both the roots are rational.

For example, consider a quadratic equation having distinct irrational coefficients

$$3\sqrt{\frac{3}{2}}x^2 + \frac{5}{\sqrt{6}}x - 2\sqrt{\frac{2}{3}} = 0$$

Now,

$$D = b^2 - 4ac$$

$$= \left[\frac{5}{\sqrt{6}}\right]^2 - 4\left[\frac{3\sqrt{3}}{\sqrt{2}}\right]\left[\frac{-2\sqrt{2}}{\sqrt{3}}\right]$$

$$\left(a = \frac{3\sqrt{3}}{\sqrt{2}}, b = \frac{5}{\sqrt{6}}, c = \frac{-2\sqrt{2}}{\sqrt{3}}\right)$$

$$= \frac{25}{6} + \frac{24}{1} = \frac{25 + 144}{6}$$

$$\Rightarrow \qquad D = \frac{169}{6} \quad \Rightarrow \quad \sqrt{D} = \frac{13}{\sqrt{6}}$$

Roots are given by

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-\frac{5}{\sqrt{6}} \pm \frac{13}{\sqrt{6}}}{2 \cdot \frac{3\sqrt{3}}{\sqrt{2}}}$$

$$= \frac{\frac{1}{\sqrt{6}}[-5 \pm 13]\sqrt{2}}{6\sqrt{3}} = \frac{(-5 \pm 13)\sqrt{2}}{\sqrt{2}\,\sqrt{3} \times 6\sqrt{3}}$$

$$\Rightarrow \qquad x = \frac{(-5 \pm 13)}{18}$$

$$\Rightarrow \qquad x = \frac{8}{18} \text{ or } \frac{-18}{18}$$

$$\Rightarrow \qquad x = \frac{4}{9} \text{ or } -1$$

Hence, the roots are rational while coefficients a, b, c were irrational.

Q6. Is 0.2 a root of equation $x^2 - 0.4 = 0$? Justify your answer.

Sol. If 0.2 is a root of equation $x^2 - 0.4 = 0$, then 0.2 must satisfy the given equation.

$$x^2 - 0.4 = 0 \qquad \text{[Given]}$$

$$\Rightarrow \qquad (0.2)^2 - 0.4 = 0$$

$$\Rightarrow \qquad 0.04 - 0.4 = 0$$

$$\Rightarrow \qquad -0.36 \neq 0$$

So, 0.2 is not a root of the given equation.

Q7. If $b = 0$, $c < 0$, is it true that the roots of $x^2 + bx + c = 0$ are numerically equal and opposite in sign? Justify your answer.

Sol. Given equation is $x^2 + bx + c = 0$

$$b = 0 \qquad \text{[Given]}$$

$$\therefore \qquad x^2 + c = 0$$

$$\Rightarrow \qquad x^2 = -c$$

$$\Rightarrow \qquad x = \sqrt{-c}$$

As c is negative so $-c$ becomes positive or $\sqrt{-c}$ is real.

So, the roots of the given equation are

$$x = \pm\sqrt{-c}$$

$$\text{or} \qquad x = +\sqrt{-c} \text{ and } -\sqrt{-c} \qquad [\because (-c) \text{ is positive}]$$

Hence, the roots of the given equation are real, equal and opposite in sign.

EXERCISE 4.3

Q1. Find the roots of the quadratic equations by using the quadratic formula in each of the following:

(*i*) $2x^2 - 3x - 5 = 0$ (*ii*) $5x^2 + 13x + 8 = 0$

(*iii*) $-3x^2 + 5x + 12 = 0$ (*iv*) $-x^2 + 7x - 10 = 0$

(*v*) $x^2 + 2\sqrt{2}x - 6 = 0$ (*vi*) $x^2 - 3\sqrt{5x} + 10 = 0$

(*vii*) $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

Sol. Main concept used: Roots of quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b \pm \sqrt{D}}{2a}$$

