EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. Let A = {– 1, 2, 3} and B = {1, 3}. Determine

(*i*) $A \times B$ (*ii*) $B \times A$ (*iii*) $B \times B$ (*iv*) $A \times A$

Sol. Given that: A = {– 1, 2, 3} and B = {1, 3}

(*i*) $A \times B = \{(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)\}$

(*ii*) $B \times A = \{(1, -1), (3, -1), (1, 2), (3, 2), (1, 3), (3, 3)\}$

(*iii*) $B \times B = \{1, 3\} \times \{1, 3\} = \{(1, 1), (1, 3), (3, 1), (3, 3)\}$

(*iv*) $A \times A = \{-1, 2, 3\} \times \{-1, 2, 3\}$

$= \{(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)\}$

Q2. If $P = \{x : x < 3, x \in N\}$, $Q = \{x : x \leq 2, x \in W\}$. Find $(P \cup Q) \times (P \cap Q)$ where W is the set of whole numbers.

Sol. Given that: $P = \{x : x < 3, x \in N\} \Rightarrow P = \{1, 2\}$

$Q = \{x : x \leq 2, x \in W\} \Rightarrow Q = \{0, 1, 2\}$

Now $(P \cup Q) = \{0, 1, 2\}$ and $(P \cap Q) = \{1, 2\}$

$\therefore (P \cup Q) \times (P \cap Q) = \{(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)\}$

Q3. If $A = \{x : x \in W, x < 2\}$, $B = \{x : x \in N, 1 < x < 5\}$, and $C = \{3, 5\}$, find

(*i*) $A \times (B \cap C)$ (*ii*) $A \times (B \cup C)$

Sol. Given that: $A = \{x : x \in W, x < 2\} \Rightarrow A = \{0, 1\}$

$B = \{x : x \in N, 1 < x < 5\} \Rightarrow B = \{2, 3, 4\}$

$C = \{3, 5\}$

(*i*) $A \times (B \cap C) = \{0, 1\} \times \{3\} = \{(0, 3), (1, 3)\}$

(*ii*) $A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\}$

$= \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$

Q4. In each of the following cases, find *a* and *b*

(*i*) $(2a + b, a - b) = (8, 3)$ (*ii*) $\left(\frac{a}{4}, a - 2b\right) = (0, 6 + b)$

Sol. (*i*) Given that: $(2a + b, a - b) = (8, 3)$

Comparing the domains and ranges, we get

$$2a + b = 8 \quad \text{...}(i)$$

$$a - b = 3 \quad \text{...}(ii)$$

Solving (*i*) and (*ii*) we get $a = \frac{11}{3}$ and $b = \frac{2}{3}$

(*ii*) Given that: $\left(\frac{a}{4}, a - 2b\right) = (0, 6 + b)$

Comparing the domains and ranges, we get

$\frac{a}{4} = 0 \Rightarrow a = 0,\ a - 2b = 6 + b$

$\Rightarrow \quad a - 3b = 6 \Rightarrow 0 - 3b = 6$

$\therefore \quad b = -2.$

So, $a = 0$, $b = -2$.

Q5. Given A = {1, 2, 3, 4, 5}, S = {$(x, y) : x \in$ A, $y \in$ A}. Find the ordered pairs which satisfy the condition given below:

(*i*) $x + y = 5$ (*ii*) $x + y < 5$ (*iii*) $x + y > 8$

Sol. Given that: A = {1, 2, 3, 4, 5}

and S = {$(x, y) : x \in$ A, $y \in$ A}

(*i*) $x + y = 5$, so, the ordered pairs satisfying the given conditions are (1, 4), (4, 1), (2, 3), (3, 2).

(*ii*) $x + y < 5$, so, the ordered pairs satisfying the given conditions are (1, 1), (1, 2), (2, 1), (1, 3), (2, 2), (3, 1).

(*iii*) $x + y > 8$, so the ordered pairs satisfying the given conditions are (4, 5), (5, 4), (5, 5).

Q6. Given R = {$(x, y) : x, y \in$ W, $x^2 + y^2 = 25$}, find the domain and range of R.

Sol. Given that: R = {$(x, y) : x, y \in$ W, $x^2 + y^2 = 25$}

So, the ordered pairs satisfying the given condition $x^2 + y^2 = 25$ are (0, 5), (3, 4), (5, 0), (4, 3) {$\because x, y \in$ W}

Hence, the domain = {0, 3, 4, 5} and the range = {0, 3, 4, 5}.

Q7. If R_1 = {$(x, y) \mid y = 2x + 7$ where $x \in$ R and $-5 \le x \le 5$} is a relation. Then find the domain and range.

Sol. Given that: R_1 = {$(x, y) \mid y = 2x + 7$ where $x \in$ R and $-5 \le x \le 5$}

Here domain is $-5 \le x \le 5 \Rightarrow$ {– 5, – 4, – 3, – 2, – 1, 0, 1, 2, 3, 4, 5}

and $y = 2x + 7$.

