## EXERCISE

### SHORT ANSWER TYPE QUESTIONS

**Q1.** Prove that: $\dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \dfrac{1 + \sin A}{\cos A}$

**Sol.** L.H.S. $\dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1}$

$$= \frac{\tan A + (\sec A - 1)}{\tan A - (\sec A - 1)}$$

$$= \frac{[\tan A + (\sec A - 1)][\tan A + (\sec A - 1)]}{[\tan A - (\sec A - 1)][\tan A + (\sec A - 1)]}$$

[Rationalizing the denominator]

$$= \frac{[\tan A + (\sec A - 1)]^2}{\tan^2 A - (\sec A - 1)^2}$$

$$= \frac{\tan^2 A + (\sec A - 1)^2 + 2\tan A(\sec A - 1)}{\tan^2 A - (\sec^2 A + 1 - 2\sec A)}$$

$$= \frac{\tan^2 A + \sec^2 A + 1 - 2\sec A + 2\tan A\sec A - 2\tan A}{\tan^2 A - \sec^2 A - 1 + 2\sec A}$$

$$= \frac{\sec^2 A + \sec^2 A - 2\sec A + 2\tan A\sec A - 2\tan A}{-1 - 1 + 2\sec A}$$

$$= \frac{2\sec^2 A - 2\sec A + 2\tan A\sec A - 2\tan A}{2\sec A - 2}$$

$$= \frac{\sec^2 A - \sec A + \sec A\tan A - \tan A}{\sec A - 1}$$

$$= \frac{\sec A(\sec A - 1) + \tan A(\sec A - 1)}{\sec A - 1}$$

$$= \frac{(\sec A - 1)(\sec A + \tan A)}{(\sec A - 1)}$$

$$= \sec A + \tan A = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$= \dfrac{1 + \sin A}{\cos A}$ R.H.S. Hence proved.

**Q2.** If $\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=y$, then prove that $\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is also equal to $y$.

**Sol.** Given that:

$$y = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$$

$$= \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}\times\frac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha-\cos\alpha}$$

$$= \frac{2\sin\alpha\,(1-\cos\alpha+\sin\alpha)}{(1+\sin\alpha)^2-\cos^2\alpha} = \frac{2\sin\alpha\,(1-\cos\alpha+\sin\alpha)}{1+\sin^2\alpha+2\sin\alpha-\cos^2\alpha}$$

$$= \frac{2\sin\alpha(1-\cos\alpha+\sin\alpha)}{(1-\cos^2\alpha)+\sin^2\alpha+2\sin\alpha} = \frac{2\sin\alpha\,(1-\cos\alpha+\sin\alpha)}{\sin^2\alpha+\sin^2\alpha+2\sin\alpha}$$

$$= \frac{2\sin\alpha\,(1-\cos\alpha+\sin\alpha)}{2\sin^2\alpha+2\sin\alpha} = \frac{2\sin\alpha\,(1-\cos\alpha+\sin\alpha)}{2\sin\alpha\,(1+\sin\alpha)}$$

$$= \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$$

$\therefore \dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}=y$. Hence proved.

**Q3.** If $m\sin\theta = n\sin(\theta+2\alpha)$, then prove that

$\tan(\theta+\alpha).\cot\alpha = \dfrac{m+n}{m-n}$.

**Sol.** Given that: $m\sin\theta = n\sin(\theta+2\alpha)$

$$\Rightarrow \frac{\sin(\theta+2\alpha)}{\sin\theta} = \frac{m}{n}$$

Using componendo and dividendo theorem we get

$$\Rightarrow \frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{2\sin\left(\dfrac{\theta+2\alpha+\theta}{2}\right).\cos\left(\dfrac{\theta+2\alpha-\theta}{2}\right)}{2\cos\left(\dfrac{\theta+2\alpha+\theta}{2}\right).\sin\left(\dfrac{\theta+2\alpha-\theta}{2}\right)} = \frac{m+n}{m-n}$$

$$\left[\begin{aligned}\because\ \sin A+\sin B &= 2\sin\frac{A+B}{2}.\cos\frac{A-B}{2}\\ \sin A-\sin B &= 2\cos\frac{A+B}{2}.\sin\frac{A-B}{2}\end{aligned}\right]$$

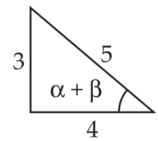
$$\Rightarrow \qquad \frac{\sin(\theta+\alpha).\cos\alpha}{\cos(\theta+\alpha).\sin\alpha} = \frac{m+n}{m-n}$$

$$\Rightarrow \qquad \tan(\theta+\alpha).\cot\alpha = \frac{m+n}{m-n}. \text{ Hence proved.}$$

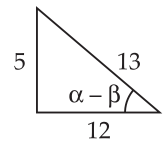
**Q4.** If $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $\alpha$ lies between 0 and $\frac{\pi}{4}$, find the value of $\tan 2\alpha$.