(*i*) Given equation is $2x^2 - 3x - 5 = 0$

$$D = b^2 - 4ac$$

$$\Rightarrow \quad D = (-3)^2 - 4(2)(-5) \qquad (a = 2, b = -3, c = -5)$$

$$= 9 + 40$$

$$\Rightarrow \quad \sqrt{D} = \sqrt{49}$$

$$\Rightarrow \quad \sqrt{D} = 7$$

Now, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-3) \pm 7}{2 \times 2} \quad \Rightarrow \quad x = \frac{3 \pm 7}{4}$$

$$\Rightarrow \quad x_1 = \frac{3+7}{4} \quad \text{and} \quad x_2 = \frac{3-7}{4}$$

$$\Rightarrow \quad x_1 = \frac{10}{4} = \frac{5}{2} \quad \text{and} \quad x_2 = \frac{-4}{4} = -1$$

$\therefore$ Roots of the given equation are $\frac{5}{2}$ and -1.

(*ii*) $5x^2 + 13x + 8 = 0$

$$D = b^2 - 4ac$$

$$= (13)^2 - 4(5)(8) \qquad (a = 5, b = 13, c = 8)$$

$$= 169 - 160$$

$$\Rightarrow \quad \sqrt{D} = \sqrt{9} = 3$$

Now, $$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(13) \pm 3}{2 \times 5}$$

$$\Rightarrow \quad x_1 = \frac{-13+3}{10} \quad \text{and} \quad x_2 = \frac{-13-3}{10}$$

$$\Rightarrow \quad x_1 = \frac{-10}{10} \quad \text{and} \quad x_2 = \frac{-16}{10}$$

$$\Rightarrow \quad x_1 = -1 \quad \text{and} \quad x_2 = \frac{-8}{5}$$

So, the roots of the given equation are -1 and $\frac{-8}{5}$.

(*iii*) $-3x^2 + 5x + 12 = 0$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (5)^2 - 4(-3)(12) \qquad (a = -3,\ b = 5,\ c = 12) \\ &= 25 + 144 \end{aligned}$$

$\Rightarrow \quad D = 169$

$\Rightarrow \quad \sqrt{D} = 13$

Now, $\quad x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-5 \pm 13}{2 \times (-3)} = \dfrac{-5 \pm 13}{-6}$

$\Rightarrow \quad x_1 = \dfrac{-5 + 13}{-6} \quad$ and $\quad x_2 = \dfrac{-5 - 13}{-6}$

$\Rightarrow \quad x_1 = \dfrac{8}{-6} \quad$ and $\quad x_2 = \dfrac{-18}{-6}$

$\Rightarrow \quad x_1 = -\dfrac{4}{3} \quad$ and $\quad x_2 = +3$

Hence, the roots of the given equation are $-\dfrac{4}{3}$ and 3.

(*iv*) $-x^2 + 7x - 10 = 0$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (7)^2 - 4(-1)(-10) \quad (a = -1,\ b = 7,\ c = -10) \\ &= 49 - 40 = 9 \end{aligned}$$

$\Rightarrow \quad \sqrt{D} = 3$

Now, $\quad x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-7 \pm 3}{2 \times (-1)}$

$\Rightarrow \quad x_1 = \dfrac{-7 + 3}{-2} \quad$ and $\quad x_2 = \dfrac{-7 - 3}{-2}$

$\Rightarrow \quad x_1 = \dfrac{-4}{-2} \quad$ and $\quad x_2 = \dfrac{-10}{-2}$

$\Rightarrow \quad x_1 = 2 \quad$ and $\quad x_2 = 5$

Hence, the roots of the given quadratic equation are 2 and 5.

(*v*) $x^2 + 2\sqrt{2}x - 6 = 0$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (2\sqrt{2})^2 - 4(1)(-6) \qquad (a = 1,\ b = 2\sqrt{2},\ c = -6) \\ &= 4 \times 2 + 24 = 8 + 24 = 32 \end{aligned}$$

$\Rightarrow \quad \sqrt{D} = \sqrt{32} \quad \Rightarrow \quad \sqrt{D} = 4\sqrt{2}$

Now, $\quad x = \dfrac{- \ \pm \sqrt{D}}{2} \quad \dfrac{-2\sqrt{2} \pm 4\sqrt{2}}{2 \times 1}$

$\Rightarrow \quad x_1 = \dfrac{-2\sqrt{2}+4\sqrt{2}}{2}$ and $x_2 = \dfrac{-2\sqrt{2}-4\sqrt{2}}{2\times 1}$

$\Rightarrow \quad x_1 = \dfrac{2\sqrt{2}}{2}$ and $x_2 = \dfrac{-6\sqrt{2}}{2}$

$\Rightarrow \quad x_1 = \sqrt{2}$ and $x_2 = -3\sqrt{2}$

Hence, the roots of the given equation are $\sqrt{2}$ and $-3\sqrt{2}$.

