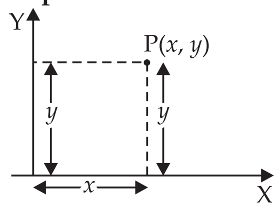
EXERCISE 7.1

Choose the correct answer from the given four options:

Q1. The distance of the point P(2, 3) from x-axis is



(*a*) 2 (*b*) 3
(*c*) 1 (*d*) 5

Sol. (*b*): The perpendicular distance of P(2, 3) from x-axis is equal to the y coordinate so, it is 3 units. verifies ans. (*b*).

Q2. The distance between the points A(0, 6) and B(0, – 2) is

(*a*) 6 (*b*) 8 (*c*) 4 (*d*) 2

Sol. (*b*): $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-0)^2 + (-2-6)^2} = \sqrt{0 + (-8)^2} = \sqrt{64}$

$\Rightarrow$ AB = 8 units

Hence, verifies Ans (*b*).

Q3. The distance of the point P(– 6, 8) from the origin is

(*a*) 8 (*b*) $2\sqrt{7}$ (*c*) 10 (*d*) 6

Sol. (*c*): Coordinates of origin are O(0, 0) and P(– 6, 8)

$\therefore \quad (OP)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= (-6-0)^2 + (8-0)^2 = 36 + 64$

$OP = \sqrt{100}$

$\Rightarrow$ OP = 10 units. verifies ans. (*c*).

Q4. The distance between the points (0, 5) and (–5, 0) is

(*a*) 5 (*b*) $5\sqrt{2}$ (*c*) $2\sqrt{5}$ (*d*) 10

Sol. (*b*): Let A(0, 5) and B(–5, 0) are the two points.

Then, $AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= (-5-0)^2 + (0-5)^2 = 25 + 25$

$\Rightarrow AB^2 = 50$

$\Rightarrow AB = 5\sqrt{2}$ units. verifies ans. (*b*).

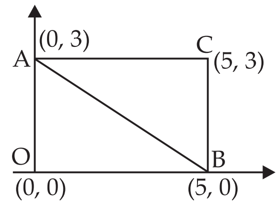
Q5. AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0), and B(5, 0). The length of its diagonal is

(*a*) 5 (*b*) 3 (*c*) $\sqrt{34}$ (*d*) 4

Sol. (*c*): A (0, 3) and B(5, 0)

The length of diagonal = AB

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (5-0)^2 + (0-3)^2$$
$$= 25 + 9$$
$$\Rightarrow \quad AB = \sqrt{34} \text{ verifies Ans. } (c).$$



Q6. The perimeter of a triangle with vertices (0, 4), (0, 0), and (3, 0) is

(*a*) 5 (*b*) 12 (*c*) 11 (*d*) $7 + \sqrt{5}$

Sol. (*b*): Perimeter of ΔABC = AB + BC + AC

Let A(0, 4), B(0, 0), C(3, 0) be the three vertices of ΔABC.

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (0-0)^2 + (0-4)^2 = 0 + 16$$
$$\Rightarrow \quad AB = \sqrt{16} = 4 \text{ cm}$$
$$AC^2 = (3-0)^2 + (0-4)^2 = 9 + 16$$
$$\Rightarrow \quad AC^2 = 25$$
$$\Rightarrow \quad AC = 5 \text{ cm}$$
$$BC^2 = (3-0)^2 + (0-0)^2 = 9 + 0$$
$$\Rightarrow \quad BC^2 = 9$$
$$\Rightarrow \quad BC = 3 \text{ cm}$$
$$\therefore \quad \text{Perimeter} = 4 \text{ cm} + 5 \text{ cm} + 3 \text{ cm} = 12 \text{ cm}$$

Hence, verifies Ans. (*b*).

Q7. The area of triangle with vertices A(3, 0), B(7, 0), and C(8, 4) is

(*a*) 14 (*b*) 28 (*c*) 8 (*d*) 6

Sol. (*c*): Area (A) of ΔABC whose vertices are A(3, 0), B(7, 0) and C(8, 4) is given by

$$\text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$= \frac{1}{2}[3(0-4) + 7(4-0) + 8(0-0)]$$
$$= \frac{1}{2}[-12 + 28 + 0] = \frac{1}{2}[16] = 8 \text{ sq.units}$$

Hence, verifies the Ans. (*c*).

Q8. The points (– 4, 0), (4, 0) and (0, 3) are the vertices of a

(*a*) right triangle (*b*) isosceles triangle

(*c*) equilateral triangle (*d*) scalene triangle

Sol. (*b*): Let the vertices of ΔABC are A(–4, 0), B(4, 0) and C(0, 3).

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$\Rightarrow \quad AB^2 = [4 - (-4)]^2 + (0-0)^2 = 64 + 0 = 64$$
$$\Rightarrow \quad AB = 8 \text{ cm}$$
$$AC^2 = [0 - (-4)]^2 + (3-0)^2 = 16 + 9 = 25$$

$\Rightarrow \qquad AC^2 = 25$

$\Rightarrow \qquad AC = 5$ cm

$\qquad BC^2 = (0-4)^2 + (3-0)^2 = 16 + 9 = 25$

$\Rightarrow \qquad BC^2 = 25$

$\Rightarrow \qquad BC = 5$ cm

$\therefore \qquad AC = BC = 5$ cm and $AB = 8$ cm

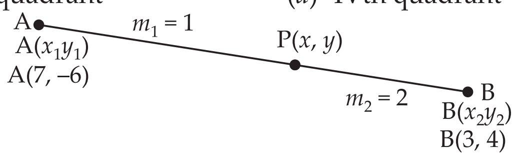
Hence, the triangle is an isosceles triangle. So, verifies ans. (*b*).

Q9. The point which divides the line segment joining the points $(7, -6)$ and $(3, 4)$ in ratio 1 : 2 internally lies in the

(*a*) Ist quadrant (*b*) IInd quadrant

(*c*) IIIrd quadrant (*d*) IVth quadrant

Sol. (*d*):



$$\boxed{x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}} \qquad \boxed{y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}}$$

$$\Rightarrow \quad x = \frac{1(3) + 2(7)}{1+2} = \frac{3+14}{3} \qquad y = \frac{1(4) + 2(-6)}{1+2} = \frac{4-12}{3}$$

$$\Rightarrow \quad x = \frac{17}{3} \qquad y = \frac{-8}{3}$$

$P\left(\frac{17}{3}, \frac{-8}{3}\right)$ verifies the Ans. (*d*).

Q10. The point which lies on the perpendicular bisector of the line segment joining the points $A(-2, -5)$ and $B(2, 5)$ is

(*a*) (0, 0) (*b*) (0, 2) (*c*) (2, 0) (*d*) (–2, 0)

Sol. (*a*): The perpendicular bisector of AB will pass through the mid- point of AB. Mid-point of $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

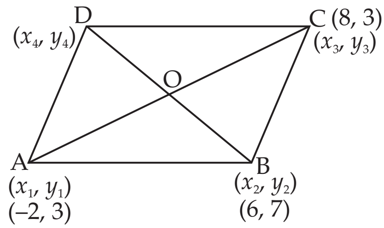
$$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0, 0)$$

So, the perpendicular bisector passes through (0, 0).

Q11. The fourth vertex D of a parallelogram ABCD whose three vertices are $A(-2, 3)$, $B(6, 7)$, and $C(8, 3)$ is

(*a*) (0, 1) (*b*) (0, –1) (*c*) (–1, 0) (*d*) (1, 0)

Sol. (*b*): We know that the diagonals AC and BD of parallelogram ABCD bisect each other.



OR

$$\begin{bmatrix}\text{The mid point} \\ \text{of diagonal AC}\end{bmatrix} = \begin{bmatrix}\text{Mid point of} \\ \text{diagonal BD}\end{bmatrix}$$

$$\Rightarrow \quad \left(\frac{-2+8}{2}, \frac{3+3}{2}\right) = \left(\frac{x_4+6}{2}, \frac{y_4+7}{2}\right)$$

$$\Rightarrow \quad \left(\frac{6}{2}, \frac{6}{2}\right) = \left(\frac{x_4+6}{2}, \frac{y_4+7}{2}\right)$$

$$\Rightarrow \quad (3, 3) = \left(\frac{x_4+6}{2}, \frac{y_4+7}{2}\right)$$

Comparing both sides, we have

$$\frac{x_4+6}{2} = 3 \quad \text{and} \quad \frac{y_4+7}{2} = 3$$

$$\Rightarrow \quad x_4 + 6 = 6 \quad \Rightarrow \quad y_4 + 7 = 6$$

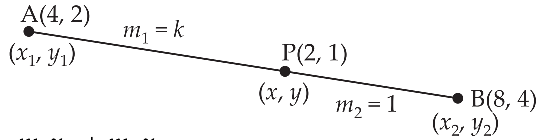
$$\Rightarrow \quad x_4 = 0 \quad \Rightarrow \quad y_4 = 6 - 7 = -1$$

$\therefore$ The fourth vertex of parallelogram is (0, – 1) verifies ans. (*b*).

Q12. If the point P(2, 1) lies on the line segment joining points A(4, 2) and B(8, 4), then

(*a*) $AP = \frac{1}{3}AB$ (*b*) $AP = PB$ (*c*) $PB = \frac{1}{3}AB$ (*d*) $AP = \frac{1}{2}AB$

Sol. (*d*):



$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \qquad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

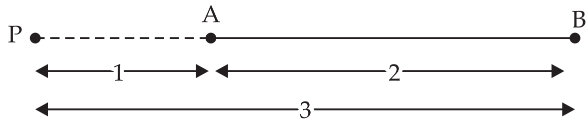
$$\Rightarrow \quad 2 = \frac{k(8) + 1(4)}{k+1} \qquad 1 = \frac{k(4) + 1(2)}{k+1}$$

$$\Rightarrow \quad 8k + 4 = 2k + 2 \qquad 4k + 2 = k + 1$$

$$\Rightarrow \quad 6k = -2 \qquad 3k = -1$$

$$\Rightarrow \quad k = \frac{-1}{3} \qquad k = \frac{-1}{3}$$

Verification:



$$\therefore \quad \frac{AP}{PB} = \frac{-1}{3}$$

$\Rightarrow$ $AP = -1$ i.e., 1 part outside AB

and $PB = 3$

$\therefore$ $AP = 1x$ unit

and $AB = 3x - 1x = 2x$ units

So, $AP = \frac{1}{2} AB$

$\Rightarrow$ $1 = \frac{1}{2} \times 2 \Rightarrow 1 = 1$, which is true

Hence, verifies the ans. (*d*).