**Sol.** Given that:

$$\cos(\alpha+\beta) = \frac{4}{5} \qquad \therefore \tan(\alpha+\beta) = \frac{3}{4}$$




and $\quad \sin(\alpha-\beta) = \frac{5}{13} \qquad \therefore \tan(\alpha-\beta) = \frac{5}{12}$




Now $\quad \tan 2\alpha = \tan[\alpha+\beta+\alpha-\beta]$

$$= \tan[(\alpha+\beta)+(\alpha-\beta)]$$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta).\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} = \frac{\frac{9+5}{12}}{\frac{48-15}{48}} = \frac{14}{12}\times\frac{48}{33} = \frac{56}{33}$$

Hence, $\quad \tan 2\alpha = \frac{56}{33}$.

**Q5.** If $\tan x = \frac{b}{a}$, then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$.

**Sol.** Given that: $\tan x = \frac{b}{a}$

$$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{\sqrt{a+b}}{\sqrt{a-b}} + \frac{\sqrt{a-b}}{\sqrt{a+b}}$$

$$= \frac{a+b+a-b}{\sqrt{(a-b)(a+b)}} = \frac{2a}{\sqrt{a^2-b^2}} = \frac{2a}{a\sqrt{1-\frac{b^2}{a^2}}}$$

$$= \frac{2}{\sqrt{1-\tan^2 x}} \qquad \left[\because \tan x = \frac{b}{a}\right]$$

$$= \frac{2}{\sqrt{1-\frac{\sin^2 x}{\cos^2 x}}} = \frac{2}{\frac{\sqrt{\cos^2 x-\sin^2 x}}{\cos x}}$$

$$= \frac{2\cos x}{\sqrt{\cos 2x}} \qquad [\because \cos^2 x - \sin^2 x = \cos 2x]$$

Hence, $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{2\cos x}{\sqrt{\cos 2x}}$.

**Q6.** Prove that: $\cos\theta\cos\frac{\theta}{2} - \cos 3\theta\cos\frac{9\theta}{2} = \sin 7\theta\sin 8\theta$.

**Sol.** L.H.S. $\cos\theta\cos\frac{\theta}{2} - \cos 3\theta\cos\frac{9\theta}{2}$

$$= \frac{1}{2}\left[2\cos\theta\cos\frac{\theta}{2}\right] - \frac{1}{2}\left[2\cos 3\theta\cos\frac{9\theta}{2}\right]$$

$$= \frac{1}{2}\left[\cos\left(\theta+\frac{\theta}{2}\right)+\cos\left(\theta-\frac{\theta}{2}\right)\right]-\frac{1}{2}\left[\cos\left(3\theta+\frac{9\theta}{2}\right)+\cos\left(3\theta-\frac{9\theta}{2}\right)\right]$$

$$= \frac{1}{2}\left[\cos\frac{3\theta}{2} + \cos\frac{\theta}{2} - \cos\frac{15\theta}{2} - \cos\left(\frac{-3\theta}{2}\right)\right]$$

$$= \frac{1}{2}\left[\cos\frac{3\theta}{2} + \cos\frac{\theta}{2} - \cos\frac{15\theta}{2} - \cos\frac{3\theta}{2}\right] \qquad [\because \cos(-\theta) = \cos\theta]$$

$$= \frac{1}{2}\left[\cos\frac{\theta}{2} - \cos\frac{15\theta}{2}\right] = \frac{1}{2}\left[-2\sin\left(\frac{\theta}{2}+\frac{15\theta}{2}\right).\sin\left(\frac{\theta}{2}-\frac{15\theta}{2}\right)\right]$$

$$= -\sin 8\theta\sin(-7\theta) = \sin 7\theta\sin 8\theta \qquad [\because \sin(-\theta) = -\sin\theta]$$

L.H.S. = R.H.S. Hence proved.

**Q7.** If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$ then show that $a^2 + b^2 = m^2 + n^2$.

**Sol.** Given that: $a\cos\theta + b\sin\theta = m$

and $a\sin\theta - b\cos\theta = n$

R.H.S. $m^2 + n^2 = (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$

$$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$$

$$= a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta$$

$$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$$

$= a^2.1 + b^2.1 = a^2 + b^2$ L.H.S.

LHS = RHS. Hence proved

**Q8.** Find the value of $\tan 22°30'$.

**Sol.** Let $22°30' = \frac{\theta}{2}$ $\quad\therefore\ \theta = 45°$

$$\tan 22°30' = \tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \frac{\sin\theta}{1+\cos\theta}$$

Put $\theta = 45°$

$$\therefore \quad \frac{\sin\theta}{1+\cos\theta} = \frac{\sin 45°}{1+\cos 45°} = \frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}+1}$$

$$= \frac{1\times\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)} = \sqrt{2}-1$$

Hence, $\tan 22°30' = \sqrt{2}-1$.