So, the values of y for the corresponding given values of x are {– 3, –1, 1, 3, 5, 7, 9, 11, 13, 15, 17}

Hence, the domain of R_1 = [– 5, 5] and range of R_1 = [– 3, 17]

Q8. If R_2 = {$(x, y) \mid x$ and y are integers and $x^2 + y^2 = 64$} is a relation, then find R_2.

Sol. Given that: $x^2 + y^2 = 64$, $x, y \in$ Z

Since the sum of the squares of two integers is 64

$\therefore$ For $x = 0$, $y = \pm 8$

For $x = \pm 8$, $y = 0$

Hence, R_2 = {(0, 8), (0, – 8), (8, 0), (– 8, 0)}

Q9. If $R_3 = \{(x, |x|) \,|\, x \text{ is a real number}\}$ is a relation, then find domain and range of R_3.

Sol. Given that: $R_3 = \{(x, |x|) \,|\, x \text{ is a real number}\}$

Clearly, domain of $R_3 = R$

and Range of $R_3 = (0, \infty)$ $[\because |x| = R_+]$

Q10. Is the given relation a function? Give reason for your answer:

(*i*) $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$

(*ii*) $f = \{(x, x) \mid x \text{ is a real number}\}$

(*iii*) $g = \left\{\left(n, \frac{1}{n}\right) \middle| n \text{ is a positive integer}\right\}$

(*iv*) $s = \{(n, n^2) \mid n \text{ is a positive integer}\}$

(*v*) $t = \{(x, 3) \mid x \text{ is a real number}\}$

Sol. Given that: (*i*) $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$

Since in the given relation 3 has two images 9 and 11. So, h is not a function.

(*ii*) $f = \{(x, x) \mid x \text{ is a real number}\}$. Here, we observe that for every element of domain has a unique image. So, f is a function.

(*iii*) Given that: $g = \left\{\left(n, \frac{1}{n}\right) \middle| n \text{ is a positive integer}\right\}$.

Here, we observe that n is a positive integer so, for every element of domain, there is a unique $\frac{1}{n}$ image. Hence g is a function.

(*iv*) Given that: $S = \{(n, n^2) \mid n \text{ is a positive integer}\}$

Here, we observe that the square of any integer is a unique number. So, for every element element in the domain there is unique image. Hence, S is a function.

(*v*) Given that: $t = \{(x, 3) \mid x \text{ is a real number}\}$

Here, we observe that for every real element in the domain, there is a constant number 3. Hence t is a constant function.

Q11. If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$, find each of the following:

(*i*) $f(3) + g(-5)$ (*ii*) $f\left(\frac{1}{2}\right) \times g(14)$ (*iii*) $f(-2) + g(-1)$

(*iv*) $f(t) - f(-2)$ (*v*) $\frac{f(t) - f(5)}{t - 5}$, if $t \neq 5$

Sol. Given that: $f(x) = x^2 + 7$ and $g(x) = 3x + 5$

(*i*) $f(3) + g(-5) = [(3)^2 + 7] + [3(-5) + 5]$

$= (9 + 7) + (-15 + 5) = 16 - 10 = 6$

Hence, $f(3) + g(-5) = 6$

(*ii*) $f\left(\frac{1}{2}\right)\times g(14) = \left[\left(\frac{1}{2}\right)^2+7\right]\times[3\times14+5]$

$= \left(\frac{1}{4}+7\right)\times(42+5) = \frac{29}{4}\times 47 = \frac{1363}{4}$

Hence, $f\left(\frac{1}{2}\right)\times g(14) = \frac{1363}{4}$

(*iii*) $f(-2)+g(-1) = [(-2)^2+7]+[3(-1)+5]$

$= (4+7)+(-3+5) = 11+2 = 13$

Hence, $f(-2)+g(-1) = 13$

(*iv*) $f(t)-f(-2) = (t^2+7)-[(-2)^2+7] = t^2+7-11$

$= t^2-4$

Hence, $f(t)-f(-2) = t^2-4$.

(*v*) $\frac{f(t)-f(5)}{t-5}, t\neq 5 = \frac{(t^2+7)-((5)^2+7)}{t-5}$

$= \frac{t^2+7-32}{t-5} = \frac{t^2-25}{t-5} = t+5$

Hence, $\frac{f(t)-f(5)}{t-5}, t\neq 5 = t+5$.

Q12. Let f and g be real functions defined by $f(x) = 2x+1$ and $g(x) = 4x-7$

(*i*) For what real numbers, $f(x) = g(x)$?

(*ii*) For what real numbers, $f(x) < g(x)$?