(*vi*) $x^2 - 3\sqrt{5}x + 10 = 0$

$D = b^2 - 4ac$

$\Rightarrow \quad D = (-3\sqrt{5})^2 - 4(1)(10) \qquad (a = 1, b = -3\sqrt{5}, c = 10)$

$= 9 \times 5 - 40$

$\Rightarrow \quad D = 45 - 40 = 5$

$\Rightarrow \quad \sqrt{D} = \sqrt{5}$

Now, $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-(-3\sqrt{5}) \pm \sqrt{5}}{2\times 1} = \dfrac{3\sqrt{5} \pm \sqrt{5}}{2}$

$\Rightarrow \quad x_1 = \dfrac{3\sqrt{5}+\sqrt{5}}{2}$ and $x_2 = \dfrac{3\sqrt{5}-\sqrt{5}}{2}$

$\Rightarrow \quad x_1 = \dfrac{4\sqrt{5}}{2}$ and $x_2 = \dfrac{2\sqrt{5}}{2}$

$\Rightarrow \quad x_1 = 2\sqrt{5}$ and $x_2 = \sqrt{5}$

Hence, the roots of the given equation are $2\sqrt{5}$ and $\sqrt{5}$.

(*vii*) $\dfrac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

$D = b^2 - 4ac$

$= (-\sqrt{11})^2 - 4\left(\dfrac{1}{2}\right)(1) \qquad \left(a = \dfrac{1}{2}, b = -\sqrt{11}, c = 1\right)$

$= +11 - 2 = 9$

$\Rightarrow \quad \sqrt{D} = \sqrt{9} \Rightarrow \sqrt{D} = 3$

Now, $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{+\sqrt{11} \pm 3}{2 \times \frac{1}{2}} = \sqrt{11} \pm 3$

$\Rightarrow \quad x_1 = \sqrt{11} + 3$ and $x_2 = \sqrt{11} - 3$

Hence, the roots of the given equation are $\sqrt{11}+3$ and $\sqrt{11}-3$.

Q2. Find the roots of the following quadratic equations by the factorization method.

(*i*) $2x^2 + \dfrac{5}{3}x - 2 = 0$ (*ii*) $\dfrac{2}{5}x^2 - x - \dfrac{3}{5} = 0$

(*iii*) $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$ (*iv*) $3x^2 + 5\sqrt{5}x - 10 = 0$

(*v*) $21x^2 - 2x + \frac{1}{21} = 0$

Sol. (*i*) $2x^2 + \frac{5}{3}x - 2 = 0$

$\Rightarrow \quad 6x^2 + 5x - 6 = 0$

$\Rightarrow \quad 6x^2 + 9x - 4x - 6 = 0$

$\Rightarrow \quad 3x(2x + 3) - 2(2x + 3) = 0$

$\Rightarrow \quad (2x + 3)(3x - 2) = 0$

$\Rightarrow \quad 2x + 3 = 0 \quad \text{or} \quad 3x - 2 = 0$

$\Rightarrow \quad 2x = -3 \quad \text{or} \quad 3x = 2$

$\Rightarrow \quad x = \frac{-3}{2} \quad \text{or} \quad x = \frac{2}{3}$

So, the roots of the given quadratic equation are $\frac{-3}{2}$ and $\frac{2}{3}$.

(*ii*) $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

$\Rightarrow \quad 2x^2 - 5x - 3 = 0$

$\Rightarrow \quad 2x^2 - 6x + 1x - 3 = 0$

$\Rightarrow \quad 2x(x - 3) + 1(x - 3) = 0$

$\Rightarrow \quad (x - 3)(2x + 1) = 0$

$\Rightarrow \quad x - 3 = 0 \quad \text{or} \quad 2x + 1 = 0$

$\Rightarrow \quad x = 3 \quad \text{or} \quad 2x = -1 \quad \text{or} \quad x = \frac{-1}{2}$

So, the roots of the quadratic equation are 3 and $\frac{-1}{2}$.