Q13. If $P\left(-, 4\right)$ is the mid point of the line segment joining the points Q(– 6, 5) and R (– 2, 3), then the value of '*a*' is

(*a*) – 4 (*b*) – 12 (*c*) 12 (*d*) – 6

Sol. (*b*): P(*x*, *y*) is mid-point of QR then

$$\left(\frac{a}{3}, 4\right) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\Rightarrow \quad \left(\frac{a}{3}, 4\right) = \left(\frac{-6 - 2}{2}, \frac{5 + 3}{2}\right)$$

$$\Rightarrow \quad \frac{a}{3} = \frac{-8}{2}$$

$$\Rightarrow \quad a = -4 \times 3 = -12$$

Verifies the ans. (*b*).

Q14. The perpendicular bisector of the line segment joining the points A(1, 5) and B(4, 6) cuts *y*-axis at

(*a*) (0, 13) (*b*) (0, – 13) (*c*) (0, 12) (*d*) (13, 0)

Sol. (*a*): The given points are A(1, 5) and B(4, 6).

The perpendicular bisector of the line segment joining the points A(1, 5) and B(4, 6) cuts the *y*-axis at P(0, *y*).

Now, $AP = BP \quad \Rightarrow \quad AP^2 = BP^2$

$$\therefore \quad 1 + (y - 5)^2 = 16 + (y - 6)^2$$

$$\Rightarrow \quad 1 + y^2 - 10y + 25 = 16 + y^2 - 12y + 36$$

$$\Rightarrow \quad -10y + 26 = -12y + 52$$

$$\Rightarrow \quad 12y - 10y = 52 - 26$$

$$\Rightarrow \quad 2y = 26$$

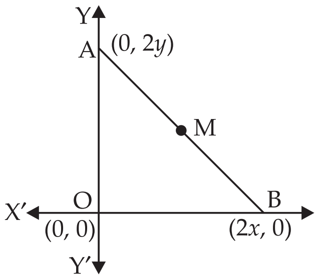
$$\Rightarrow \quad y = 26 \div 2 = 13$$

So, the required point is (0, 13).
Hence, (*a*) is the correct answer.

Q15. The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the figure is

(*a*) (x, y) (*b*) (y, x)

(*c*) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (*d*) $\left(\frac{y}{2}, \frac{x}{2}\right)$



Sol. (*a*): In a right triangle, the mid-point of the hypotenuse is equidistant from the three vertices of triangle.

Mid-point of A(2*x*, 0) and B(0, 2*y*) is

$$= \left(\frac{2x+10}{2}, \frac{0+2y}{2}\right) = (x, y)$$

Hence, (*a*) is the correct answer.

Q16. A circle drawn with origin as the centre passes through $\left(\frac{13}{2}, 0\right)$. The point which does not lie in the interior of the circle is

(*a*) $\left(\frac{-3}{4}, 1\right)$ (*b*) $\left(2, \frac{7}{3}\right)$ (*c*) $\left(5, \frac{-1}{2}\right)$ (*d*) $\left(-6, \frac{5}{2}\right)$

Sol. (*d*): Radius of circle = $\sqrt{\left(\frac{13}{2} - 0\right)^2 + (0-0)^2} = \frac{13}{2} = 6.5$ units

(*a*) Distance of point $\left(\frac{-3}{4}, 1\right)$ from (0, 0) is

$$= \sqrt{\left(\frac{-3}{4} - 0\right)^2 + (1-0)^2} = \sqrt{\frac{9}{16} + 1} = \sqrt{\frac{25}{16}} = \frac{5}{4} = 1.25 \text{ units}$$

The distance $1.25 < 6.5$. So, the point $\left(\frac{-3}{4}, 1\right)$ lies in the interior of the circle.

(*b*) Distance of point $\left(2, \frac{7}{3}\right)$ from (0, 0) is

$$= \sqrt{(2-0)^2 + \left(\frac{7}{3} - 0\right)^2} = \sqrt{4 + \frac{49}{9}} = \sqrt{\frac{85}{9}} = \frac{9.2195}{3} = 3.0731 < 6.25$$

So, the point $\left(2, \frac{7}{3}\right)$ lies in the interior of the circle.

(*c*) Distance of point $\left(5, -\frac{1}{2}\right)$ from (0, 0) is

$$= \sqrt{(5-0)^2 + \left(-\frac{1}{2} - 0\right)} = \sqrt{25 + \frac{1}{4}} = \sqrt{\frac{101}{4}} = \frac{10.0498}{2} = 5.0249 < 6.5$$

So, the point $\left(5, \frac{-1}{2}\right)$ lies in the interior of the circle.

(*d*) Distance of point $\left(-6, \frac{5}{2}\right)$ from (0, 0) is

$$= \sqrt{(-6-0)^2 + \left(\frac{5}{2}-0\right)^2} = \sqrt{36 + \frac{25}{4}} = \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5 \text{ units}$$

So, $\left(-6, \frac{5}{2}\right)$ lies on the circle. It does not lie in the interior of the circle.

Hence, (*d*) is the correct answer.

Q17. A line intersects the *y*-axis and *x*-axis at points P and Q respectively. If (2, – 5) is the mid-point of PQ, then co-ordinates of P and Q are respectively.

(*a*) (0, – 5) and (2, 0) (*b*) (0, 10) and (– 4, 0)
(*c*) (0, 4) and (– 10, 0) (*d*) (0, – 10) and (4, 0)

Sol. (*d*): P lies on *y*-axis so co-ordinates of P are (0, *y*).

Similarly, co-ordinates of Q lies on *x*-axis = Q(*x*, 0)

Mid-point of PQ is

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M(2, -5), \text{ which is given}$$

$$\Rightarrow \quad M\left(\frac{0 + x}{2}, \frac{y + 0}{2}\right) = M(2, -5)$$

$$\Rightarrow \quad \left(\frac{x}{2}, \frac{y}{2}\right) = (2, -5)$$

Comparing both sides, we get

$$\frac{x}{2} = 2 \quad \text{and} \quad \frac{y}{2} = -5$$

$$\Rightarrow \quad x = 4 \quad \text{and} \quad y = -10$$

Hence, the co-ordinates of P(0, – 10) and Q(4, 0) verifies ans. (*d*).

Q18. The area of the triangle with vertices (*a*, *b* + *c*) (*b*, *c* + *a*) and (*c*, *a* + *b*) is

(*a*) $(a + b + c)^2$ (*b*) 0 (*c*) $a + b + c$ (*d*) abc

Sol. (*b*): If the vertices of ΔABC are

$$A(x_1, y_1) = A(a, b + c)$$
$$B(x_2, y_2) = B(b, c + a)$$
$$C(x_3, y_3) = C(c, a + b)$$

Then, Area of $\Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}[a\{c + a - (a + b)\} + b\{a + b - (b + c)\} + c\{b + c - (c + a)\}]$$

$$= \frac{1}{2}[a(c - b) + b(a - c) + c(b - a)]$$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}[ac - ab + ab - bc + bc - ac]$

$\Rightarrow$Area of $\Delta ABC = 0$ So, verifies the option (*b*).

Q19. If the distance between the points (4, *p*) and (1, 0) is 5, then the value of *p* is

(*a*) 4 only (*b*) ± 4 (*c*) -4 only (*d*) 0

Sol. (*b*): According to the question, the distance between A(4, *p*) and B(1, 0) is 5 units.

$$\begin{aligned}
\therefore \quad AB &= 5 \text{ units} \\
\Rightarrow \quad (AB)^2 &= (5)^2 \\
\Rightarrow \quad (4-1)^2 + (p-0)^2 &= 25 \\
\Rightarrow \quad (3)^2 + (p)^2 &= 25 \\
\Rightarrow \quad p^2 &= 25 - 9 \\
\Rightarrow \quad p^2 &= 16 \\
\Rightarrow \quad p &= \pm 4
\end{aligned}$$

Hence, verifies the ans. (*b*).

Q20. If the points A(1, 2), O(0, 0) and C(*a*, *b*) are collinear, then

(*a*) $a = b$ (*b*) $a = 2b$ (*c*) $2a = b$ (*d*) $a = -b$

Sol. (*c*): Points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3\ y_3)$ will be collinear if the area of ΔABC is zero so, A(1, 2), B(0, 0), C(*a*, *b*) will collinear if area $\Delta ABC = 0$

$$\begin{aligned}
\text{or} \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] &= 0 \\
\Rightarrow \quad \frac{1}{2}[1(0 - b) + 0(b - 2) + a(2 - 0)] &= 0 \\
\Rightarrow \quad \frac{1}{2}(-b + 2a) &= 0 \\
\Rightarrow \quad \frac{-b}{2} + a &= 0 \\
\Rightarrow \quad -b + 2a &= 0 \\
\Rightarrow \quad 2a &= b
\end{aligned}$$

Hence, verifies the ans. (*c*).

EXERCISE 7.2

State whether the following statements are true or false. Justify your answer.

Q1. ΔABC with vertices A(– 2, 0), B(2, 0) and C(0, 2) is similar to ΔDEF with vertices D(– 4, 0), E(4, 0) and F(0, 4).

Sol. True: $\Delta ABC \sim \Delta DEF$ if $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF} = k$

In ΔABC,

$$AB^2 = [2 - (-2)]^2 + [0 - (0)]^2 = (4)^2 + 0 = (4)^2$$

$$\Rightarrow \quad AB = 4 \text{ units}$$

$$BC^2 = (0 - 2)^2 + (2 - 0)^2 = 4 + 4 = 8$$

$\Rightarrow \quad BC = 2\sqrt{2}$ units

$AC^2 = [0 - (-2)]^2 + (2 - 0)^2 = 2^2 + 2^2 = 4 + 4 = 8$

$\Rightarrow \quad AC = 2\sqrt{2}$ units

In ΔDEF,

$DE^2 = [4 - (-4)]^2 + (0 - 0)^2 = (8)^2$

$\Rightarrow \quad DE = 8$ units

$EF^2 = (0 - 4)^2 + (4 - 0)^2 = 4^4 + 4^2 = 16 + 16 = 32$

$\Rightarrow \quad EF = 4\sqrt{2}$ units

$DF^2 = [0 - (-4)]^2 + (4 - 0)^2 = 16 + 16 = 32$

$\Rightarrow \quad DF = 4\sqrt{2}$ units

Now, $\dfrac{AB}{DE} = \dfrac{4}{8} = \dfrac{1}{2}$

$\dfrac{BC}{EF} = \dfrac{2\sqrt{2}}{4\sqrt{2}} = \dfrac{1}{2}$

$\dfrac{AC}{DF} = \dfrac{2\sqrt{2}}{4\sqrt{2}} = \dfrac{1}{2}$

$\therefore \quad \dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF} = \dfrac{1}{2}$

Hence, $\Delta ABC \sim \Delta DEF$.

