

MATHEMATICS

TIME: 3 Hrs

MAXIMUM MARKS : 80

General instructions:

- All questions are compulsory.
- The question paper consists of 30 questions divided into 4 sections, A, B, C and D.
- Section-A consists of 6 questions of 1 mark each. Section-B consists of 6 questions of 2 marks each. Section-C consists of 10 questions of 3 marks each and Section-D consists of 8 questions of 4 marks each.
- Use of calculator is not permitted.

SECTION-A (1 × 6 = 6 Marks)

1. Find the value of k if one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3 .
2. If positive integer a and b are written as $a = x^3y^2$ and $b = xy^3$, where x and y are prime numbers, then find the H.C.F. of a and b .
3. For what value of k , do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines?
4. If the n^{th} term of an a.p. is $3n-2$, find the sum of first 20 terms.
5. If angle between two tangents drawn from a point P to a circle of radius $\sqrt{3}$ cm and centre O is 60° . Find the length of OP .
6. State the following statement true or false with reasons: " $\sqrt{1.44}$ is an irrational number."

SECTION - B (2 × 6 = 12 Marks)

7. If one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is 0, then find the product of other two zeroes.
8. Find the altitude of an equilateral triangle of side 8 cm.
9. If a pole 6m high casts a shadow $2\sqrt{3}$ m long on the ground, then find the Sun's elevation.
10. Prove that $(\sqrt{3} + 1)(3 - \cot 30^\circ) = \tan^3 60^\circ - 2\sin 60^\circ$.
11. Prove that $(\sin \alpha + \cos \alpha)(\tan \alpha + \cot \alpha) = \sec \alpha + \operatorname{cosec} \alpha$.
12. If $\sqrt{3}\tan \theta = 1$, then find the value of $\sin^2 \theta - \cos^2 \theta$.

SECTION - C (3 × 10 = 30 Marks)

13. Find the largest number which divides 70 and 125 leaving remainder 5 and 8 respectively.
14. Show that one and only one out of $n, n+4, n+8, n+12$ and $n+16$ is divisible by 5, where n is any positive integer.
15. If the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form $a, a + b$ and $a + 2b$ for some real numbers a and b , find the values of a and b .

16. Two numbers are in the ratio 5: 6. If 8 is subtracted from each of the numbers, the ratio becomes 4: 5. Find the numbers.

17. How many numbers lie between 10 and 300, which ^{if} divided by 4 leave a remainder 3.

18. If A and B are respectively the points on the sides PQ and PR of a ΔPQR such that $PQ = 12.5\text{cm}$, $PA = 5\text{cm}$, $BR = 6\text{cm}$ and $PB = 4\text{cm}$. Is $AB \parallel QR$?

19. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

20. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

21. A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is lost at random. Find the probability that it is a

- i) Triangle
- ii) Square of blue colour
- iii) Square

22. Show that: $\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{\operatorname{cosec} A + \cot A - 1} = 1$

SECTION - D (4 × 8 = 32 Marks)

23. If $(x - \sqrt{5})$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, then find all the zeroes of the polynomial.

24. Draw the graph of the pair of equations $2x + y = 4$ and $2x - y = 4$. Write the vertices of the triangle formed by these lines and the y-axis. Find the area of this triangle.

25. If the first term of an AP is a, the second term is b and the last term is c, then show that the sum of the AP is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$.

26. If s denotes the semi-perimeter of a ΔABC in which $BC = a$, $CA = b$ and $AB = c$ and if a circle touches the sides BC, CA, AB at D, E, F respectively. Prove that $BD = s - b$.

27. In ΔPQR , $PD \perp QR$ such that D lies on QR, if $PQ = a$, $PR = b$, $QD = c$ and $DR = d$, then prove that $(a + b)(a - b) = (c + d)(c - d)$.

28. Prove that the area of the equilateral triangle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the equilateral triangle drawn on the other two sides of the triangle.

29. Prove that: $\left(\frac{1 + \sin x - \cos x}{1 + \sin x + \cos x}\right)^2 = \frac{1 - \cos x}{1 + \cos x}$

30. Prove that $\frac{\sin \theta}{\cos \theta + 1} + \frac{1 + \cos \theta}{\sin \theta} = 2 \operatorname{cosec} \theta$.