**Q9.** Prove that: $\sin 4A = 4\sin A\cos^3 A - 4\cos A\sin^3 A$.

**Sol.** L.H.S. $\sin 4A = \sin(A+3A)$

$= \sin A\cos 3A + \cos A\sin 3A$

$= \sin A(4\cos^3 A - 3\cos A) + \cos A(3\sin A - 4\sin^3 A)$

$= 4\sin A\cos^3 A - 3\sin A\cos A + 3\sin A\cos A - 4\cos A\sin^3 A$

$= 4\sin A\cos^3 A - 4\cos A\sin^3 A$. R.H.S.

L.H.S. = R.H.S. Hence proved.

**Q10.** If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$ then prove that $m^2 - n^2 = 4\sin\theta\tan\theta$.

**Sol.** Given that: $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$

L.H.S. $m^2 - n^2 = (m+n)(m-n)$

$= [(\tan\theta + \sin\theta) + (\tan\theta - \sin\theta)].[(\tan\theta + \sin\theta) - (\tan\theta - \sin\theta)]$

$= (\tan\theta + \sin\theta + \tan\theta - \sin\theta).(\tan\theta + \sin\theta - \tan\theta + \sin\theta)$

$= 2\tan\theta . 2\sin\theta = 4\sin\theta\tan\theta$. R.H.S.

L.H.S. = R.H.S. Hence proved.

**Q11.** If $\tan(A+B) = p$, $\tan(A-B) = q$, then show that $\tan 2A = \frac{p+q}{1-pq}$.

**Sol.** Given that: $\tan(A+B) = p$, $\tan(A-B) = q$

$\tan 2A = \tan(A+B+A-B) = \tan[(A+B)+(A-B)]$

$$= \frac{\tan(A+B) + \tan(A-B)}{1-\tan(A+B).\tan(A-B)}$$

$= \frac{p+q}{1-pq}$ Hence proved.

**Q12.** If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then prove that $\cos 2\alpha + \cos 2\beta = -2\cos(\alpha+\beta)$.

**Sol.** Given that: $\cos\alpha + \cos\beta = 0$

and $\sin\alpha + \sin\beta = 0$

$\therefore (\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2 = 0$

$\Rightarrow (\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta) - (\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta) = 0$

$\Rightarrow \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta - \sin^2\alpha - \sin^2\beta - 2\sin\alpha\sin\beta = 0$

$\Rightarrow (\cos^2\alpha - \sin^2\alpha) + (\cos^2\beta - \sin^2\beta) + 2(\cos\alpha\cos\beta - \sin\alpha\sin\beta) = 0$

$\Rightarrow \cos 2\alpha + \cos 2\beta + 2\cos(\alpha + \beta) = 0$

Hence, $\cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$.

Hence proved.

**Q13.** If $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$, then show that $\dfrac{\tan x}{\tan y} = \dfrac{a}{b}$.

**Sol.** Given that: $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$

$$\Rightarrow \frac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$$

(Using componendo and dividendo theorem)

$$\Rightarrow \frac{2\sin\left(\dfrac{x+y+x-y}{2}\right)\cos\left(\dfrac{x+y-x+y}{2}\right)}{2\cos\left(\dfrac{x+y+x-y}{2}\right)\sin\left(\dfrac{x+y-x+y}{2}\right)} = \frac{2a}{2b}$$

$$\Rightarrow \frac{\sin x.\cos y}{\cos x.\sin y} = \frac{a}{b} \Rightarrow \tan x.\cot y = \frac{a}{b}$$

$\Rightarrow \dfrac{\tan x}{\tan y} = \dfrac{a}{b}$. Hence proved.

**Q14.** If $\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then show that $\sin\alpha + \cos\alpha = \sqrt{2}\cos\theta$.

**Sol.** Given that: $\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$

$$\Rightarrow \tan\theta = \frac{\tan\alpha - 1}{\tan\alpha + 1} = \frac{\tan\alpha - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{4}\tan\alpha}$$

$$\Rightarrow \tan\theta = \tan\left(\alpha - \frac{\pi}{4}\right)$$

$$\therefore \theta = \alpha - \frac{\pi}{4} \Rightarrow \cos\theta = \cos\left(\alpha - \frac{\pi}{4}\right)$$

$$\Rightarrow \cos\theta = \cos\alpha\cos\frac{\pi}{4} + \sin\alpha\sin\frac{\pi}{4}$$

$$\Rightarrow \cos\theta = \cos\alpha.\frac{1}{\sqrt{2}} + \sin\alpha.\frac{1}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2}\cos\theta = \cos\alpha + \sin\alpha$$

$\Rightarrow \sin\alpha + \cos\alpha = \sqrt{2}\cos\theta$. Hence proved.