Q2. Point P(– 4, 2) lies on the line segment joining the points A(– 4, 6) and B(– 4, – 6).

Sol. True: We observe that x-coordiante is same i.e., equal to (– 4) so line is parallel to y-axis. y-coordinate of P i.e., 2 lies between 6 and – 6 of A and B respectively. Hence, P lies between and on AB.

OR

Point P(– 4, 2) will lie on the line AB if area of ΔABP is zero.

$\therefore$ i.e., ar(ΔABP) = 0

$$\Rightarrow \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[-4(-6-2) - 4(2-6) - 4(6+6)] = 0$$

$$\Rightarrow \quad [-4(-8) - 4(-4) - 4(12)] = 0$$

$$\Rightarrow \quad 32 + 16 - 48 = 0$$

$$\Rightarrow \quad 48 - 48 = 0, \text{ which is true.}$$

Hence, point P lies on the line joining A and B.

Q3. The points (0, 5), (0, – 9) and (3, 6) are collinear.

Sol. False: Three points A, B, and C will be collinear if the area of $\Delta ABC = 0$

$$\Rightarrow \quad \frac{1}{2}[0(-9-6) + 0(6-5) + 3(5-(-9))] = 0$$

$\Rightarrow \quad 0 + 0 + 3(14) = 0$

$\Rightarrow \quad 42 \neq 0$, which is false.

Hence, the given points are not collinear.

Q4. Point P(0, 2) is the point of intersection of y-axis and perpendicular bisector of line segment joining the points A(– 1, 1) and B(3, 3).

Sol. False: As the point P(0, 2) is the point of intersection of y-axis and perpendicular bisector of the line joining the points A(– 1, 1) and B(3, 3), then point P must be equidistant from A and B. So, we must write PA = PB.

$$PA = \sqrt{(-1-0)^2 + (1-2)^2} = \sqrt{1+1} = \sqrt{2} \text{ units}$$

$$PA = \sqrt{(3-0)^2 + (3-2)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

$\therefore \quad PA \neq PB$

Hence, the given statement is false.

Q5. Points A(3, 1), B(12, – 2) and C(0, 2) cannot be the vertices of a triangle.

Sol. True: Points A, B, C can form a triangle if the sum of any two sides is greater than the third side.

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$\Rightarrow \quad AB^2 = (12-3)^2 + (-2-1)^2 = 81 + 9 = 90$

$\Rightarrow \quad AB = 3\sqrt{10}$ units

$BC^2 = (0-12)^2 + [2-(-2)]^2 = 144 + 16 = 160$

$\Rightarrow \quad BC = 4\sqrt{10}$ units

$AC^2 = (0-3)^2 + (2-1)^2 = 9 + 1 = 10 \Rightarrow AC = \sqrt{10}$ units

$\therefore \quad AC = \sqrt{10}$ units, $\quad AB = 3\sqrt{10}$ units $\quad$ and $\quad BC = 4\sqrt{10}$ units

Now, $\quad AB + AC = \sqrt{10} + 3\sqrt{10} = 4\sqrt{10}$ units $= BC$

So, A, B, C points cannot form a Δ.

Q6. Points A(4, 3), B(6, 4), C(5, – 6) and D(–3, 5) are the vertices of a parallelogram.

Sol. False: The diagonals of parallelogram bisect each other so, ABCD will be a parallelogram if

mid-point of diagonal AC = mid-point of diagonal BD

$$\Rightarrow \quad \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{x_1' + x_2'}{2}, \frac{y_1' + y_2'}{2}\right)$$

$$\Rightarrow \quad \left(\frac{4+5}{2}, \frac{-6+3}{2}\right) = \left(\frac{6-3}{2}, \frac{4+5}{2}\right)$$

$$\Rightarrow \quad \left(\frac{9}{2}, \frac{-3}{2}\right) \neq \left(\frac{3}{2}, \frac{9}{2}\right)$$

Hence, ABCD is not a parallelogram.

Q7. A circle has its centre at the origin and a point P(5, 0) lies on it. The point Q(6, 8) lies outside the circle.

Sol. True: If the distance of Q from the cente O(0, 0) is greater than the radius then point Q lies in the exterior of the circle. Point P(5, 0) lies on the circle and centre is at O(0, 0) so radius = OP

$$OP^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (5 - 0)^2 + (0 - 0)^2$$
$$\Rightarrow \quad OP^2 = 5^2$$
$$\Rightarrow \quad OP = 5 \text{ units}$$
Now, $$OQ^2 = (6 - 0)^2 + (8 - 0)^2 = 36 + 64 = 100$$
$$\Rightarrow \quad OQ = 10 \text{ units}$$
$$\therefore \quad OQ > OP \text{ (radius)}$$
So, point Q lies exterior to circle.

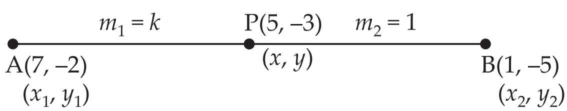
Q8. The point A(2, 7) lies on the perpendicular bisector of line segment joining the points P(6, 5) and Q(0, – 4).

Sol. False: Any point (A) on perpendicular bisector will be equidistant from P and Q so

$$PA = QA$$
or $$PA^2 = QA^2$$
$$\Rightarrow (2-6)^2 + [7-(5)]^2 = (2-0)^2 + [7-(-4)]^2$$
$$\Rightarrow \quad (-4)^2 + (2)^2 = 2^2 + (11)^2$$
$$\Rightarrow \quad 16 + 4 = 4 + 121$$
$$\Rightarrow \quad 20 \neq 125$$
So, A does not lie on the perpendicular bisector of PQ.

Q9. Point P(5, – 3) is one of the two points of trisection of the line segment joining the points A(7, – 2) and B(1, – 5).

Sol. True



Let point P divides the line AB in ratio $k : 1$ then

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \quad \text{and} \quad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$
$$\Rightarrow \quad x = \frac{k(1) + 1(7)}{(k+1)}, \quad y = \frac{k(-5) + 1(-2)}{k+1}$$
$$\Rightarrow \quad 5 = \frac{k+7}{k+1}, \quad -3 = \frac{-5k-2}{k+1}$$
$$\Rightarrow \quad 5k + 5 = k + 7, \quad -5k - 2 = -3k - 3$$
$$\Rightarrow \quad 4k = 7 - 5, \quad -2k = -3 + 2$$
$$\Rightarrow \quad k = \frac{2}{4} = \frac{1}{2}, \quad k = \frac{-1}{-2} = \frac{1}{2}$$

So, P divides AB in 1 : 2 ratio.
Hence, P is one point of trisection of AB.

Q10. Points A(– 6, 10), B(–4, 6) and C(3, – 8) are collinear such that AB = $\frac{2}{9}$ AC.

Sol. True: Points A, B and C will be collinear if ar (ΔABC) = 0

$$\text{ar } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[-6\{6-(-8)\} - 4(-8-10) + 3(10-6)] = 0$$

$$\Rightarrow \quad -6(14) - 4(-18) + 3(4) = 0$$

$$\Rightarrow \quad -84 + 72 + 12 = 0$$

$$\Rightarrow \quad -84 + 84 = 0, \text{ which is true}$$

So, points A, B and C are collinear.

$$AC^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$= (3+6)^2 + (-8-10)^2 = 81 + 324$$

$$\Rightarrow \quad AC = \sqrt{405} = 9\sqrt{5} \text{ units}$$

$$AB^2 = [-4-(-6)]^2 + (6-10)^2$$

$$= (-4+6)^2 + (-4)^2$$

$$= (2)^2 + (-4)^2 = 4 + 16$$

$$\Rightarrow \quad AB^2 = 20$$

$$\Rightarrow \quad AB = 2\sqrt{5} \text{ units}$$

Now, $$AB = \frac{2}{9} AC$$

$$\text{R.H.S.} = \frac{2}{9} \times 9\sqrt{5}$$

$$= 2\sqrt{5}$$

$$= AB$$

Hence, AB = $\frac{2}{9}$ AC is true.

Q11. The point P(– 2, 4) lies on a circle of radius 6 and centre (3, 5).

Sol. False: The point P(– 2, 4) lies on a circle if distance between P and centre is equal to the radius so distance of P from centre O(3, 5) will be

$$OP^2 = (-2-3)^2 + (4-5)^2$$

$$\Rightarrow \quad OP^2 = 25 + (-1)^2$$

$$\Rightarrow \quad OP = \sqrt{26} \neq \text{radius } 6$$

So, P does not lie on the circle. It will lie inside the circle.

Q12. The points A(– 1, – 2), B(4, 3), C(2, 5) and D(– 3, 0) in that order form a rectangle.

Sol. True: ABCD will form a rectangle if

(*i*) it is a parallelogram. (*ii*) diagonals are equal.

For parallelogram: Diagonals bisect each other.

i.e., Mid point of AC = Mid point of BD is

i.e., $$\left(\frac{-1+2}{2}, \frac{-2+5}{2}\right) = \left(\frac{4-3}{2}, \frac{3+0}{2}\right)$$

$$\Rightarrow \quad \left(\frac{1}{2}, \frac{3}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$$

Hence, ABCD is a parallelogram.

Now, $$\text{Diagonal AC} = \sqrt{(2+1)^2 + (5+1)^2} = \sqrt{9+49}$$

$$\Rightarrow \quad AC = \sqrt{58} \text{ units}$$

and $$\text{Diagonal BD} = \sqrt{(-3-4)^2 + (0-3)^2}$$

$$\Rightarrow \quad BD = \sqrt{49+9} \text{ units}$$

$$\Rightarrow \quad BD = \sqrt{58} \text{ units}$$

$\therefore$ Diagonal AC = Diagonal BD

Hence, ABCD is a rectangle.

For parallelogram: Diagonals bisect each other.

i.e., Mid point of AC = Mid point of BD is

i.e., $\left(\dfrac{-1+2}{2}, \dfrac{-2+5}{2}\right) = \left(\dfrac{4-3}{2}, \dfrac{3+0}{2}\right)$

$\Rightarrow \left(\dfrac{1}{2}, \dfrac{3}{2}\right) = \left(\dfrac{1}{2}, \dfrac{3}{2}\right)$

Hence, ABCD is a parallelogram.