(*iii*) $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$

$\Rightarrow \quad 3\sqrt{2}x^2 - 6x + 1x - \sqrt{2} = 0$

$\Rightarrow 3\sqrt{2}x(x - \sqrt{2}) + 1(x - \sqrt{2}) = 0$

$\Rightarrow \quad (x - \sqrt{2})(3\sqrt{2}x + 1) = 0$

$\Rightarrow \quad x - \sqrt{2} = 0 \quad \text{or} \quad 3\sqrt{2}x + 1 = 0$

$\Rightarrow \quad x = \sqrt{2} \quad \text{or} \quad 3\sqrt{2}x = -1$

$\Rightarrow \quad x = \sqrt{2} \text{ or } x = \frac{-1}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\Rightarrow \quad x = \sqrt{2} \text{ or } x = \frac{-\sqrt{2}}{6}$

Hence, the roots of the given equation are $\sqrt{2}$ and $\frac{-\sqrt{2}}{6}$.

(*iv*) $3x^2 + 5\sqrt{5}x - 10 = 0$

$$\Rightarrow \quad 3x^2 + 6\sqrt{5}x - \sqrt{5}x - 10 = 0$$
$$\Rightarrow 3x(x + 2\sqrt{5}) - \sqrt{5}(x + 2\sqrt{5}) = 0$$
$$\Rightarrow \quad (3x - \sqrt{5})(x + 2\sqrt{5}) = 0$$
$$\Rightarrow \quad 3x - \sqrt{5} = 0 \quad \text{or} \quad x + 2\sqrt{5} = 0$$
$$\Rightarrow \quad 3x = \sqrt{5} \quad \text{or} \quad x = -2\sqrt{5}$$
$$\Rightarrow \quad x = \frac{\sqrt{5}}{3} \quad \text{or} \quad x = -2\sqrt{5}$$

Hence, the roots of the given quadratic equation are $\frac{\sqrt{5}}{3}$ and $-2\sqrt{5}$.

(*v*) $21x^2 - 2x + \frac{1}{21} = 0$

$$\Rightarrow \quad 441x^2 - 42x + 1 = 0$$
$$\Rightarrow \quad 441x^2 - 21x - 21x + 1 = 0$$
$$\Rightarrow 21x(21x - 1) - 1(21x - 1) = 0$$
$$\Rightarrow \quad (21x - 1)(21x - 1) = 0$$
$$\Rightarrow \quad (21x - 1) = 0 \quad \text{or} \quad (21x - 1) = 0$$
$$\Rightarrow \quad 21x = 1 \quad \text{or} \quad 21x = 1$$
$$\Rightarrow \quad x = \frac{1}{21} \quad \text{or} \quad x = \frac{1}{21}$$

So, the roots of the given equation are $\frac{1}{21}$ and $\frac{1}{21}$.

OR

The given equation is $21x^2 - 2x + \frac{1}{21} = 0$

$$\Rightarrow 441x^2 - 42x + 1 = 0$$

[Multiplying by 21 (LCM of equation) to both sides]

As 441 and 1 are perfect squares so

$$(21x)^2 + (1)^2 - 2(21x)(1) = 0$$
$$\Rightarrow \quad (21x - 1)^2 = 0$$
$$\Rightarrow \quad (21x - 1)(21x - 1) = 0$$
$$\Rightarrow \quad 21x - 1 = 0 \quad \text{or} \quad 21x - 1 = 0$$
$$\Rightarrow \quad 21x = 1 \quad \text{or} \quad 21x = 1$$
$$\Rightarrow \quad x = \frac{1}{21} \quad \text{or} \quad x = \frac{1}{21}$$

Hence, the roots of the given equation are $\frac{1}{21}$ and $\frac{1}{21}$.

EXERCISE 4.4

Q1. Find whether the following equations have real roots. If real roots exist, find them.