**Q15.** If $\sin\theta + \cos\theta = 1$, then find the general value of $\theta$.

**Sol.** Given that: $\sin\theta + \cos\theta = 1$

Dividing both sides by $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$, we get

$$\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}} \qquad \text{...}(i)$$

$$\Rightarrow \sin\frac{\pi}{4}\sin\theta + \cos\frac{\pi}{4}\cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$$\Rightarrow \theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4},\ n \in Z \qquad \left[\begin{array}{l}\text{If } \cos\theta = \cos\alpha \\ \qquad \theta = 2n\pi \pm \alpha\end{array}\right]$$

$$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$$

$$\Rightarrow \theta = 2n\pi + \frac{\pi}{4} + \frac{\pi}{4} \text{ or } \theta = 2n\pi - \frac{\pi}{4} + \frac{\pi}{4}$$

$$\therefore \theta = 2n\pi + \frac{\pi}{2} \text{ or } \theta = 2n\pi,\ n \in Z$$

Hence, the general values of $\theta$ are $2n\pi + \frac{\pi}{2}$ and $2n\pi$.

**Alternate method:**

From eqn. $(i)$ we get

$$\cos\frac{\pi}{4}\sin\theta + \sin\frac{\pi}{4}\cos\theta = \sin\frac{\pi}{4}$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \sin\frac{\pi}{4}$$

$$\Rightarrow \theta + \frac{\pi}{4} = n\pi + (-1)^n . \frac{\pi}{4} \qquad \left[\begin{array}{l}\text{If } \sin\theta = \sin\alpha \\ \therefore \qquad \theta = n\pi + (-1)^n . \alpha\end{array}\right]$$

$$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}.$$

Hence, the general value of $\theta$ is $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$, $n \in Z$.

**Q16.** Find the most general value of $\theta$ satisfying the equation $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$.

**Sol.** Given that: $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$

$$\tan\theta = -1$$

$$\Rightarrow \tan\theta = \tan\left(\frac{-\pi}{4}\right)$$

$$\Rightarrow \quad \tan\theta = \tan\left(2\pi - \frac{\pi}{4}\right) \Rightarrow \tan\theta = \tan\frac{7\pi}{4}$$

$$\therefore \quad \theta = \frac{7\pi}{4}$$

Now $\quad \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \cos\theta = \cos\frac{\pi}{4}$

$$\Rightarrow \cos\theta = \cos\left(2\pi - \frac{\pi}{4}\right)$$

$$\Rightarrow \cos\theta = \cos\frac{7\pi}{4}$$

$$\therefore \quad \theta = \frac{7\pi}{4}$$

[tan θ and cos θ are positive in 4th quadrant]

Hence, the most general value of $\theta = 2n\pi + \frac{7\pi}{4}$.

**Q17.** If cot θ + tan θ = 2 cosec θ, then find the general value of θ.

**Sol.** Given that: cot θ + tan θ = 2 cosec θ

$$\Rightarrow \quad \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{2}{\sin\theta}$$

$$\Rightarrow \quad \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \frac{2}{\sin\theta}$$

$$\Rightarrow \quad \frac{1}{\sin\theta\cos\theta} = \frac{2}{\sin\theta}$$

$$\Rightarrow \quad 2\sin\theta\cos\theta = \sin\theta$$

$$\Rightarrow \quad 2\sin\theta\cos\theta - \sin\theta = 0$$

$$\Rightarrow \quad \sin\theta(2\cos\theta - 1) = 0$$

$$\Rightarrow \quad \sin\theta \neq 0 \quad \text{or} \quad 2\cos\theta - 1 = 0 \text{ or } \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \cos\theta = \cos\frac{\pi}{3}$$

$$\therefore \quad \theta = 2n\pi \pm \frac{\pi}{3}$$

Hence, the general values of θ is $2n\pi \pm \frac{\pi}{3}$.