Now, Diagonal AC $= \sqrt{(2+1)^2 + (5+1)^2} = \sqrt{9+49}$

$\Rightarrow$ AC $= \sqrt{58}$ units

and Diagonal BD $= \sqrt{(-3-4)^2 + (0-3)^2}$

$\Rightarrow$ BD $= \sqrt{49+9}$ units

$\Rightarrow$ BD $= \sqrt{58}$ units

$\therefore$ Diagonal AC = Diagonal BD

Hence, ABCD is a rectangle.

EXERCISE 7.3

Q1. Name the type of triangle formed by the points A(–5, 6), B(– 4, – 2) and C(7, 5).

Sol. A(– 5, 6), B(– 4, – 2), C(7, 5)

$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\Rightarrow AB^2 = (-4+5)^2 + (-2-6)^2$

$= (1)^2 + (-8)^2 = 1 + 64 = 65$

$\Rightarrow AB = \sqrt{65}$ units

$AC^2 = (7+5)^2 + (5-6)^2$

$\Rightarrow AC^2 = (12)^2 + (-1)^2 \Rightarrow AC^2 = 144 + 1$

$\Rightarrow AC = \sqrt{145}$ units

$BC^2 = (7+4)^2 + (5+2)^2 = 11^2 + 7^2 = 121 + 49$

$\Rightarrow BC = \sqrt{170}$ units

As $AB \neq BC \neq AC$ so scalene triangle.

$\because$ $AC^2 + AB^2 = 145 + 65 = 210 \neq BC^2$, so it is not a right angled Δ

So, a scalene Δ will be formed.

Q2. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ from point (7, – 4). How many such points are there?

Sol. Let point P(x, 0) be a point on x-axis, and A be the point (7, – 4).

So, $AP = 2\sqrt{5}$ [Given]

$\Rightarrow AP^2 = 4 \times 5 = 20$

$\Rightarrow (x-7)^2 + [0-(-4)]^2 = 20$

$\Rightarrow x^2 + 49 - 14x + 16 = 20$

$$\Rightarrow \quad x^2 - 14x - 20 + 65 = 0$$
$$\Rightarrow \quad x^2 - 14x + 45 = 0$$
$$\Rightarrow \quad x^2 - 9x - 5x + 45 = 0$$
$$\Rightarrow \quad x(x - 9) - 5(x - 9) = 0$$
$$\Rightarrow \quad (x - 9)(x - 5) = 0$$
$$\Rightarrow \quad x - 9 = 0 \quad \text{or} \quad x - 5 = 0$$
$$\Rightarrow \quad x = 9 \quad \text{or} \quad x = 5$$

Hence, there are two such points on x-axis whose distance from $(7, -4)$ is $2\sqrt{5}$. Hence, required points are $(9, 0)$, $(5, 0)$.

Q3. What type of quadrilateral do the points A(2, -2), B(7, 3), C(11, -1) and D(6, -6) taken in that order, form?

Sol. (*i*) A qudrilateral is a parallelogram, if mid points of diagonals AC and BD are same.

(*ii*) A parallelogram is not a rectangle, if diagonals AC $\neq$ BD.

(*iii*) A parallelogram may be a rhombus if AB = BC.

(*iv*) If in a parallelogram diagonals are equal, then it is rectangle.

In a rectangle if the sides AB = BC, then the rectangle is a square.

For parallelogram with vertices A(2, -2), B(7, 3), C(11, -1), D(6, -6).

mid point of AC = mid point of BD

$$\Rightarrow \quad \left(\frac{2+11}{2}, \frac{-2-1}{2}\right) = \left(\frac{7+6}{2}, \frac{3-6}{2}\right)$$

$$\Rightarrow \quad \left(\frac{13}{2}, \frac{-3}{2}\right) = \left(\frac{13}{2}, \frac{-3}{2}\right), \text{ which is true.}$$

Hence, ABCD is a parallelogram.

Now, we will check whether AC = BD

$$\text{or} \quad AC^2 = BD^2$$
$$\Rightarrow \quad (11 - 2)^2 + (-1 + 2)^2 = (6 - 7)^2 + (-6 - 3)^2$$
$$\Rightarrow \quad (9)^2 + (1)^2 = (-1)^2 + (-9)^2$$
$$\Rightarrow \quad 81 + 1 = 1 + 81$$
$$\Rightarrow \quad 82 = 82, \text{ which is true.}$$

As the diagonals are equal so it is a rectangle or square.

Now, we will check whether adjacent sides AB = BC

$$\text{or} \quad AB^2 = BC^2$$
$$\Rightarrow \quad (7 - 2)^2 + (3 + 2)^2 = (11 - 7)^2 + (-1 - 3)^2$$
$$\Rightarrow \quad 5^2 + 5^2 = (4)^2 + (-4)^2$$
$$\Rightarrow \quad 25 + 25 = 16 + 16$$
$$\Rightarrow \quad 50 \neq 32, \text{ which is false.}$$

So, ABCD is not a square. Hence, ABCD is a rectangle.

Q4. Find the value of a, if the distance between the points A(-3, -14) and B(a, -5) is 9 units.

Sol. Consider A(-3, -14) and B(a, -5).

According to the question, $\quad AB = 9$

$$\Rightarrow \quad AB^2 = 81$$
$$\Rightarrow \quad (a + 3)^2 + (-5 + 14)^2 = 81$$
$$\Rightarrow \quad a^2 + 9 + 6a + (9)^2 = 81$$
$$\Rightarrow \quad a^2 + 6a + 9 = 81 - 81$$
$$\Rightarrow \quad (a + 3)^2 = 0$$
$$\Rightarrow \quad a + 3 = 0$$
$$\Rightarrow \quad a = -3$$

Q5. Find a point which is equidistant from the points A(– 5, 4) and B(– 1, 6). How many such points are there?

Sol. Let P(x, y) is equidistant from A(–5, 4) and B(– 1, 6), then

$$PA = PB$$
$$\Rightarrow \quad PA^2 = PB^2$$
$$\Rightarrow \quad (x + 5)^2 + (y - 4)^2 = (x + 1)^2 + (y - 6)^2$$
$$\Rightarrow \quad x^2 + 25 + 10x + y^2 + 16 - 8y = x^2 + 1 + 2x + y^2 + 36 - 12y$$
$$\Rightarrow \quad 41 + 10x - 8y = 37 + 2x - 12y$$
$$\Rightarrow \quad 8x + 4y + 4 = 0$$
$$\Rightarrow \quad 2x + 1y + 1 = 0 \qquad \text{(I)}$$

The above equation shows that infinite points are equidistant from AB, because all the points on perpendicular bisector of AB will be equidistant from AB.

$\Rightarrow$ One such point which is equidistant from A and B is the mid-point M of AB *i.e.*,

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$M\left(\frac{-5 - 1}{2}, \frac{4 + 6}{2}\right)$$
$$M\left(\frac{-6}{2}, \frac{10}{2}\right)$$
$$M(-3, 5)$$

So, (– 3, 5) is equidistant from points A and B.

Q6. Find the coordinates of the point Q on the x-axis which lies on the perpendicular bisector of the line segment joining the points A(– 5, – 2) and B(4, – 2). Name the type of triangle formed by the points Q, A and B.

Sol. Let Q(x, 0) be a point on x-axis which lies on the perpendicular bisector of AB.

$$\therefore \quad QA = QB$$
$$\Rightarrow \quad QA^2 = QB^2$$
$$\Rightarrow \quad (-5 - x)^2 + (-2 - 0)^2 = (4 - x)^2 + (-2 - 0)^2$$
$$\Rightarrow \quad (x + 5)^2 + (-2)^2 = (4 - x)^2 + (-2)^2$$
$$\Rightarrow \quad x^2 + 25 + 10x + 4 = 16 + x^2 - 8x + 4$$

$$\Rightarrow \quad 10x + 8x = 16 - 25$$
$$\Rightarrow \quad 18x = -9$$
$$\Rightarrow \quad x = \frac{-9}{18} = \frac{-1}{2}$$

Hence, the point Q is $\left(\frac{-1}{2}, 0\right)$.

Now, $$QA^2 = \left[-5 + \frac{1}{2}\right]^2 + [-2 - 0]^2$$
$$= \left(\frac{-9}{2}\right)^2 + \frac{4}{1}$$
$$\Rightarrow \quad QA^2 = \frac{81}{4} + \frac{4}{1} = \frac{81 + 16}{4} = \frac{97}{4}$$
$$\Rightarrow \quad QA = \sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2} \text{ units}$$

Now, $$QB^2 = \left(4 + \frac{1}{2}\right)^2 + (-2 - 0)^2 = \left(\frac{9}{2}\right)^2 + (-2)^2$$
$$\Rightarrow \quad QB^2 = \frac{81}{4} + \frac{4}{1} = \frac{81 + 16}{4} = \frac{97}{4}$$
$$\Rightarrow \quad QB = \sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2} \text{ units}$$

and $$AB = \sqrt{(4 + 5)^2 + [-2 - (-2)]^2} = \sqrt{(9)^2} = 9 \text{ units}$$
$$\Rightarrow \quad AB = 9 \text{ units}$$

As $QA = QB$

So, ΔQAB is an isosceles Δ.

Q7. Find the value of m if the points $(5, 1)$, $(-2, -3)$ and $(8, 2m)$ are collinear.

Sol. Points A, B, C will be collinear if the area of $\Delta ABC = 0$.

i.e., $$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$
$$\Rightarrow \quad \frac{1}{2}[5(-3 - 2m) - 2(2m - 1) + 8(1 + 3)] = 0$$
$$\Rightarrow \quad -15 - 10m - 4m + 2 + 32 = 0$$
$$\Rightarrow \quad -14m - 15 + 34 = 0$$
$$\Rightarrow \quad -14m + 19 = 0$$
$$\Rightarrow \quad -14m = -19$$
$$\Rightarrow \quad m = \frac{19}{14}$$

Hence, the required value of $m = \frac{19}{14}$.

Q8. If the point A(2, – 4) is equidistant from P(3, 8) and Q(–10, y), then find the values of y. Also find distance PQ.