(*i*) $8x^2 + 2x - 3 = 0$ (*ii*) $-2x^2 + 3x + 2 = 0$

(*iii*) $5x^2 - 2x - 10 = 0$ (*iv*) $\dfrac{1}{(2x-3)} + \dfrac{1}{(x-5)} = 1,\ x \neq \dfrac{3}{2}, 5$

(*v*) $x^2 + 5\sqrt{5}x - 70 = 0$

Sol. Main concept used: For real roots of quadratic equation $ax^2 + bx + c = 0,\ b^2 - 4ac > 0$

(*i*) The given equation is $8x^2 + 2x - 3 = 0$

Discriminant (D) $= b^2 - 4ac$

$\Rightarrow \quad D = (2)^2 - 4(8)(-3) \quad (a = 8, b = 2, c = -3)$

$\Rightarrow \quad D = 4 + 96 \quad \Rightarrow \quad D = 100$

As $D > 0$, so, roots are real.

Now, Discriminant $\sqrt{D} = 10$

So, roots are $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-2 \pm 10}{2 \times 8} = \dfrac{-2 \pm 10}{16}$

$\Rightarrow \quad x_1 = \dfrac{-2+10}{16}$ and $x_2 = \dfrac{-2-10}{16}$

$\Rightarrow \quad x_1 = \dfrac{8}{16}$ and $x_2 = \dfrac{-12}{16}$

$\Rightarrow \quad x_1 = \dfrac{1}{2}$ and $x_2 = \dfrac{-3}{4}$

So, the roots of the given equation are $\dfrac{1}{2}$ and $\dfrac{-3}{4}$.

(*ii*) $-2x^2 + 3x + 2 = 0$

Discriminant $D = b^2 - 4ac$

$\Rightarrow \quad D = (3)^2 - 4(-2)(2) \quad (a = -2, b = 3, c = 2)$

$\Rightarrow \quad D = 9 + 16$

$\Rightarrow \quad D = 25 > 0$

So, the given equation has real and distinct roots.

Now, $\sqrt{D} = 5$

And, $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-3 \pm 5}{2(-2)} = \dfrac{-3 \pm 5}{-4}$

$\Rightarrow \quad x_1 = \dfrac{-3+5}{-4}$ and $x_2 = \dfrac{-3-5}{-4}$

$\Rightarrow \quad x_1 = \dfrac{2}{-4}$ and $x_2 = \dfrac{-8}{-4}$

$\Rightarrow \quad x_1 = \frac{-1}{2} \quad$ and $\quad x_2 = 2$

Hence, the roots of the given equation are 2 and $\frac{-1}{2}$.

(*iii*) $5x^2 - 2x - 10 = 0$

Discriminant $D = b^2 - 4ac$

$\Rightarrow \quad D = (-2)^2 - 4(5)(-10) \quad (a = 5, b = -2, c = -10)$

$= 4 + 200$

$\Rightarrow \quad D = 204 > 0$

So, the roots of the given equation are real and distinct.

Now, $\quad \sqrt{D} = \sqrt{204} \quad \Rightarrow \quad \sqrt{D} = 2\sqrt{51}$

And, $\quad x = \frac{-b \pm \sqrt{D}}{2a} = \frac{+2 \pm 2\sqrt{51}}{2 \times 5}$

$= \frac{2[1 \pm \sqrt{51}]}{10} = \frac{1 \pm \sqrt{51}}{5}$

$\Rightarrow \quad x_1 = \frac{1 + \sqrt{51}}{5} \quad$ and $\quad x_2 = \frac{1 - \sqrt{51}}{5}$

Hence, the roots of the given equation are $\frac{1+\sqrt{51}}{5}, \frac{1-\sqrt{51}}{5}$.

(*iv*) $\frac{1}{2x-3} + \frac{1}{x-5} = 1, \quad x \neq \frac{3}{2}, 5$

$\Rightarrow \quad \frac{(x-5)+(2x-3)}{(2x-3)(x-5)} = 1$

$\Rightarrow \quad 2x^2 - 10x - 3x + 15 = x - 5 + 2x - 3$

$\Rightarrow \quad 2x^2 - 13x + 15 = 3x - 8$

$\Rightarrow \quad 2x^2 - 13x + 15 - 3x + 8 = 0$

$\Rightarrow \quad 2x^2 - 16x + 23 = 0$

Now, $\quad D = b^2 - 4ac$

$= (-16)^2 - 4(2)(23) \, (a = 2, b = -16, c = 23)$

$\Rightarrow \quad D = 256 - 184 = 72 > 0$

$\Rightarrow \quad \sqrt{D} = \sqrt{72}$

$\Rightarrow \quad \sqrt{D} = 6\sqrt{2}$

Now, $\quad x = \frac{-b \pm \sqrt{D}}{2a} = \frac{+16 \pm 6\sqrt{2}}{2 \times 2} = \frac{16}{4} \pm \frac{6\sqrt{2}}{4}$