**Q18.** If $2\sin^2\theta = 3\cos\theta$, where $0 \le \theta \le 2\pi$, then find the value of θ.

**Sol.** Given that: $\quad 2\sin^2\theta = 3\cos\theta$

$$\Rightarrow \quad 2(1 - \cos^2\theta) = 3\cos\theta$$

$$\Rightarrow \quad 2 - 2\cos^2\theta - 3\cos\theta = 0$$

$$\Rightarrow \quad 2\cos^2\theta + 3\cos\theta - 2 = 0$$

$$\Rightarrow \quad 2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$$
$$\Rightarrow \quad 2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$$
$$\Rightarrow \quad (\cos\theta + 2)(2\cos\theta - 1) = 0$$
$$\Rightarrow \quad \cos\theta + 2 = 0 \text{ or } 2\cos\theta - 1 = 0$$
$$\Rightarrow \quad \cos\theta \neq -2 \qquad [-1 \le \cos\theta \le 1]$$
$$\therefore \quad 2\cos\theta - 1 = 0$$
$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$
$$\Rightarrow \quad \cos\theta = \cos\frac{\pi}{3}, \cos\left(2\pi - \frac{\pi}{3}\right)$$
$$\Rightarrow \quad \cos\theta = \cos\frac{\pi}{3}, \cos\frac{5\pi}{3}$$
$$\therefore \quad \theta = 2n\pi \pm \frac{\pi}{3}$$
$$\text{and} \quad \theta = 2n\pi \pm \frac{5\pi}{3} \qquad (n \in Z)$$

Hence, the value of $\theta$ are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

**Q19.** If $\sec x \cos 5x + 1 = 0$, where $0 < x \le \frac{\pi}{2}$, then find the value of $x$.

**Sol.** Given that: $\sec x \cos 5x + 1 = 0$

$$\Rightarrow \quad \frac{1}{\cos x}.\cos 5x + 1 = 0$$
$$\Rightarrow \quad \cos 5x + \cos x = 0$$
$$\Rightarrow \quad 2\cos\left(\frac{5x + x}{2}\right).\cos\left(\frac{5x - x}{2}\right) = 0$$
$$\Rightarrow \quad \cos 3x \,.\, \cos 2x = 0$$
$$\Rightarrow \quad \cos 3x = 0 \quad \text{or} \quad \cos 2x = 0$$
$$\Rightarrow \quad 3x = \frac{\pi}{2} \quad \text{or} \quad 2x = \frac{\pi}{2}$$
$$x = \frac{\pi}{6} \quad \text{or} \quad x = \frac{\pi}{4}$$

Hence, the values of $x$ are $\frac{\pi}{6}, \frac{\pi}{4}$.

## LONG ANSWER TYPE QUESTIONS

**Q20.** If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that $\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$.

**Sol.** Given that:

$\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$

$\cos(\alpha - \beta) = \cos[\theta + \alpha - \theta - \beta] = \cos[(\theta + \alpha) - (\theta + \beta)]$

$$= \cos(\theta+\alpha)\cos(\theta+\beta)+\sin(\theta+\alpha)\sin(\theta+\beta)$$

$$= \sqrt{1-\sin^2(\theta+\alpha)}\sqrt{1-\sin^2(\theta+\beta)}+a.b$$

$$= \sqrt{(1-a^2)(1-b^2)}+ab = ab+\sqrt{1-a^2-b^2+a^2b^2}$$

Now $\cos 2(\alpha-\beta)-4ab\cos(\alpha-\beta)$

$= 2\cos^2(\alpha-\beta)-1-4ab\cos(\alpha-\beta)$ $\quad [\because \cos 2\theta = 2\cos^2\theta - 1]$

$$= 2\left[ab+\sqrt{1-a^2-b^2+a^2b^2}\right]^2-1-4ab\left[ab+\sqrt{1-a^2-b^2+a^2b^2}\right]$$

$$= 2\left[a^2b^2+1-a^2-b^2+a^2b^2+2ab\sqrt{1-a^2-b^2+a^2b^2}\right]-1$$
$$-4a^2b^2-4ab\sqrt{1-a^2-b^2+a^2b^2}$$

$$= 2a^2b^2+2-2a^2-2b^2+2a^2b^2+4ab\sqrt{1-a^2-b^2+a^2b^2}-1$$
$$-4a^2b^2-4ab\sqrt{1-a^2-b^2+a^2b^2}$$

$= 1-2a^2-2b^2$

Hence, $\cos 2(\alpha-\beta)-4ab\cos(\alpha-\beta) = 1-2a^2-2b^2$.

Hence proved.

**Q21.** If $\cos(\theta+\phi) = m\cos(\theta-\phi)$, then prove that $\tan\theta = \dfrac{1-m}{1+m}\cot\phi$.

**Sol.** Given that: $\cos(\theta+\phi) = m\cos(\theta-\phi)$

$$\Rightarrow \quad \frac{\cos(\theta+\phi)}{\cos(\theta-\phi)} = \frac{m}{1}$$

Using componendo and dividendo theorem, we get

$$\frac{\cos(\theta+\phi)+\cos(\theta-\phi)}{\cos(\theta+\phi)-\cos(\theta-\phi)} = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{2\cos\left(\dfrac{\theta+\phi+\theta-\phi}{2}\right).\cos\left(\dfrac{\theta+\phi-\theta+\phi}{2}\right)}{-2\sin\left(\dfrac{\theta+\phi+\theta-\phi}{2}\right).\sin\left(\dfrac{\theta+\phi-\theta+\phi}{2}\right)} = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{\cos\theta.\cos\phi}{-\sin\theta.\sin\phi} = \frac{m+1}{m-1} \Rightarrow -\cot\theta.\cot\phi = \frac{m+1}{m-1}$$

$$\Rightarrow \quad \frac{-\cot\phi}{\tan\theta} = \frac{m+1}{m-1} = -\frac{1+m}{1-m}$$

$$\Rightarrow \quad \tan\theta = \frac{1-m}{1+m}\cot\phi.$$ Hence proved.