Sol. Given that: $f(x) = 2x+1$ and $g(x) = 4x-7$

(*i*) For $f(x) = g(x)$, we get

$2x+1 = 4x-7 \Rightarrow 2x-4x = -7-1$

$\Rightarrow -2x = -8$

$\Rightarrow x = 4$. Hence, the required real number is 4.

(*ii*) For $f(x) < g(x)$, we get

$2x+1 < 4x-7$

$\Rightarrow 2x-4x < -1-7 \Rightarrow -2x < -8 \Rightarrow 2x > 8$

$\therefore x > 4$

Hence, the required real number is $x > 4$.

Q13. If f and g are two real valued functions defined as $f(x) = 2x+1$, $g(x) = x^2+1$ then find

(*i*) $f+g$ (*ii*) $f-g$ (*iii*) $f.g$ (*iv*) $\frac{f}{g}$

Sol. Given that: $f(x) = 2x+1$ and $g(x) = x^2+1$

(*i*) $f+g = f(x)+g(x) \Rightarrow 2x+1+x^2+1 \Rightarrow x^2+2x+2$

(*ii*) $f-g = f(x)-g(x) \Rightarrow (2x+1)-(x^2+1) = 2x+1-x^2-1 \Rightarrow 2x-x^2$

(*iii*) $f.g = f(x).g(x) \Rightarrow (2x+1)(x^2+1) \Rightarrow 2x^3 + x^2 + 2x + 1$

(*iv*) $\dfrac{f}{g} = \dfrac{f(x)}{g(x)} = \dfrac{2x+1}{x^2+1}$

Q14. Express the following functions as set of ordered pairs and determine their range

$f : X \rightarrow R, f(x) = x^3 + 1$, where $X = \{-1, 0, 3, 9, 7\}$.

Sol. Given that: $f : X \rightarrow R, f(x) = x^3 + 1$, where $X = \{-1, 0, 3, 9, 7\}$

Here $X = \{-1, 0, 3, 9, 7\}$

For $x = -1$, $f(-1) = (-1)^3 + 1 = 0$

For $x = 0$, $f(0) = (0)^3 + 1 = 1$

For $x = 3$, $f(3) = (3)^3 + 1 = 28$

For $x = 9$, $f(9) = (9)^3 + 1 = 730$

For $x = 7$ $f(7) = (7)^3 + 1 = 344$

$\therefore$ The ordered pairs are $(-1, 0), (0, 1), (3, 28), (7, 344), (9, 730)$ and the range $= \{0, 1, 28, 344, 730\}$.

Q15. Find the values of x for which the functions $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal?

Sol. Given that: $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$

Since $f(x) = g(x)$ (given)

$\Rightarrow$ $3x^2 - 1 = 3 + x \Rightarrow 3x^2 - x - 4 = 0$

$\Rightarrow$ $3x^2 - 4x + 3x - 4 = 0 \Rightarrow x(3x-4) + 1(3x-4) = 0$

$\Rightarrow$ $(3x-4)(x+1) = 0 \Rightarrow 3x - 4 = 0$ or $x + 1 = 0$

$\Rightarrow$ $3x = 4$ or $x = -1$

$\therefore$ $x = \dfrac{4}{3}$

Hence, the value of x are -1 and $\dfrac{4}{3}$.

LONG ANSWER TYPE QUESTIONS

Q16. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? Justify: If this is described by the relation $g(x) = \alpha x + \beta$, then what values should be assigned to α and β?

Sol. Given that: $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$

Since every element of the domain in this relations has unique image, so g is a function.

Now $g(x) = \alpha x + \beta$

For (1, 1) $g(1) = \alpha(1) + \beta = 1 \Rightarrow \alpha + \beta = 1$ ...(*i*)

For (2, 3) $g(2) = \alpha(2) + \beta = 3 \Rightarrow 2\alpha + \beta = 3$ ...(*ii*)

Solving eqn. (*i*) and (*ii*) we have

$\alpha = 2$ and $\beta = -1$

[Note: We can take any other two ordered pairs]

Hence, the value of $\alpha = 2$ and $\beta = -1$.

Q17. Find the domain of each of the following functions given by:

(*i*) $f(x) = \dfrac{1}{\sqrt{1 - \cos x}}$ (*ii*) $f(x) = \dfrac{1}{\sqrt{x + |x|}}$ (*iii*) $f(x) = x|x|$

(*iv*) $f(x) = \dfrac{x^3 - x + 3}{x^2 - 1}$ (*v*) $f(x) = \dfrac{3x}{28 - x}$

Sol. (*i*) Given that: $f(x) = \dfrac{1}{\sqrt{1 - \cos x}}$

We know that $-1 \le \cos x \le 1$

$\Rightarrow \quad 1 \ge -\cos x \ge -1$

$\Rightarrow \quad 1 + 1 \ge 1 - \cos x \ge -1 + 1$

$\Rightarrow \quad 2 \ge 1 - \cos x \ge 0$

$\Rightarrow \quad 0 \le 1 - \cos x \le 2$

For real value of domain

$1 - \cos x \ne 0 \quad \Rightarrow \quad \cos x \ne 1$

$\Rightarrow \quad x \ne 2n\pi \quad \forall\, n \in Z$

Hence, the domain of $f = R - \{2n\pi, n \in Z\}$

(*ii*) Given that: $f(x) = \dfrac{1}{\sqrt{x + |x|}}$

$\because \quad x + |x| = x + x = 2x$ if $x \ge 0$

and $\quad x + |x| = x - x = 0$ if $x < 0$

So far $x < 0$, f is not defined.