$\Rightarrow \quad x_1 = 4 + \frac{3}{2}\sqrt{2} \quad$ and $\quad x_2 = 4 - \frac{3}{2}\sqrt{2}$

Hence, the roots of the given quadratic equation are

$\left(4 + \frac{3}{2}\sqrt{2}\right)$ and $\left(4 - \frac{3}{2}\sqrt{2}\right)$

(v) $x^2 + 5\sqrt{5}x - 70 = 0$

$$D = b^2 - 4ac$$
$$= (5\sqrt{5})^2 - 4(1)(-70) \quad (a = 1, b = 5\sqrt{5}, c = -70)$$
$$= 25 \times 5 + 280 = 125 + 280$$
$$\Rightarrow \quad D = 405 > 0$$

So, the roots of the given equation are real and distinct.

For roots $\sqrt{D} = \sqrt{405} \Rightarrow \sqrt{D} = \sqrt{9 \times 9 \times 5}$

$$\Rightarrow \quad \sqrt{D} = 9\sqrt{5}$$

Now, $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-5\sqrt{5} \pm 9\sqrt{5}}{2 \times 1} = \dfrac{(-5 \pm 9)\sqrt{5}}{2}$

$$\Rightarrow \quad x_1 = \frac{(-5+9)\sqrt{5}}{2} \quad \text{and} \quad x_2 = \frac{(-5-9)\sqrt{5}}{2}$$
$$= \frac{4\sqrt{5}}{2} \qquad\qquad = \frac{-14\sqrt{5}}{2}$$
$$= 2\sqrt{5} \qquad\qquad = -7\sqrt{5}$$

Hence, the roots of the given quadratic equation are $2\sqrt{5}$ and $-7\sqrt{5}$.

Q2. Find a natural number whose square diminished by 84, is equal to thrice of 8 more than the given number.

Sol. Let the required number be x.

According to the question,

$$x^2 - 84 = 3 \times (x + 8)$$
$$\Rightarrow \quad x^2 - 84 = 3x + 24$$
$$\Rightarrow \quad x^2 - 3x - 84 - 24 = 0$$
$$\Rightarrow \quad x^2 - 3x - 108 = 0$$
$$\Rightarrow \quad x^2 - 12x + 9x - 108 = 0$$
$$\Rightarrow \quad x(x - 12) + 9(x - 12) = 0$$
$$\Rightarrow \quad (x - 12)(x + 9) = 0$$
$$\Rightarrow \quad x - 12 = 0 \quad \text{or} \quad x + 9 = 0$$
$$\Rightarrow \quad x = 12 \quad \text{or} \quad x = -9$$

($x = -9$ is not a natural number so it is rejected.)

Hence, the required number is 12.

Q3. A natural number, when increased by 12 equals 160 times its reciprocal. Find the number.

Sol. Let the required number be x. (where $x \neq 0$)

According to the question,

$$x + 12 = \frac{1}{x} \times 160$$
$$\Rightarrow \quad x^2 + 12x = 160$$

$$\Rightarrow \quad x^2 + 12x - 160 = 0$$
$$\Rightarrow \quad x^2 + 20x - 8x - 160 = 0$$
$$\Rightarrow \quad x(x + 20) - 8(x + 20) = 0$$
$$\Rightarrow \quad (x + 20)\,(x - 8) = 0$$
$$\Rightarrow \quad x + 20 = 0 \quad \text{or} \quad x - 8 = 0$$
$$\Rightarrow \quad x = -20 \quad \text{or} \quad x = 8$$

But, $x = -20$ is not a natural number.
Hence, the required number is 8.