**Q22.** Find the value of the expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$$

**Sol.** Given that:

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$$

$= 3[\cos^4\alpha + \sin^4(\pi+\alpha)] - 2[\cos^6\alpha + \sin^6(\pi-\alpha)]$

$= 3[\cos^4\alpha + \sin^4\alpha] - 2[\cos^6\alpha + \sin^6\alpha]$

$= 3[\cos^4\alpha + \sin^4\alpha + 2\sin^2\alpha\cos^2\alpha - 2\sin^2\alpha\cos^2\alpha]$

$\quad - 2[(\cos^2\alpha + \sin^2\alpha)^3 - 3\cos^2\alpha\sin^2\alpha(\cos^2\alpha+\sin^2\alpha)]$

$= 3[(\cos^2\alpha+\sin^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha] - 2[1 - 3\cos^2\alpha\sin^2\alpha]$

$= 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[1 - 3\cos^2\alpha\sin^2\alpha]$

$= 3 - 6\sin^2\alpha\cos^2\alpha - 2 + 6\cos^2\alpha\sin^2\alpha$

$= 3 - 2 = 1$

Hence, the value of the given expression is 1.

**Q23.** If $a\cos 2\theta + b\sin 2\theta = c$ has $\alpha$ and $\beta$ as its roots, then prove that $\tan\alpha + \tan\beta = \dfrac{2b}{a+c}$.

**Sol.** Given that: $a\cos 2\theta + b\sin 2\theta = c$

$$\Rightarrow \quad a\left[\frac{1-\tan^2\theta}{1+\tan^2\theta}\right] + b\left[\frac{2\tan\theta}{1+\tan^2\theta}\right] = c$$

$$\Rightarrow \quad a - a\tan^2\theta + 2b\tan\theta = c(1+\tan^2\theta)$$

$$\left[\because \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}, \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right]$$

$\Rightarrow \quad a - a\tan^2\theta + 2b\tan\theta = c + c\tan^2\theta$

$\Rightarrow \quad a - a\tan^2\theta + 2b\tan\theta - c\tan^2\theta - c = 0$

$\Rightarrow \quad -(a+c)\tan^2\theta + 2b\tan\theta + (a-c) = 0$

$\Rightarrow \quad (a+c)\tan^2\theta - 2b\tan\theta + (c-a) = 0$

Since $\alpha$ and $\beta$ are the roots of this equation

$$\Rightarrow \quad \tan\alpha + \tan\beta = \frac{-(-2b)}{a+c}$$

$$\Rightarrow \quad \tan\alpha + \tan\beta = \frac{2b}{a+c}.$$ Hence proved.

**Q24.** If $x = \sec\phi - \tan\phi$ and $y = \operatorname{cosec}\phi + \cot\phi$, then show that $xy + x - y + 1 = 0$.

**Sol.** Given that: $x = \sec\phi - \tan\phi$

and $y = \operatorname{cosec}\phi + \cot\phi$

$xy + x - y + 1 = 0$

L.H.S. $xy + x - y + 1$

$= (\sec\phi - \tan\phi)(\operatorname{cosec}\phi + \cot\phi) + (\sec\phi - \tan\phi) - (\operatorname{cosec}\phi + \cot\phi) + 1$

$$= \left(\frac{1}{\cos\phi} - \frac{\sin\phi}{\cos\phi}\right)\left(\frac{1}{\sin\phi} + \frac{\cos\phi}{\sin\phi}\right) + \left(\frac{1}{\cos\phi} - \frac{\sin\phi}{\cos\phi}\right) - \left(\frac{1}{\sin\phi} + \frac{\cos\phi}{\sin\phi}\right) + 1$$

$$= \left(\frac{1-\sin\phi}{\cos\phi}\right)\left(\frac{1+\cos\phi}{\sin\phi}\right) + \frac{1-\sin\phi}{\cos\phi} - \frac{1+\cos\phi}{\sin\phi} + 1$$

$$= \frac{1-\sin\phi+\cos\phi-\sin\phi\cos\phi}{\cos\phi\sin\phi} + \frac{\sin\phi-\sin^2\phi-\cos\phi-\cos^2\phi}{\cos\phi\sin\phi} + 1$$

$$= \frac{1-\sin\phi+\cos\phi-\sin\phi\cos\phi+\sin\phi-\cos\phi-(\sin^2\phi+\cos^2\phi)+\sin\phi\cos\phi}{\cos\phi\sin\phi}$$

$$= \frac{1-1}{\cos\phi\sin\phi} = 0. \text{ R.H.S.}$$

L.H.S. = R.H.S. Hence proved.