Hence, the domain $f = R^+$.

(*iii*) Given that: $f(x) = x|x|$

It is clear that $f(x)$ is defined for all $x \in R$.

Hence, the domain of $f = R$.

(*iv*) Given that: $f(x) = \dfrac{x^3 - x + 3}{x^2 - 1}$

Here, $f(x)$ is only defined if $x^2 - 1 \ne 0$

$(x - 1)(x + 1) \ne 0$

$\therefore \quad x \ne 1, x \ne -1$

Hence, the domain of $f = R - \{-1, 1\}$

(*v*) Given that: $f(x) = \dfrac{3x}{28 - x}$

Here, $f(x)$ is only defined if $28 - x \ne 0 \Rightarrow x \ne 28$

Hence, the domain $= R - \{28\}$.

Q18. Find the range of the following functions given by

(*i*) $f(x) = \dfrac{3}{2 - x^2}$ (*ii*) $f(x) = 1 - |x - 2|$

(*iii*) $f(x) = |x - 3|$ (*iv*) $f(x) = 1 + 3\cos 2x$

Sol. (*i*) Given that: $f(x) = \dfrac{3}{2-x^2}$

Let $y = f(x)$ $\therefore$ $y = \dfrac{3}{2-x^2}$

$\Rightarrow$ $y(2-x^2) = 3 \Rightarrow 2y - yx^2 = 3$

$\Rightarrow$ $yx^2 = 2y - 3$

$\Rightarrow$ $x^2 = \dfrac{2y-3}{y} \Rightarrow x = \sqrt{\dfrac{2y-3}{y}}$

Here, x is real if $2y - 3 \geq 0$ and $y \geq 0$

$\Rightarrow$ $y \geq \dfrac{3}{2}$

Hence, the range of $f = \left[\dfrac{3}{2}, \infty\right)$.

(*ii*) Given that: $f(x) = 1 - |x-2|$

We know that $|x-2| = -(x-2)$ if $x < 2$

and $|x-2| = (x-2)$ if $x \geq 2$

$\therefore$ $-|x-2| \leq 0 \Rightarrow 1 - |x-2| \leq 1$

Hence, the range of $f = (-\infty, 1]$.

(*iii*) Given that: $f(x) = |x-3|$

We know that $|x-3| \geq 0 \Rightarrow f(x) \geq 0$

Hence, the range of $f = [0, \infty)$

(*iv*) Given that: $f(x) = 1 + 3\cos 2x$

We know that $-1 \leq \cos 2x \leq 1$

$\Rightarrow$ $-3 \leq 3\cos 2x \leq 3 \Rightarrow -3 + 1 \leq 1 + 3\cos 2x \leq 3 + 1$

$\Rightarrow$ $-2 \leq 1 + 3\cos 2x \leq 4 \Rightarrow -2 \leq f(x) \leq 4$

Hence, the range of $f = [-2, 4]$.

Q19. Redefine the function $f(x) = |x-2| + |2+x|,\ -3 \leq x \leq 3$.

Sol. Given that: $f(x) = |x-2| + |2+x|,\ -3 \leq x \leq 3$

Since $|x-2| = -(x-2),\ x < 2$

and $|x-2| = (x-2),\ x \geq 2$

$|2+x| = -(2+x),\ x < -2$

and $|2+x| = (2+x),\ x \geq -2$

Now $f(x) = |x-2| + |2+x|,\ -3 \leq x \leq 3$.

$$= \begin{cases} -(x-2)-(2+x), & -3 \leq x < -2 \\ -(x-2)+(2+x), & -2 \leq x < 2 \\ (x-2)+(2+x), & 2 \leq x \leq 3 \end{cases}$$

$$\therefore \quad f(x) = \begin{cases} -2x, & -3 \leq x < -2 \\ 4, & -2 \leq x < 2 \\ 2x, & 2 \leq x \leq 3 \end{cases}$$

Q20. If $f(x) = \frac{x-1}{x+1}$, then show that

(*i*) $f\left(\frac{1}{x}\right) = -f(x)$ (*ii*) $f\left(\frac{-1}{x}\right) = \frac{-1}{f(x)}$