Sol. According to the question,

$$PA = QA$$
$$\Rightarrow \quad PA^2 = QA^2$$
$$\Rightarrow \quad (3-2)^2 + (8+4)^2 = (-10-2)^2 + (y+4)^2$$
$$\Rightarrow \quad 1^2 + 12^2 = (-12)^2 + y^2 + 16 + 8y$$
$$\Rightarrow \quad y^2 + 8y + 16 - 1 = 0$$
$$\Rightarrow \quad y^2 + 8y + 15 = 0$$
$$\Rightarrow \quad y^2 + 5y + 3y + 15 = 0$$
$$\Rightarrow \quad y(y+5) + 3(y+5) = 0$$
$$\Rightarrow \quad (y+5)(y+3) = 0$$
$$\Rightarrow \quad y + 5 = 0 \quad \text{or } y + 3 = 0$$
$$\Rightarrow \quad y = -5 \quad \text{or} \quad y = -3$$

So, the co-ordinates are P(3, 8), Q_1(– 10, – 3), Q_2(– 10, – 5).

Now, $$PQ_1^2 = (3+10)^2 + (8+3)^2 = 13^2 + 11^2$$
$$\Rightarrow \quad PQ_1^2 = 169 + 121$$
$$\Rightarrow \quad PQ_1 = \sqrt{290} \text{ units}$$
and $$PQ_2^2 = (3+10)^2 + (8+5)^2 = 13^2 + 13^2$$
$$= 13^2[1+1]$$
$$\Rightarrow \quad PQ_2^2 = 13^2 \times 2$$
$$\Rightarrow \quad PQ_2 = 13\sqrt{2} \text{ units}$$

Hence, $y = -3, -5$, and PQ = $\sqrt{290}$ units and $13\sqrt{2}$ units.

Q9. Find the area of the triangle whose vertices are (– 8, 4), (– 6, 6) and (– 3, 9).

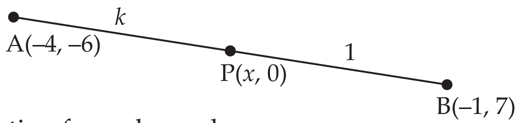
Sol. Vertices of ΔABC are A(– 8, 4), B(– 6, 6) and C(– 3, 9).

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}[-8(6-9) - 6(9-4) - 3(4-6)]$$
$$= \frac{1}{2}[-8(-3) - 6(5) - 3(-2)]$$
$$= \frac{1}{2}[24 - 30 + 6] = 0$$

Hence, the area of given triangle is zero.

Q10. In what ratio does the x-axis divides the line segment joining the points (– 4, – 6) and (– 1, 7)? Find the coordinates of the point of division.

Sol. Point P(x, 0) on x-axis intersects the line joining the points A(– 4, – 6) and B(– 1, 7). Let P divides the line in the ratio k : 1.



Using the section formula, we have

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \qquad \text{(I)}$$

$$\Rightarrow \quad \frac{0}{1} = \frac{k(7) + 1(-6)}{k+1}$$

$$\Rightarrow \quad 7k - 6 = 0$$

$$\Rightarrow \quad k = \frac{6}{7}$$

$$\Rightarrow \quad m_1 = 6 \text{ and } m_2 = 7$$

Again, using the section formula, we have

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$\Rightarrow \quad x = \frac{6(-1) + 7(-4)}{6+7} = \frac{-6-28}{13}$$

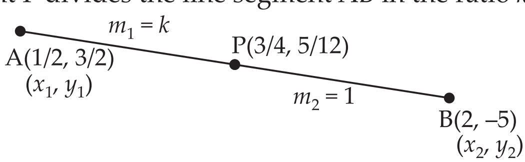
$$\Rightarrow \quad x = \frac{-34}{13}$$

Now, $\quad y = \dfrac{6(7) + 7(-6)}{6+7} = \dfrac{42-42}{13} = 0 \qquad$ [From (I)]

$\therefore$ Hence, the required point of intersection is $\left(\dfrac{-34}{13}, 0\right)$.

Q11. Find the ratio in which the point $P\left(\dfrac{3}{4}, \dfrac{5}{12}\right)$ divides the line segment joining the points $A\left(\dfrac{1}{3}, \dfrac{3}{2}\right)$ and $B(2, -5)$.

Sol. Let point P divides the line segment AB in the ratio $k : 1$, then



The coordinates of P, by section formula are

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \; y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\therefore \quad x = \frac{k(2) + 1\left(\frac{1}{2}\right)}{k+1}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{2k+\frac{1}{2}}{k+1}$$

$$\Rightarrow \quad 8k+2 = 3k+3$$

$$\Rightarrow \quad 8k-3k = 3-3$$

$$\Rightarrow \quad 5k = 1$$

$$\Rightarrow \quad k = \frac{1}{5}$$

$$\Rightarrow \quad m_1 = 1 \text{ and } m_2 = 5$$

Now, $$y = \frac{m_1(y_2)+m_2(y_1)}{m_1+m_2}$$

$\Rightarrow$

$\Rightarrow$

$\therefore$ y-coordinate of P is $\left(\frac{5}{12}\right)$ $$y = \frac{1(-5)+5\left(\frac{3}{2}\right)}{1+5} = \frac{-5+\frac{15}{2}}{6}$$

$$= \frac{\frac{-10+15}{2}}{6}$$

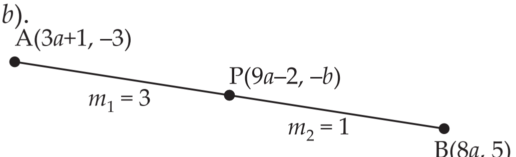
$$\Rightarrow \quad y = \frac{5}{2}\times\frac{1}{6} = \frac{5}{12}$$

y-coordinate of P is $\left(\frac{5}{12}\right)$.

Hence, P divides AB in ratio 1 : 5.

Q12. If point P($9a-2, -b$) divides the line segment joining the points A($3a+1, -3$) and B($8a$, 5) in the ratio 3 : 1, then find the values of a and b.

Sol. Point P($9a-2, -b$) divides the line segment joining the points A($3a+1, -3$) and B($8a$, 5) in the ratio 3 : 1. But, the coordinates of P are ($9a-2, -b$).



Using section formula, we have

$$9a-2 = \frac{3(8a)+1(3a+1)}{3+1}$$

$$= \frac{24a+3a+1}{4}$$

$$\Rightarrow \quad 36a-8 = 27a+1$$

$$\Rightarrow \quad 36a-27a = 8+1$$

$$\Rightarrow \quad 9a = 9$$

$$\Rightarrow \quad a = \frac{9}{9} = 1$$

$$-b = \frac{3(+5)+1(-3)}{3+1}$$

$$\Rightarrow \quad -b = \frac{+15-3}{4} = \frac{12}{4}$$

$$\Rightarrow \quad b = -3$$

Hence, $a = +1$ and $b = -3$

Q13. If (a, b) is mid-point of the line segment joining points A(10, – 6) and B(k, 4) and $a - 2b = 18$, then find the value of k and the distance AB.

Sol. Let P(a, b) is the mid-point of the line-segment joining the points A(10, –6) and B(k, 4). Therefore, P(a, b) divides the line segment joining the points A(10, – 6) and B(k, 4) in the ratio 1 : 1.

$$\Rightarrow \quad a = \frac{10+k}{2} \quad \text{(I)} \quad \text{and} \quad b = \frac{-6+4}{2}$$

$$\Rightarrow \quad b = \frac{-2}{2}$$

$$\Rightarrow \quad b = -1 \qquad \text{(II)}$$

But, $\quad a - 2b = 18 \qquad$ (III) [Given]

$\Rightarrow \quad a - 2(-1) = 18 \qquad$ [Using (II)]

$\Rightarrow \quad a = 18 - 2 \Rightarrow a = 16$

But, $\quad a = \dfrac{10+k}{2} \qquad$ [From (I)]

$\Rightarrow \quad 16 = \dfrac{10+k}{2}$

$\Rightarrow \quad 10 + k = 32$

$\Rightarrow \quad k = 32 - 10$

$\Rightarrow \quad k = 22$

Now, the co-ordinates of A and B are given by A (10, – 6) and B(22, 4).

$\therefore \quad AB^2 = (22-10)^2 + (4+6)^2$

$\quad = 12^2 + 10^2 = 144 + 100$

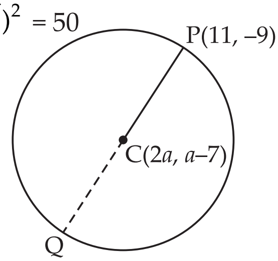
$\Rightarrow \quad AB^2 = 244$

$\Rightarrow \quad AB = 2\sqrt{61}$ units

Hence, the required value of $k = 22$, $a = 16$, $b = -1$ and AB = $2\sqrt{61}$ units.

Q14. If the centre of circle is $(2a, a - 7)$ then find the values of a if the circle passes through the point (11, – 9) and has diameter $10\sqrt{2}$ units.

Sol. Let C($2a$, $a - 7$) be the centre of the circle and it passes through the point P(11, – 9).



$\therefore \quad PQ = 10\sqrt{2}$

$\Rightarrow \quad CP = 5\sqrt{2}$

$\Rightarrow \quad CP^2 = (5\sqrt{2})^2 = 50$

$\Rightarrow \quad (2a - 11)^2 + (a - 7 + 9)^2 = 50$

$\Rightarrow \quad (2a)^2 + (11)^2 - 2(2a)(11) + (a + 2)^2 = 50$

$\Rightarrow \quad 4a^2 + 121 - 44a + (a)^2 + (2)^2 + 2(a)(2) = 50$

$\Rightarrow \quad 5a^2 - 40a + 125 = 50$

$\Rightarrow \quad a^2 - 8a + 25 = 10$

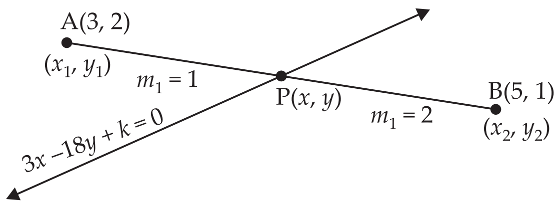
$\Rightarrow \quad a^2 - 8a + 25 - 10 = 0$

$$\Rightarrow \quad a^2 - 8a + 15 = 0$$
$$\Rightarrow \quad a^2 - 5a - 3a + 15 = 0$$
$$\Rightarrow \quad a(a-5) - 3(a-5) = 0$$
$$\Rightarrow \quad (a-5)(a-3) = 0$$
$$\Rightarrow \quad a - 5 = 0 \quad \text{or} \quad a - 3 = 0$$
$$\Rightarrow \quad a = 5 \quad \text{or} \quad a = 3$$

Hence, the required values of a are 5 and 3.