Q4. A train travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

Sol. Let the original speed of train = x km/hr
So, the new increased speed of train = $(x + 5)$ km/hr
Time taken by train in covering 360 km with original speed

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{360}{x}\text{ hr}$$

Time taken by train in covering 360 km with new speed = $\frac{360}{x+5}$ hr
According to the question,

$$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}\text{ hr}$$
$$\Rightarrow \quad 360\left[\frac{1}{x} - \frac{1}{(x+5)}\right] = \frac{4}{5}$$
$$\Rightarrow \quad 360\left[\frac{x+5-x}{x(x+5)}\right] = \frac{4}{5}$$
$$\Rightarrow \quad \frac{90[5]}{x^2 + 5x} = \frac{1}{5}$$
$$\Rightarrow \quad x^2 + 5x = 90 \times 25$$
$$\Rightarrow \quad x^2 + 5x - 90 \times 25 = 0$$
$$\Rightarrow \quad x^2 + 50x - 45x - 90 \times 25 = 0$$
$$\Rightarrow \quad x(x + 50) - 45[x + 10 \times 5] = 0$$
$$\Rightarrow \quad (x + 50)\,(x - 45) = 0$$
$$\Rightarrow \quad x + 50 = 0 \quad \text{or} \quad x - 45 = 0$$
$$\Rightarrow \quad x = -50 \quad \text{or} \quad x = 45$$

$x = -50$ is rejected as it is negative.
Hence, the original speed of train is 45 km/hr.

Q5. If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?

Sol. Let Zeba's actual (real) age now = x years

$\therefore$ Zeba's age when she was 5 years younger than now = $(x - 5)$ years

According to the question,

$$(x-5)^2 = 5x + 11$$

$\Rightarrow \quad (x)^2 + (5)^2 - 2(x)(5) - 5x - 11 = 0$

$\Rightarrow \quad x^2 + 25 - 10x - 5x - 11 = 0$

$\Rightarrow \quad x^2 - 15x + 14 = 0$

$\Rightarrow \quad x^2 - 14x - 1x + 14 = 0$

$\Rightarrow \quad x(x-14) - 1(x-14) = 0$

$\Rightarrow \quad (x-14)(x-1) = 0$

$\Rightarrow \quad x - 14 = 0 \quad$ or $\quad x - 1 = 0$

$\Rightarrow \quad x = 14 \quad$ or $\quad x = 1$ year

When 5 is subtracted from 1, we get negative age so $x = 1$ is rejected.

Hence, the age of Zeba is 14 years.

Q6. At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Nisha and Asha.

Sol. Let the present age of Asha = x years

and the present age of her daughter Nisha = y years

At present, Asha's age, $x = (y^2) + 2$ (I)

Age of Nisha will be equal to age of her mother (x) after

= Age of Mother – Age of Daughter

$= x - y$

$= y^2 + 2 - y = y^2 - y + 2$

$\therefore$ Age of (Nisha) daughter after $(y^2 - y + 2)$ years

$= y^2 - y + 2 + y = (y^2 + 2)$ years

Age of Asha (mother) after $(y^2 - y + 2)$ years

$= x + y^2 - y + 2$

$= y^2 + 2 + y^2 - y + 2$ [From I]