Sol. Given that: $f(x) = \frac{x-1}{x+1}$

(*i*) $$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1} = \frac{1-x}{1+x} = \frac{-(x-1)}{x+1} = -f(x)$$

Hence, $f\left(\frac{1}{x}\right) = -f(x)$

(*ii*) $$f\left(\frac{-1}{x}\right) = \frac{-\frac{1}{x}-1}{-\frac{1}{x}+1} = \frac{-\left(\frac{1}{x}+1\right)}{-\left(\frac{1}{x}-1\right)} = \frac{1+x}{1-x} = \frac{1}{\frac{1-x}{1+x}}$$

$$= \frac{1}{-\left(\frac{x-1}{x+1}\right)} = \frac{-1}{f(x)}$$

Hence, $f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$.

Q21. Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined in the domain $R^+ \cup \{0\}$.

(*i*) $(f+g)(x)$ (*ii*) $(f-g)(x)$ (*iii*) $(f.g)(x)$ (*iv*) $\left(\frac{f}{g}\right)(x)$

Sol. Given that: $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined in the domain $R^+ \cup \{0\}$

(*i*) $(f+g)(x) = f(x) + g(x) = \sqrt{x} + x$

(*ii*) $(f-g)(x) = f(x) - g(x) = \sqrt{x} - x$

(*iii*) $(fg)(x) = f(x).g(x) = \sqrt{x}\,.x = x^{3/2}$

(*iv*) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$.

Q22. Find the domain and range of the function $f(x) = \frac{1}{\sqrt{x-5}}$.

Sol. Given that: $f(x) = \frac{1}{\sqrt{x-5}}$

Here, it is clear that $f(x)$ is real when $x - 5 > 0 \Rightarrow x > 5$

Hence, the domain = $(5, \infty)$

Now to find the range put

$$f(x) = y = \frac{1}{\sqrt{x-5}}$$

$$\Rightarrow \quad \sqrt{x-5} = \frac{1}{y} \Rightarrow x - 5 = \frac{1}{y^2}$$

$$\Rightarrow \quad x = \frac{1}{y^2} + 5$$

For $x \in (5, \infty)$, $y \in R^+$.

Hence, the range of f = R^+.

Q23. If $f(x) = y = \dfrac{ax-b}{cx-a}$, then prove that $f(y) = x$.

Sol. Given that: $f(x) = y = \dfrac{ax-b}{cx-a}$

$$f(y) = \frac{ay-b}{cy-a} = \frac{a\left[\dfrac{ax-b}{cx-a}\right]-b}{c\left[\dfrac{ax-b}{cx-a}\right]-a}$$

$$\Rightarrow \quad f(y) = \frac{a^2x - ab - bcx + ab}{cax - bc - cax + a^2} = \frac{a^2x - bcx}{a^2 - bc}$$

$$\Rightarrow \quad f(y) = \frac{x(a^2 - bc)}{a^2 - bc} = x.$$

Hence, $f(y) = x$.

OBJECTIVE TYPE QUESTIONS

Choose the correct answer out of the given four options in each of the Exercises from 24 to 35 (M.C.Q.)

Q24. Let $n(A) = m$ and $n(B) = n$, then the total number of non-empty relations that can be defined from A to B is

(*a*) m^n (*b*) $n^m - 1$ (*c*) $mn - 1$ (*d*) $2^{mn} - 1$

Sol. Given that: $n(A) = m$ and $n(B) = n$

$\therefore \quad n(A \times B) = n(A) \,.\, n(B) = mn$

So, the total number of relations from A to B = $2^{mn} - 1$.

Hence, the correct option is (*d*).

Q25. If $[x]^2 - 5[x] + 6 = 0$, where [.] denotes the greatest integer function, then

(*a*) $x \in [3, 4]$ (*b*) $x \in (2, 3]$ (*c*) $x \in [2, 3]$ (*d*) $x \in [2, 4)$

Sol. We have $[x]^2 - 5[x] + 6 = 0$

$\Rightarrow$ $[x]^2 - 3[x] - 2[x] + 6 = 0$

$\Rightarrow$ $[x]([x]-3) - 2([x]-3) = 0$

$\Rightarrow$ $([x]-3)([x]-2) = 0 \Rightarrow [x] = 2, 3$

So, $x \in [2, 3]$.

Hence, the correct option is (*c*).