Q15. The line segment joining the points A(3, 2) and B(5, 1) is divided at the point P in the ratio of 1 : 2 and it lies on the line $3x - 18y + k = 0$. Find the value of k.

Sol.



P divides AB in the ratio 1 : 2. Then, the coordinates of P(x, y) are given by

$$x = \frac{m_1(x_2) + m_2(x_1)}{m_1 + m_2} \quad \text{and} \quad y = \frac{m_1(y_2) + m_2(y_1)}{m_1 + m_2}$$

$$\Rightarrow \quad x = \frac{1(5) + 2(3)}{1+2} = \frac{5+6}{3} \qquad \Rightarrow \quad y = \frac{1(1) + 2(2)}{1+2} = \frac{1+4}{3}$$

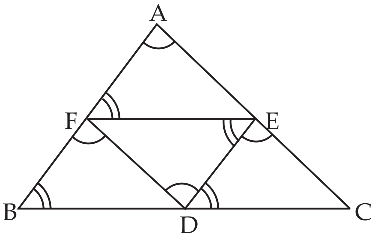
$$\Rightarrow \quad x = \frac{11}{3} \qquad \Rightarrow \quad y = \frac{5}{3}$$

But, $P\left(\frac{11}{3}, \frac{5}{3}\right)$ P(x, y) lies on the line $3x - 18y + k = 0$

$$\therefore \quad 3\left(\frac{11}{3}\right) - 18\left(\frac{5}{3}\right) + k = 0$$
$$\Rightarrow \quad \frac{33}{3} - \frac{90}{3} + k = 0$$
$$\Rightarrow \quad 33 - 90 + 3k = 0$$
$$\Rightarrow \quad 3k = 90 - 33$$
$$\Rightarrow \quad 3k = 57$$
$$\Rightarrow \quad k = \frac{57}{3}$$
$$\Rightarrow \quad k = 19$$

Hence, the required value of $k = 19$.

Q16. If $D\left(\frac{-1}{2}, \frac{5}{2}\right)$, $E(7, 3)$ and $F\left(\frac{7}{2}, \frac{7}{2}\right)$ are the mid-points of sides of ΔABC, find the area of ΔABC.



Sol. In ΔABC, D is mid point of BC, E is mid point of AC, and F is mid point of AB.

$\therefore \quad \Delta DEF \cong \Delta AFE \cong \Delta FBD \cong \Delta EDC$

So, area of ΔABC = 4 (area of ΔDEF)

The mid-points of sides of ΔABC are given by $D\left(\frac{-1}{2}, \frac{5}{2}\right)$, $E(7, 3)$, and $F\left(\frac{7}{2}, \frac{7}{2}\right)$.

$$\therefore \quad \text{Area } \Delta DEF = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}\left[-\frac{1}{2}\left(3 - \frac{7}{2}\right) + 7\left(\frac{7}{2} - \frac{5}{2}\right) + \frac{7}{2}\left(\frac{5}{2} - 3\right)\right]$$

$$= \frac{1}{2}\left[-\frac{1}{2}\left(\frac{-1}{2}\right) + 7(1) + \frac{7}{2}\left(\frac{-1}{2}\right)\right]$$

or

$$= \frac{1}{2}\left[\frac{1}{4} + 7 - \frac{7}{4}\right]$$

$$= \frac{1}{2}\left[\frac{1 + 28 - 7}{4}\right]$$

$$= \frac{1}{2}\left(\frac{29 - 7}{4}\right)$$

$$= \frac{22}{8} = \frac{11}{4}$$

$$\therefore \quad \text{Area of } \Delta ABC = 4 \times \text{Area } \Delta DEF$$

$$= 4 \times \frac{11}{4}$$

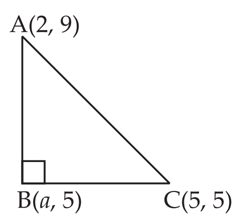
$$= 11 \text{ square units}$$

Hence, the required area of ΔABC is 11 square units.

Q17. The points A(2, 9), B(a, 5) and C(5, 5) are the vertices of a ΔABC right angled at B. Find the values of a and hence the area of ΔABC.

Sol. ΔABC is right angled at B.

$\therefore$ By Pythagoras theorem,



$$AB^2 + BC^2 = AC^2 \quad \text{...(I)}$$

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow \quad AB^2 = (a - 2)^2 + (5 - 9)^2$$

$$\Rightarrow \quad AB^2 = (a)^2 + (2)^2 - 2(a)(2) + (-4)^2$$

$$= a^2 + 4 - 4a + 16$$

$$\Rightarrow \quad AB^2 = a^2 - 4a + 20$$

$$BC^2 = (a-5)^2 + (5-5)^2$$
$$= (a)^2 + (5)^2 - 2(a)(5) + 0^2$$
$$\Rightarrow \quad BC^2 = a^2 + 25 - 10a$$
$$AC^2 = (5-2)^2 + (5-9)^2$$
$$= 3^2 + (-4)^2$$
$$= 9 + 16 = 25$$
$$\Rightarrow \quad AC = \sqrt{25} = 5 \text{ units}$$
$$\therefore \quad a^2 - 4a + 20 + a^2 + 25 - 10a = (5)^2 \qquad \text{[From (I)]}$$
$$\Rightarrow \quad 2a^2 - 14a + 45 - 25 = 0$$
$$\Rightarrow \quad 2a^2 - 14a + 20 = 0$$
$$\Rightarrow \quad a^2 - 7a + 10 = 0$$
$$\Rightarrow \quad a^2 - 5a - 2a + 10 = 0$$
$$\Rightarrow \quad a(a-5) - 2(a-5) = 0$$
$$\Rightarrow \quad (a-5)(a-2) = 0$$
$$\Rightarrow \quad a - 5 = 0 \quad \text{or} \quad a - 2 = 0$$
$$\Rightarrow \quad a = 5 \quad \text{or} \quad a = 2$$

If $a = 5$ then B(5, 5) and C(5, 5) and BC = 0, which is not possible. Hence, $a = 2$.

Now,
$$AB^2 = a^2 - 4a + 20$$
$$= (2)^2 - 4(2) + 20$$
$$= 4 - 8 + 20$$
$$\Rightarrow \quad AB^2 = 24 - 8$$
$$\Rightarrow \quad AB^2 = 16$$
$$\Rightarrow \quad AB = 4 \text{ units}$$

And,
$$BC^2 = a^2 + 25 - 10a$$
$$= (2)^2 + 25 - 10(2) \qquad [\because a = 2]$$
$$= 4 + 25 - 20 = 29 - 20 = 9$$
$$\Rightarrow \quad BC^2 = 9$$
$$\Rightarrow \quad BC = 3 \text{ units}$$

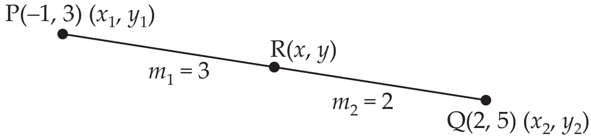
$$\therefore \quad \text{Area of right angled triangle ABC} = \frac{1}{2}\,\text{base} \times \text{altitude}$$
$$= \frac{1}{2}\,BC \times AB$$
$$= \frac{1}{2} \times 3 \times 4$$
$$= 6 \text{ square units}$$

Hence, the value of $a = 2$ and area of ΔABC is 6 sq. units.

Q18. Find the coordinates of the point R on the line segment joining the points P(– 1, 3) and Q(2, 5) such that $PR = \frac{3}{5} PQ$.

Sol. $\quad \text{PR} = \dfrac{3}{5}\text{PQ}$ [Given]

$\Rightarrow \quad \dfrac{5}{3} = \dfrac{\text{PQ}}{\text{PR}}$



$\Rightarrow \quad \dfrac{5}{3} = \dfrac{\text{PR} + \text{RQ}}{\text{PR}}$

$\Rightarrow \quad \dfrac{5}{3} = \dfrac{\text{PR}}{\text{PR}} + \dfrac{\text{RQ}}{\text{PR}}$

$\Rightarrow \quad \dfrac{\text{QR}}{\text{PR}} = \dfrac{5}{3} - 1 = \dfrac{5-3}{3}$

$\Rightarrow \quad \dfrac{\text{QR}}{\text{PR}} = \dfrac{2}{3}$

or $\quad \dfrac{\text{PR}}{\text{QR}} = \dfrac{3}{2}$ or PR : QR = 3 : 2

$\therefore \quad m_1 = 3$ and $m_2 = 2$

Now, the coordinates of point R are given by

$$x = \frac{m_1(x_2) + m_2(x_1)}{m_1 + m_2} \quad \text{and} \quad y = \frac{m_1(y_2) + m_2(y_1)}{m_1 + m_2}$$

$$\Rightarrow \quad x = \frac{3(2) + 2(-1)}{3+2} = \frac{6-2}{5} \qquad \Rightarrow \quad y = \frac{3(5) + 2(3)}{3+2} = \frac{15+6}{5}$$

$$\Rightarrow \quad x = \frac{4}{5} \qquad \Rightarrow \quad y = \frac{21}{5}$$

Hence, the required coordinates of R are $\left(\dfrac{4}{5}, \dfrac{21}{5}\right)$.

Q19. Find the value of k if the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 1, 5k) are collinear.

Sol. Points A, B, and C will be collinear if area of ΔABC = 0

$$\Rightarrow \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(k+1)\{2k + 3 - 5k\} + 3k\{5k - 2k\} + (5k-1)\{2k - (2k+3)\}] = 0$$

$$\Rightarrow \quad (k+1)(-3k+3) + 3k(3k) + (5k-1)(2k - 2k - 3) = 0$$

$\Rightarrow -3(k+1)(k-1)+3(3k^2)-3(5k-1)=0$

Divide by 3 on both sides, we have

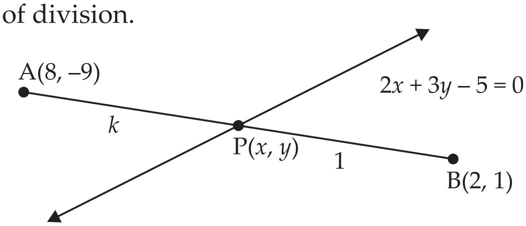
$$[(k+1)(-k+1)+3k^2+(5k-1)(-1)]=0$$

$$\begin{aligned}
\Rightarrow\quad 1-k^2+3k^2-5k+1 &= 0\\
\Rightarrow\quad 2k^2-5k+2 &= 0\\
\Rightarrow\quad 2k^2-4k-1k+2 &= 0\\
\Rightarrow\quad 2k(k-2)-1(k-2) &= 0\\
\Rightarrow\quad (k-2)(2k-1) &= 0\\
\Rightarrow\quad k-2 = 0 \quad &\text{or} \quad 2k-1=0\\
\Rightarrow\quad k = 2 \quad &\text{or} \quad 2k=1\\
\Rightarrow\quad k = 2 \quad &\text{or} \quad k=\frac{1}{2}
\end{aligned}$$

Hence, the required value of k are 2 and $\frac{1}{2}$.