$= 2y^2 - y + 4$ years

After $(y^2 - y - 2)$ years, age of Asha $= 2y^2 - y + 4 = 10y - 1$

$\Rightarrow \quad 2y^2 - y - 10y + 5 = 0$

$\Rightarrow \quad 2y^2 - 11y + 5 = 0$

$\Rightarrow \quad 2y^2 - 10y - 1y + 5 = 0$

$\Rightarrow \quad 2y(y-5) - 1(y-5) = 0$

$\Rightarrow \quad (y-5)(2y-1) = 0$

$\Rightarrow \quad y - 5 = 0 \quad$ or $\quad 2y - 1 = 0$

$\Rightarrow \quad y = 5 \quad$ or $\quad y = \frac{1}{2}$ years

From I, we have

$$x = y^2 + 2$$

Putting $y = 5$, we have

$$x = (5)^2 + 2 = 25 + 2 = 27$$

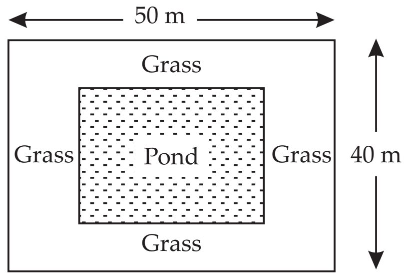
Putting $y = \frac{1}{2}$, we have

$$x = \left(\frac{1}{2}\right)^2 + 2 = 2\frac{1}{4} \text{ years}$$

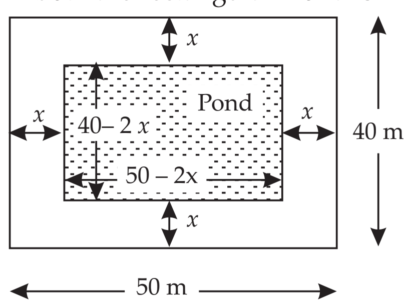
Mother's age can never be $2\frac{1}{4}$ years, so it is rejected.

Hence, the ages of Asha and Nisha are 27 years and 5 years respectively.

Q7. In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed, so that the area of grass surrounding the pond would be 1184 m^2 (see figure). Find the length and breadth of the pond.



Sol. Pond and lawn both are rectangular. Pond is inside the lawn.



Let the length of pond = $(50 - 2x)$ m

and the breadth of pond = $(40 - 2x)$ m

But, Area of grass around the pond = 1184 m^2

$\Rightarrow$ Area of Lawn – Area of Pond = 1184

$\Rightarrow$ $50 \times 40 - (50 - 2x)(40 - 2x) = 1184$

$\Rightarrow$ $2000 - (2000 - 100x - 80x + 4x^2) - 1184 = 0$

$\Rightarrow$ $2000 - (2000 - 180x + 4x^2) - 1184 = 0$

$$\begin{aligned}
\Rightarrow \quad 2000 - 2000 + 180x - 4x^2 - 1184 &= 0\\
\Rightarrow \quad 4x^2 - 180x + 1184 &= 0\\
\Rightarrow \quad x^2 - 45x + 296 &= 0\\
\Rightarrow \quad x^2 - 37x - 8x + 296 &= 0\\
\Rightarrow \quad x(x - 37) - 8(x - 37) &= 0\\
\Rightarrow \quad (x - 37)(x - 8) &= 0
\end{aligned}$$

$\Rightarrow \quad x - 37 = 0 \quad \text{or} \quad x - 8 = 0$

$\Rightarrow \quad x = 37 \quad \text{or} \quad x = 8$

When $x = 37$, then
the length of pond $= 50 - 2 \times 37$
$= 50 - 74$
$= -24$ m

Length cannot be negative. So, $x = 37$ is rejected.

When $x = 8$, then
the length of pond $= 50 - 2x$
$= 50 - 2 \times 8$
$= 50 - 16$
$= 34$ m

and the breadth of the pond
$= 40 - 2x$
$= 40 - 2 \times 8$
$= 40 - 16$
$= 24$ m

Hence, the length and breadth of the pond are 34 m and 24 m respectively.

Q8. At t minutes past 2 p.m., the time needed by minute hand of a clock to show 3 p.m. was found to be 3 min. less than $\frac{t^2}{4}$ min. Find t.

Sol. Total time taken by min hand from 2 p.m. to 3 p.m. is 60 min.
After t min past 2 p.m. the time needed by min. hand of a clock to show 3 p.m. is given by 3 min less than $\frac{t^2}{4}$ min.

$$\therefore \quad t + \left(\frac{t^2}{4} - 3\right) = 60$$

$$\begin{aligned}
\Rightarrow \quad 4t + t^2 - 12 &= 240\\
\Rightarrow \quad t^2 + 4t - 252 &= 0\\
\Rightarrow \quad t^2 + 18t - 14t - 252 &= 0\\
\Rightarrow \quad t(t + 18) - 14(t + 18) &= 0\\
\Rightarrow \quad (t + 18)(t - 14) &= 0
\end{aligned}$$

$\Rightarrow \quad t + 18 = 0 \quad \text{or} \quad t - 14 = 0$

$\Rightarrow \quad t = -18 \quad \text{or} \quad t = 14$ min.

Being, negative value, t = – 18 is rejected.
Hence, $t = 14$ min.