**Q25.** If $\theta$ lies in first quadrant and $\cos\theta = \frac{8}{17}$, then find the value of $\cos(30° + \theta) + \cos(45° - \theta) + \cos(120° - \theta)$.

**Sol.** Given that: $\cos\theta = \frac{8}{17}$

$$\therefore \sin\theta = \sqrt{1-\left(\frac{8}{17}\right)^2} = \sqrt{1-\frac{64}{289}} = \sqrt{\frac{289-64}{289}} = \sqrt{\frac{225}{289}} = \frac{15}{17}$$

But $\theta$ lies in I quadrant.

$$\therefore \quad \sin\theta = \frac{15}{17}$$

Now $\cos(30° + \theta) + \cos(45° - \theta) + \cos(120° - \theta)$

$= \cos 30° \cos\theta - \sin 30° \sin\theta + \cos 45° \cos\theta + \sin 45° \sin\theta + \cos 120° \cos\theta + \sin 120° \sin\theta$

$$= \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta - \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta$$

$$= \left(\frac{\sqrt{3}}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right) - \frac{1}{2}(\sin\theta + \cos\theta) + \frac{1}{\sqrt{2}}(\cos\theta + \sin\theta)$$

$$= \frac{\sqrt{3}}{2}(\cos\theta + \sin\theta) - \frac{1}{2}(\sin\theta + \cos\theta) + \frac{1}{\sqrt{2}}(\cos\theta + \sin\theta)$$

$$= \left(\frac{\sqrt{3}}{2} - \frac{1}{2} + \frac{1}{\sqrt{2}}\right)(\cos\theta + \sin\theta) = \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{8}{17} + \frac{15}{17}\right)$$

$$= \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{23}{17}\right)$$

Hence, the required solution $= \frac{23}{17}\left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right)$.

**Q26.** Find the value of the expression

$$\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$$

**Sol.** $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$

$$= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4\left(\pi - \frac{3\pi}{8}\right) + \cos^4\left(\pi - \frac{\pi}{8}\right)$$

$$= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{\pi}{8}$$

$$= 2\cos^4 \frac{\pi}{8} + 2\cos^4 \frac{3\pi}{8} = 2\left[\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8}\right]$$

$$= 2\left[\cos^4 \frac{\pi}{8} + \cos^4\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right] = 2\left[\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8}\right]$$

$$= 2\left[\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} + 2\sin^2 \frac{\pi}{8}.\cos^2 \frac{\pi}{8} - 2\sin^2 \frac{\pi}{8}.\cos^2 \frac{\pi}{8}\right]$$

$$= 2\left[\left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8}\right)^2 - 2\sin^2 \frac{\pi}{8}.\cos^2 \frac{\pi}{8}\right]$$

$$= 2\left[1 - 2\sin^2 \frac{\pi}{8}\cos^2 \frac{\pi}{8}\right] = 2 - 4\sin^2 \frac{\pi}{8}.\cos^2 \frac{\pi}{8}$$

$$= 2 - \left(2\sin \frac{\pi}{8}.\cos \frac{\pi}{8}\right)^2 = 2 - \left(\sin \frac{\pi}{4}\right)^2$$

$$= 2 - \left(\frac{1}{\sqrt{2}}\right)^2 = 2 - \frac{1}{2} = \frac{3}{2}$$

Hence, the required value of the expression $= \frac{3}{2}$.

**Q27.** Find the general solution of the equation

$5\cos^2\theta + 7\sin^2\theta - 6 = 0$

**Sol.** $5\cos^2\theta + 7\sin^2\theta - 6 = 0$

$\Rightarrow 5\cos^2\theta + 7(1 - \cos^2\theta) - 6 = 0$

$\Rightarrow 5\cos^2\theta + 7 - 7\cos^2\theta - 6 = 0 \Rightarrow -2\cos^2\theta + 1 = 0$

$\Rightarrow 2\cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{1}{2}$

$\Rightarrow \cos^2\theta = \cos^2 \frac{\pi}{4}$

$\therefore \quad \theta = n\pi \pm \dfrac{\pi}{4} \quad \left[\begin{array}{l} \because \text{If } \cos^2\theta = \cos^2\alpha \\ \therefore \quad \theta = n\pi \pm \alpha \end{array}\right]$

Hence, the general solution of $\theta = n\pi \pm \dfrac{\pi}{4}$, $n \in Z$.