Q20. Find the ratio in which the line $2x + 3y - 5 = 0$ divides the line segment joining the points $(8, -9)$ and $(2, 1)$. Also find the coordinates of the point of division.

Sol.



$$2x+3y-5=0 \qquad \text{...(I)}$$

Let the line given by equation I divides AB at P(x, y) in the ratio k : 1. Then, using the section formula, the coordinates of P are given by

$$x=\frac{m_1(x_2)+m_2(x_1)}{m_1+m_2} \quad \text{and} \quad y=\frac{m_1(y_2)+m_2(y_1)}{m_1+m_2}$$

$$\Rightarrow \quad x=\frac{k(2)+1(8)}{(k+1)} \quad \text{and} \quad y=\frac{k(1)+1(-9)}{k+1}$$

$$\Rightarrow \quad x=\frac{2k+8}{k+1} \quad \text{and} \quad y=\frac{k-9}{k+1}$$

$\Rightarrow$ P$(x, y)=\left(\frac{2k+8}{k+1}, \frac{k-9}{k+1}\right)$ lies on line I so P must satisfy equation (I)

So substitute $x=\frac{2k+8}{k+1}$ and $y=\frac{k-9}{k+1}$ in equation I

$$\Rightarrow \quad 2\left(\frac{2k+8}{k+1}\right)+3\left(\frac{k-9}{k+1}\right)-5=0$$

On multiplying by $(k + 1)$ in above equation both sides, we get

$$2(2k + 8) + 3(k - 9) - 5(k + 1) = 0$$

$$\Rightarrow \quad 4k + 16 + 3k - 27 - 5k - 5 = 0$$

$$\Rightarrow \quad 2k - 16 = 0$$

$$\Rightarrow \quad k = \frac{16}{2} = 8$$

$\therefore$ Point of intersection is given by $P\left(\frac{2k+8}{k+1}, \frac{k-9}{k+1}\right)$

$$= P\left(\frac{2 \times 8 + 8}{8+1}, \frac{8-9}{8+1}\right)$$

$$= P\left(\frac{16+8}{9}, \frac{-1}{9}\right)$$

$$= P\left(\frac{24}{9}, \frac{-1}{9}\right)$$

$$= P\left(\frac{8}{3}, \frac{-1}{9}\right)$$

Hence, line of eqn. (I) divides AB in ratio 8 : 1 at $P\left(\frac{8}{3}, \frac{-1}{9}\right)$.

EXERCISE 7.4

Q1. If (– 4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle.

Sol. Let A(– 4, 3), B(4, 3) and C(x, y) are the three vertices of ΔABC.

As the triangle is equilateral,

so $\quad AC = BC = AB$

or $\quad AC^2 = BC^2 = AB^2 \quad$ (I)

Now, $\quad AB^2 = (4 + 4)^2 + (3 - 3)^2$

$\Rightarrow \quad AB^2 = (8)^2 = 64$

$\Rightarrow \quad AB = 8$ units $\quad$ (II)

$$AC^2 = (x + 4)^2 + (y - 3)^2$$
$$= (x)^2 + (4)^2 + 2(x)(4) + (y)^2 + (3)^2 - 2(y)(3)$$
$$= x^2 + y^2 + 8x - 6y + 16 + 9$$

$\Rightarrow \quad AC^2 = x^2 + y^2 + 8x - 6y + 25 \quad$ (III)

$$BC^2 = (x - 4)^2 + (y - 3)^2$$
$$= (x)^2 + (4)^2 - 2(x)(4) + (y)^2 + (3)^2 - 2(y)(3)$$
$$= x^2 + y^2 - 8x - 6y + 16 + 9$$

$\Rightarrow \quad BC^2 = x^2 + y^2 - 8x - 6y + 25 \quad$ (IV)

Now, $AC^2 = AB^2$ [From (I)]

$\Rightarrow \quad x^2 + y^2 + 8x - 6y + 25 = 64$ [From (III), (II)]

$\Rightarrow \quad x^2 + y^2 + 8x - 6y = 64 - 25$

$\Rightarrow \quad x^2 + y^2 + 8x - 6y = 39$ (V)

Again, $BC^2 = AB^2$ [From (I)]

$\Rightarrow \quad x^2 + y^2 - 8x - 6y + 25 = 64$ [From (II), (IV)]

$\Rightarrow \quad x^2 + y^2 - 8x - 6y = 64 - 25$

$\Rightarrow \quad x^2 + y^2 - 8x - 6y = 39$ (VI)

Subtracting (V) from (VI), we have

$$\begin{array}{rcl} x^2 + y^2 - 8x - 6y &=& 39 \quad \text{(VI)} \\ x^2 + y^2 + 8x - 6y &=& 39 \quad \text{(V)} \\ - \quad - \quad - \quad + \quad & & - \\ \hline -16x &=& 0 \end{array}$$

$\Rightarrow \quad x = 0$

Putting $x = 0$ in (V), we have

$(0)^2 + y^2 + 8(0) - 6y = 39$

$\Rightarrow \quad y^2 - 6y - 39 = 0$

$D = b^2 - 4ac$ $(a = 1, b = -6, c = -39)$

$= (-6)^2 - 4(1)(-39) = 36 + 156$

$\Rightarrow \quad D = 192$

$\Rightarrow \quad \sqrt{D} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3}$

$\Rightarrow \quad \sqrt{D} = 8\sqrt{3}$

$\therefore \quad y = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{6 \pm 8\sqrt{3}}{2 \times 1} = \dfrac{2(3 \pm 4\sqrt{3})}{2}$

$\Rightarrow \quad y_1 = 3 + 4\sqrt{3}$ and $y_2 = 3 - 4\sqrt{3}$

Hence, the third vertex of ΔABC may be $C(0, 3 + 4\sqrt{3})$ and $C'(0, 3 - 4\sqrt{3})$.

Now, $C(0, 3 + 4\sqrt{3})$

$= C(0, 3 + 4 \times 1.732)$

$= C(0, 3 + 6.9)$

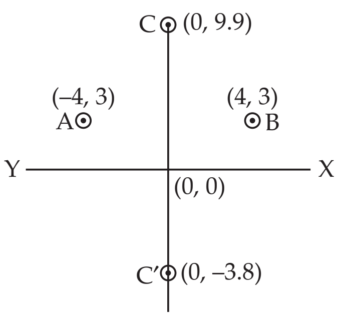
$= C(0, 9.9)$

and $C'(0, 3 - 4\sqrt{3})$

$= C'(0, 3 - 4 \times 1.732)$

$= C'(0, 3 - 6.9)$

$= C'(0, -3.9)$

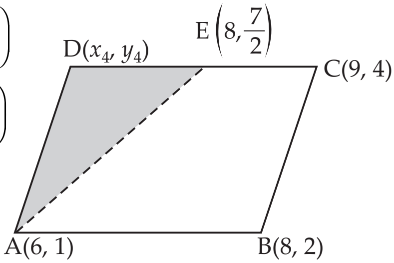


So, the required point so that origin lies inside it is $(0, 3 - 4\sqrt{3})$.

Q2. A(6, 1), B(8, 2) and C(9, 4) are three vertices of a parallelogram ABCD. If E is the mid point of DC, then find the area of ΔADE.

Sol. ABCD is a parallelogram so

[Mid point of diagonal BD] = [Mid point of diagonal AC]



$\therefore$ Mid point of BD $= \left(\dfrac{x_4+8}{2}, \dfrac{y_4+2}{2}\right)$

and Mid point of AC $= \left(\dfrac{6+9}{2}, \dfrac{1+4}{2}\right)$

$\Rightarrow \dfrac{x_4+8}{2} = \dfrac{15}{2}$ and $\dfrac{y_4+2}{2} = \dfrac{5}{2}$

$\Rightarrow \quad x_4 = 15 - 8$ and $y_4 = 5 - 2$

$\Rightarrow \quad x_4 = 7$ and $y_4 = 3$

$\therefore \quad D = (7, 3)$

$$\text{Mid point of DC is } E\left(\frac{x_4+9}{2}, \frac{y_4+4}{2}\right)$$

$$= E\left(\frac{7+9}{2}, \frac{3+4}{2}\right)$$

$$= E\left(\frac{16}{2}, \frac{7}{2}\right) = E\left(8, \frac{7}{2}\right)$$

$$\text{Now, Area of } \Delta ADE = \frac{1}{2}\left[6\left(3-\frac{7}{2}\right)+7\left(\frac{7}{2}-1\right)+8(1-3)\right]$$

$$= \frac{1}{2}\left[6\left(\frac{-1}{2}\right)+7\left(\frac{5}{2}\right)+8(-2)\right]$$

$$= \frac{1}{2}\left(3+\frac{35}{2}-16\right) = \frac{1}{2}\left(\frac{-6+35-32}{2}\right)$$

$$= \frac{1}{2}\times\frac{(-3)}{2} = \frac{-3}{4} \text{ sq units} = \frac{3}{4} \text{ sq. units}$$

[In magnitude]

Hence, the area of ΔADE is $\dfrac{3}{4}$ sq. units.

Q3. The points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of ΔABC.

(*i*) The median from A meets BC at D. Find the coordinates of the point D.

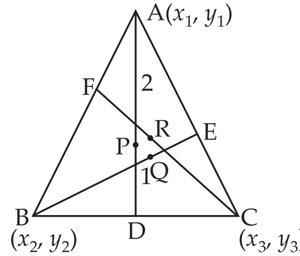
(*ii*) Find the coordinates of the point P on AD such that AP : PD = 2 : 1.

(*iii*) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.

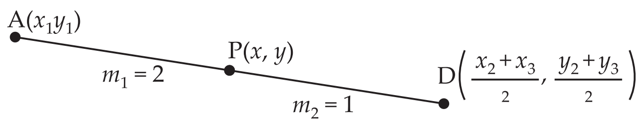
(*iv*) What are the coordinates of the centroid of the ΔABC?