Q26. Range of $f(x) = \dfrac{1}{1-2\cos x}$ is

(*a*) $\left[\dfrac{1}{3}, 1\right]$ (*b*) $\left[-1, \dfrac{1}{3}\right]$

(*c*) $(-\infty, -1] \cup \left[\dfrac{1}{3}, \infty\right)$ (*d*) $\left[-\dfrac{1}{3}, 1\right]$

Sol. Given that: $f(x) = \dfrac{1}{1-2\cos x}$

We know that $-1 \le \cos x \le 1$

$\Rightarrow$ $1 \ge \cos x \ge -1 \Rightarrow -1 \le -\cos x \le 1$

$\Rightarrow$ $-2 \le -2\cos x \le 2 \Rightarrow -2+1 \le 1-2\cos x \le 2+1$

$\Rightarrow$ $-1 \le 1-2\cos x \le 3 \Rightarrow -1 \le \dfrac{1}{1-2\cos x} \le \dfrac{1}{3}$

$\Rightarrow$ $-1 \le f(x) \le \dfrac{1}{3}$

So the range of $f(x) = \left[-1, \dfrac{1}{3}\right]$

Hence, the correct option is (*b*).

Q27. Let $f(x) = \sqrt{1+x^2}$, then

(*a*) $f(xy) = f(x).f(y)$ (*b*) $f(xy) \ge f(x).f(y)$

(*c*) $f(xy) \le f(x).f(y)$ (*d*) None of these

Sol. Given that: $f(x) = \sqrt{1+x^2} \Rightarrow f(xy) = \sqrt{1+x^2y^2}$

and $f(x) \,.\, f(y) = \sqrt{1+x^2}.\sqrt{1+y^2} = \sqrt{1+x^2+y^2+x^2y^2}$

$\because$ $\sqrt{1+x^2y^2} \le \sqrt{1+x^2+y^2+x^2y^2}$

$\Rightarrow$ $f(xy) \le f(x).f(y)$

Hence, the correct option is (*c*).

Q28. Domain of $\sqrt{a^2-x^2}$ $(a > 0)$ is

(*a*) $(-a, a)$ (*b*) $[-a, a]$ (*c*) $[0, a]$ (*d*) $(-a, 0]$

Sol. Let $f(x) = \sqrt{a^2-x^2}$

$f(x)$ is defined if $a^2 - x^2 \ge 0$

$\Rightarrow \quad x^2 - a^2 \leq 0 \quad \Rightarrow \quad x^2 \leq a^2$

$\Rightarrow \quad x \leq \pm a \quad \Rightarrow \quad -a \leq x \leq a$

$\therefore$ Domain of $f(x) = [-a, a]$

Hence, the correct option is (*b*).

Q29. If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then a and b are equal to

(*a*) $a = -3, b = -1$ (*b*) $a = 2, b = -3$

(*c*) $a = 0, b = 2$ (*d*) $a = 2, b = 3$

Sol. Given that: $f(x) = ax + b$

$\Rightarrow \quad f(-1) = a(-1) + b$

$\Rightarrow \quad -5 = -a + b$

$\Rightarrow \quad a - b = 5 \quad$ ...(*i*)

$f(3) = 3a + b$

$\Rightarrow \quad 3 = 3a + b$

$\Rightarrow \quad 3a + b = 3 \quad$ ...(*ii*)

On solving eqn. (*i*) and (*ii*), we get $a = 2, b = -3$

Hence, the correct option is (*b*).

Q30. The domain of the function f defined by

$f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2 - 1}}$ is equal to

(*a*) $(-\infty, -1) \cup (1, 4]$ (*b*) $(-\infty, -1] \cup (1, 4]$

(*c*) $(-\infty, -1) \cup [1, 4]$ (*d*) $(-\infty, -1) \cup [1, 4)$

Sol. Given that: $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2 - 1}}$

$f(x)$ is defined if

$4 - x \geq 0 \quad$ or $\quad x^2 - 1 > 0$

$\Rightarrow \quad -x \geq -4 \quad$ or $\quad (x-1)(x+1) > 0$

$\Rightarrow \quad x \leq 4 \quad$ or $\quad x < -1$ and $x > 1$

$\therefore$ Domain of $f(x)$ is $(-\infty, -1) \cup (1, 4]$

Hence, the correct option is (*a*).

Q31. The domain and range of the real function f defined by

$f(x) = \dfrac{4-x}{x-4}$ is given by

(*a*) Domain = **R**, Range = $\{-1, 1\}$

(*b*) Domain = **R** – $\{1\}$, Range = **R**

(*c*) Domain = **R** – $\{4\}$, Range = **R** – $\{-1\}$

(*d*) Domain = **R** – $\{-4\}$, Range = $\{-1, 1\}$

Sol. Given that: $f(x) = \dfrac{4-x}{x-4}$

We know that $f(x)$ is defined if $x - 4 \neq 0 \Rightarrow x \neq 4$

So, the domain of $f(x)$ is = R – {4}

Let $\quad f(x) = y = \dfrac{4 - x}{x - 4}$

$\Rightarrow \quad yx - 4y = 4 - x \Rightarrow yx + x = 4y + 4$

$\Rightarrow \quad x(y + 1) = 4y + 4 \quad \Rightarrow \quad x = \dfrac{4(1 + y)}{1 + y}$

If x is real number, then $1 + y \neq 0 \Rightarrow y \neq -1$

$\therefore$ Range of $f(x)$ = R – {– 1}

Hence, the correct option is (*c*).