Sol. (*i*) Median from A meets BC at D *i.e.*, D is the mid-point of BC.

So, the coordinates of D are given by $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$



(*ii*)



The coordinates of the point P on AD such that AP : PD = 2 : 1 are given by

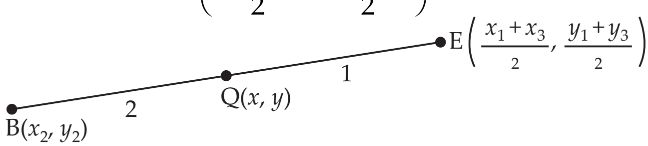
$$x = \frac{2\left(\frac{x_2+x_3}{2}\right)+1(x_1)}{2+1}, \qquad y = \frac{2\left(\frac{y_2+y_3}{2}\right)+1(y_1)}{2+1}$$

$$\Rightarrow \quad x = \frac{x_2+x_3+x_1}{3}, \qquad y = \frac{y_2+y_3+y_1}{3}$$

$\therefore$ P$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ is the required point.

(*iii*) (*a*) Median BE meets the side AC at its mid-point E.

$\therefore$ Coordinates of E are $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$.



Now, the coordinates of Q such that BE is median and BQ : QE = 2 : 1 are given by

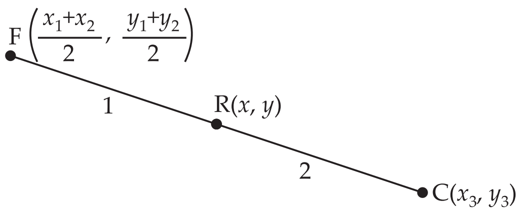
$$x = \frac{2\left(\frac{x_1+x_3}{2}\right)+1(x_2)}{2+1}, \qquad y = \frac{2\left(\frac{y_1+y_3}{2}\right)+1(y_2)}{2+1}$$

$$\Rightarrow \quad x = \frac{x_1+x_3+x_2}{3}, \qquad y = \frac{y_1+y_3+y_2}{3}$$

$\therefore$ The coordinates of point *Q* on median BE such at QB : QE = 2 : 1 are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

(*b*) Median CF meets the side AB at its mid-point F.

$\therefore$ Coordinate of F are $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$.



Now, the coordinates of R such that CF is median and CR : RF = 2 : 1 are given by

$$x = \frac{1(x_3) + 2\left(\dfrac{x_1+x_2}{2}\right)}{1+2}, \qquad y = \frac{1(y_3) + 2\left(\dfrac{y_1+y_2}{2}\right)}{1+2}$$

$$\Rightarrow \quad x = \frac{x_3+x_1+x_2}{3}, \qquad y = \frac{y_3+y_1+y_2}{3}$$

So, the coordinates of point R on the median CF such that CR : RF = 2 : 1 are $\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$

(*iv*) Coordinates of centroid G of ΔABC are

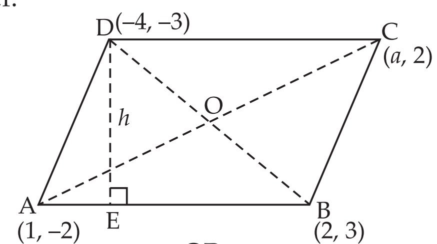
$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

It is observed that coordinates of P, Q, R and G are same.

Hence, the medians intersect at the same point *i.e.*, centroid which divides the medians in the ratio 2 : 1.

Q4. If the points A(1, –2), B(2, 3), C(*a*, 2) and D(–4, –3) form a parallelogram, find the value of *a* and height of the parallelogram taking AB as base.

Sol. As ABCD is a parallelogram and diagonals of parallelogram bisect each other.



OR

The mid points of diagonals of parallelogram will coincide i.e.,

Mid-point of diagonal AC = Mid-point of diagonal BD

$$\Rightarrow \quad \left(\frac{1+a}{2}, \frac{-2+2}{2}\right) = \left(\frac{-4+2}{2}, \frac{-3+3}{2}\right)$$

$$\Rightarrow \quad \left(\frac{1+a}{2}, 0\right) = \left(\frac{-2}{2}, 0\right)$$

$$\Rightarrow \quad \frac{1+a}{2} = \frac{-2}{2}$$

$$\Rightarrow \quad a = -2 - 1 = -3$$

Hence, the value of a is -3.

Now, Area of ΔABD $= \frac{1}{2}$ base $\times$ altitude

$$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = \frac{1}{2}\text{AB} \times h$$

$$\Rightarrow \frac{1}{2}[1\{3-(-3)\} + 2\{-3-(-2)\} - 4(-2-3)] = -\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow \quad \frac{1}{2}[(3+3) + 2(-3+2) - 4(-5)] = \frac{h}{2}\sqrt{(2-1)^2 + (3+2)^2}$$

$$\Rightarrow \quad \frac{1}{2}[6 + 2(-1) + 20] = \frac{h}{2}\sqrt{(1)^2 + (5)^2}$$

$$\Rightarrow \quad \frac{1}{2}[6 - 2 + 20] = \frac{h}{2}\sqrt{1+25}$$

$$\Rightarrow \quad \frac{1}{2}[26 - 2] = \frac{h}{2}\sqrt{26}$$

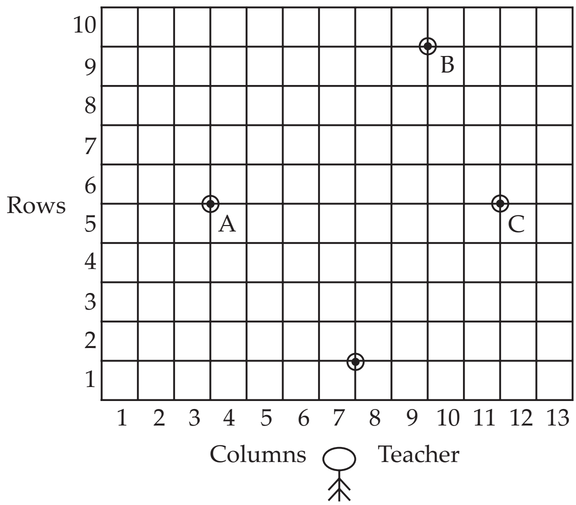
$$\Rightarrow \quad h\sqrt{26} = 24$$

$$\Rightarrow \quad h = \frac{24}{\sqrt{26}} \times \frac{\sqrt{26}}{\sqrt{26}} = \frac{24\sqrt{26}}{26}$$

$$\Rightarrow \quad h = \frac{12}{13}\sqrt{26} \text{ units}$$

Hence, the perpendicular distance between parallel sides AB and CD is $\frac{12\sqrt{26}}{13}$ units.

Q5. Student of a school are standing in rows and columns in their playground for a drill practice. A, B, C, D are the positions of four students as shown in the figure. Is it possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B C and D? If so, what should be his position?



Sol. Coordinates of A, B, C and D from graph are A(3, 5), B(7, 9), C(11, 5), and D(7, 1).

To find the shape of □ABCD:

$$AB^2 = (7-3)^2 + (9-5)^2 = 4^2 + 4^2 = 4^2(1+1)$$

$\Rightarrow \quad AB = 4\sqrt{2}$ units

$$BC^2 = (11-7)^2 + (5-9)^2 = (4)^2 + (-4)^2 = 4^2(1+1)$$

$\Rightarrow \quad BC = 4\sqrt{2}$ units

$$CD^2 = (7-11)^2 + (1-5)^2 = (-4)^2 + (-4)^2 = 4^2 + 4^2$$

$\Rightarrow \quad CD = 4\sqrt{2}$ units

$$DA^2 = (7-3)^2 + (1-5)^2 = 4^2 + (-4)^2 = 4^2 + 4^2$$

$\Rightarrow \quad DA = \sqrt{4^2(1+1)} = 4\sqrt{2}$ units

$\therefore \quad AB = BC = CD = DA = 4\sqrt{2}$ units.

So, ABCD will be either square or rhombus.

Now, Diagonal AC = $\sqrt{(11-3)^2 + (5-5)^2}$

$\Rightarrow \quad AC = \sqrt{(8)^2 + (0)^2}$

$\Rightarrow \quad AC = 8$ units

and diagonal BD = $\sqrt{(7-7)^2 + (1-9)^2} = \sqrt{(0)^2 + (8)^2} = \sqrt{0 + (8)^2} = \sqrt{8^2}$

$\Rightarrow \quad BD = 8$ units

$\therefore \quad$ Diagonal AC = Diagonal BD

So, the given quadrilateral ABCD is a square. The point which is equidistant from point A, B, C, D of a square ABCD will be at the intersecting point of diagonals and diagonals bisect each other.

Hence, the required point O equidistant from A, B, C, D is mid point of any diagonal $= \left(\frac{7+7}{2}, \frac{9+1}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$.

Hence, the required point is (7, 5).

Q6. Ayush starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. What is the extra distance travelled by Ayush in reaching his office? (Assume that all distances covered are in straight lines). If the house is situated at (2, 4), bank at (5, 8) school at (13, 14) and office at (13, 26) and coordinates are in km.

Sol. Consider the coordinates of house H(2, 4), bank B(5, 8), school S(13, 14) and office O(13, 26).

$$\text{Distance } HB^2 = (5-2)^2 + (8-4)^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$\Rightarrow \quad HB = 5 \text{ km}$$

$$\text{Distance } BS^2 = (13-5)^2 + (14-8)^2 = 8^2 + 6^2 = 64 + 36$$

$$\Rightarrow \quad BS^2 = 100$$

$$\Rightarrow \quad BS = 10 \text{ km}$$

$$\text{Distance } SO^2 = (13-13)^2 + (26-14)^2 = 0^2 + 12^2 = 12^2$$

$$\Rightarrow \quad SO = 12 \text{ km}$$

Total distance travelled by Ayush from house to bank to school and then to office

$$= HB + BS + SO$$

$$= 5 + 10 + 12 = 27 \text{ km}$$

Direct distance from house to office = HO

$$\Rightarrow \quad HO^2 = (13-2)^2 + (26-4)^2 = (11)^2 + (22)^2$$

$$\Rightarrow \quad HO^2 = 121 + 484$$

$$\Rightarrow \quad HO = \sqrt{605} = 24.6 \text{ km}$$

So, extra distance travelled by Ayush = 27 km – 24.6 km = 2.4 km.

Hence, extra distance travelled by Ayush = 2.4 km