Q32. The domain and range of real function f defined by $f(x) = \sqrt{x - 1}$ is given by

(*a*) Domain = $(1, \infty)$, Range = $(0, \infty)$

(*b*) Domain = $[1, \infty)$, Range = $(0, \infty)$

(*c*) Domain = $[1, \infty)$, Range = $[0, \infty)$

(*d*) Domain = $[1, \infty)$, Range = $[0, \infty)$

Sol. Given that: $\quad f(x) = \sqrt{x - 1}$

$f(x)$ is defined if $x - 1 \geq 0 \Rightarrow x \geq 1$

$\therefore$ Domain of $f(x) = [0, \infty)$

Let $\quad f(x) = y = \sqrt{x - 1} \Rightarrow y^2 = x - 1$

$\Rightarrow \quad x = y^2 + 1$

If x is real then $y \in$ R

$\therefore$ Range of $f(x) = [0, \infty)$

Hence, the correct option is (*d*).

Q33. The domain of the function f given by $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - x - 6}$ is

(*a*) R – {3, – 2} (*b*) R – {– 3, 2} (*c*) R – {3, – 2} (*d*) R – (3, – 2)

Sol. Given that: $\quad f(x) = \dfrac{x^2 + 2x + 1}{x^2 - x - 6}$

$f(x)$ is defined if $\quad x^2 - x - 6 \neq 0$

$\Rightarrow \quad x^2 - 3x + 2x - 6 \neq 0$

$\Rightarrow \quad (x - 3)(x + 2) \neq 0 \quad \Rightarrow \quad x \neq -2, x \neq 3$

So, the domain of $f(x)$ = R – {– 2, 3}

Hence, the correct option is (*a*).

Q34. The domain and range of the function f given by $f(x) = 2 - |x - 5|$ is

(*a*) Domain = R^+, Range = $(-\infty, 1]$

(*b*) Domain = R, Range = $(-\infty, 2]$

(*c*) Domain = R, Range = $(-\infty, 2)$

(*d*) Domain = R^+, Range = $(-\infty, 2]$

Sol. Given that: $f(x) = 2 - |x-5|$

Here, $f(x)$ is defined for $x \in R$

$\therefore$ Domain of $f(x) = R$

Now, $|x-5| \geq 0 \quad \Rightarrow \quad -|x-5| \leq 0$

$$\Rightarrow \quad 2 - |x-5| \leq 2$$

$$\Rightarrow \quad f(x) \leq 2$$

$\therefore$ Range of $f(x) = (-\infty, 2]$

Hence, the correct option is (*b*).

Q35. The domain for which the functions defined by $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal is

(*a*) $\left\{-1, \frac{4}{3}\right\}$ (*b*) $\left\{-1, \frac{4}{3}\right\}$ (*c*) $\left\{-1, \frac{4}{3}\right\}$ (*d*) $\left\{-1, \frac{4}{3}\right\}$

Sol. Given that: $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$

$$f(x) = g(x)$$

$$\Rightarrow \quad 3x^2 - 1 = 3 + x$$

$$\Rightarrow \quad 3x^2 - x - 4 = 0 \quad \Rightarrow \quad 3x^2 - 4x + 3x - 4 = 0$$

$$\Rightarrow \quad x(3x-4) + 1(3x-4) = 0 \quad \Rightarrow \quad (x+1)(3x-4) = 0$$

$$\Rightarrow \quad x + 1 = 0 \quad \text{or} \quad 3x - 4 = 0$$

$$\Rightarrow \quad x = -1, \quad \text{or} \quad x = \frac{4}{3}$$

$$\therefore \quad \text{Domain} = \left\{-1, \frac{4}{3}\right\}$$

Hence, the correct option is (*a*).

Fill in the Blanks

Q36. Let f and g be two real functions given by

$$f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)\}$$
$$g = \{(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)\}$$

then the domain of $f.g$ is given by ....

Sol. Given that: $f(x) = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)\}$

and $g(x) = \{(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)\}$

$\therefore$ Domain of $f = \{0, 2, 3, 4, 5\}$

and domain of $g = \{1, 2, 3, 4, 5\}$

So, domain of $f.g$ = Domain of $f \cap$ Domain of g

$= \{2, 3, 4, 5\}$

Hence, the filler is $\{2, 3, 4, 5\}$.

Q37. Let $f = \{(2, 4), (5, 6), (8, -1), (10, -3)\}$

and $g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, 5)\}$

be two real functions. Then match the following:

(*a*)	$f-g$	(*i*)	$\left\{\left(2, \frac{4}{5}\right), \left(8, -\frac{1}{4}\right), \left(10, \frac{-3}{13}\right)\right\}$
(*b*)	$f+g$	(*ii*)	{(2, 20), (8, – 4), (10, – 39}
(*c*)	$f \cdot g$	(*iii*)	{(2, – 1), (8, – 5), (10, –16)}
(*d*)	$\frac{f}{g}$	(*iv*)	{(2, 9), (8, 3), (10, 10)}

Sol. Given that: $f = \{(2, 4), (5, 6), (8, -1), (10, -3)\}$
and $g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, 5)\}$

$f-g, f+g, f.g, \frac{f}{g}$ are defined in the domain

(domain of $f \cap$ domain of g)

$\Rightarrow \{2, 5, 8, 10\} \cap \{2, 7, 8, 10, 11\}$

$\Rightarrow \{2, 8, 10\}$

(*i*)
$$(f-g)2 = f(2) - g(2) = 4 - 5 = -1$$
$$(f-g)8 = f(8) - g(8) = -1 - 4 = -5$$
$$(f-g)10 = f(10) - g(10) = -3 - 13 = -16$$
$$\therefore \quad (f-g) = \{(2, -1), (8, -5), (10, -16)\}$$

(*ii*)
$$(f+g)2 = f(2) + g(2) = 4 + 5 = 9$$
$$(f+g)8 = f(8) + g(8) = -1 + 4 = 3$$
$$(f+g)10 = f(10) + g(10) = -3 + 13 = 10$$
$$\therefore \quad (f+g) = \{(2, 9), (8, 3), (10, 10)\}$$

(*iii*)
$$(f \cdot g)2 = f(2) \cdot g(2) = 4 \cdot 5 = 20$$
$$(f \cdot g)8 = f(8) \cdot g(8) = (-1) \cdot (4) = -4$$
$$(f \cdot g)10 = f(10) \cdot g(10) = -3 \cdot 13 = -39$$
$$\therefore \quad (f \cdot g) = \{(2, 20), (8, -4), (10, -39)\}$$

(*iv*)
$$\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{4}{5}$$
$$\left(\frac{f}{g}\right)(8) = \frac{f(8)}{g(8)} = \frac{-1}{4}$$
$$\left(\frac{f}{g}\right)(10) = \frac{f(10)}{g(10)} = \frac{-3}{13}$$
$$\therefore \quad \left(\frac{f}{g}\right) = \left\{\left(2, \frac{4}{5}\right), \left(8, \frac{-1}{4}\right), \left(10, \frac{-3}{13}\right)\right\}$$

Hence, the correct option is

(*a*) $\leftrightarrow$ (*iii*), (*b*) $\leftrightarrow$ (*iv*), (*c*) $\leftrightarrow$ (*ii*), (*d*) $\leftrightarrow$ (*i*)

State True or False for the Statements in Each of the Exercises 38 to 42.

Q38. The ordered pair (5, 2) belongs to the relation
$R = \{(x, y) : y = x - 5, x, y \in Z\}$

Sol. Given that: $R = \{(x, y) : y = x - 5, x, y \in Z\}$
For (5, 2), $y = x - 5$
Put $x = 5$, $y = 5 - 5 = 0 \neq 2$
So (5, 2) is not the ordered pair of R.
Hence, the statement is 'False'.

Q39. If $P = \{1, 2\}$, then
$$P \times P \times P = \{(1, 1, 1), (2, 2, 2), (1, 2, 2), (2, 1, 1)\}$$

Sol. Given that $P = \{1, 2\}$
$$\therefore \quad P \times P = \{1, 2\} \times \{1, 2\} = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$
$$P \times P \times P = \{(1, 1), (1, 2), (2, 1), (2, 2)\} \times \{1, 2\}$$
$$= \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}$$
So, given statement is 'False'.

Q40. If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$ then
$(A \times B) \cup (A \times C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$

Sol. Given that: $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$
$$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$$
and $$A \times C = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$$
$$(A \times B) \cup (A \times C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$$
Hence, the given statement is 'True'.

Q41. If $(x - 2, y + 5) = \left(-2, \frac{1}{3}\right)$ are two equal ordered pairs, then $x = 4, y = \frac{-14}{3}$.

Sol. Given that: $(x - 2, y + 5) = \left(-2, \frac{1}{3}\right)$
$$\Rightarrow \quad x - 2 = -2 \Rightarrow x = 0$$
and $$y + 5 = \frac{1}{3} \Rightarrow y = \frac{1}{3} - 5 \Rightarrow y = -\frac{14}{3}$$
Hence, the given statement is 'False'.

Q42. If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$ then $A = \{a, b\}$ and $B = \{x, y\}$.

Sol. Given that: $A = \{a, b\}$ and $B = \{x, y\}$
$$\therefore \quad A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$$
Hence, the statement is 'True'.