**Q28.** Find the general solution of the equation
$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$.

**Sol.** Given that: $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$\Rightarrow (\sin 3x + \sin x) - 3\sin 2x = (\cos 3x + \cos x) - 3\cos 2x$

$\Rightarrow 2\sin\left(\dfrac{3x+x}{2}\right).\cos\left(\dfrac{3x-x}{2}\right) - 3\sin 2x$

$= 2\cos\left(\dfrac{3x+x}{2}\right).\cos\left(\dfrac{3x-x}{2}\right) - 3\cos 2x$

$\Rightarrow 2\sin 2x.\cos x - 3\sin 2x = 2\cos 2x.\cos x - 3\cos 2x$

$\Rightarrow 2\sin 2x\cos x - 2\cos 2x.\cos x = 3\sin 2x - 3\cos 2x$

$\Rightarrow 2\cos x(\sin 2x - \cos 2x) = 3(\sin 2x - \cos 2x)$

$\Rightarrow 2\cos x(\sin 2x - \cos 2x) - 3(\sin 2x - \cos 2x) = 0$

$\Rightarrow (\sin 2x - \cos 2x)(2\cos x - 3) = 0$

$\Rightarrow \sin 2x - \cos 2x = 0$ and $2\cos x - 3 \neq 0 \quad [\because\ -1 \le \cos x \le 1]$

$\Rightarrow \dfrac{\sin 2x}{\cos 2x} - 1 = 0 \Rightarrow \tan 2x = 1$

$\Rightarrow \tan 2x = \tan\dfrac{\pi}{4} \Rightarrow 2x = n\pi + \dfrac{\pi}{4} \quad \therefore x = \dfrac{n\pi}{2} + \dfrac{\pi}{8}$

Hence, the general solution of the equation is

$x = \dfrac{n\pi}{2} + \dfrac{\pi}{8}$, $n \in Z$.

**Q29.** Find the general solution of the equation
$\left(\sqrt{3}-1\right)\cos\theta + \left(\sqrt{3}+1\right)\sin\theta = 2$

**Sol.** Given that: $\left(\sqrt{3}-1\right)\cos\theta + \left(\sqrt{3}+1\right)\sin\theta = 2$

Put $\sqrt{3}-1 = r\sin\alpha$, $\sqrt{3}+1 = r\cos\alpha$

Squaring and adding, we get

$$r^2 = 3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3}$$

$$\Rightarrow \quad r^2 = 8 \Rightarrow r = \pm 2\sqrt{2}$$

Now the given equation can be written as

$r\sin\alpha\cos\theta + r\cos\alpha\sin\theta = 2$

$\Rightarrow \quad r(\sin\alpha\cos\theta + \cos\alpha\sin\theta) = 2$

$\Rightarrow \quad 2\sqrt{2}\sin(\alpha+\theta) = 2$

$\Rightarrow \quad \sin(\alpha+\theta) = \dfrac{2}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$$\Rightarrow \quad \sin(\alpha + \theta) = \sin\frac{\pi}{4}$$

$$\therefore \quad \alpha + \theta = n\pi + (-1)^n \cdot \frac{\pi}{4} \quad \text{...}(i)$$

Now $\quad \dfrac{r\sin\alpha}{r\cos\alpha} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$$\Rightarrow \quad \tan\alpha = \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{4}.\tan\frac{\pi}{3}}$$

$$\Rightarrow \quad \tan\alpha = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$\Rightarrow \quad \tan\alpha = \tan\frac{\pi}{12} \quad \therefore \alpha = \frac{\pi}{12}$$

Putting the value of α in equation (*i*) we get

$$\frac{\pi}{12} + \theta = n\pi + (-1)^n \cdot \frac{\pi}{4}$$

$$\therefore \quad \theta = n\pi + (-1)^n \cdot \frac{\pi}{4} - \frac{\pi}{12}$$

Hence, the general solution of the given equation is

$$\theta = n\pi + (-1)^n \cdot \frac{\pi}{4} - \frac{\pi}{12}, \; n \in Z$$

## OBJECTIVE TYPE QUESTIONS

**Choose the correct answer out of the given four options in each of the Exercises from 30 to 59 (M.C.Q.)**

**Q30.** If $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^2\theta + \operatorname{cosec}^2\theta$ is equal to

(*a*) 1 (*b*) 4

(*c*) 2 (*d*) None of these

**Sol.** Given that: $\sin\theta + \operatorname{cosec}\theta = 2$

Squaring both sides, we get

$$(\sin\theta + \operatorname{cosec}\theta)^2 = (2)^2$$

$$\Rightarrow \quad \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta = 4$$

$$\Rightarrow \quad \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta \times \frac{1}{\sin\theta} = 4$$

$$\Rightarrow \quad \sin^2\theta + \operatorname{cosec}^2\theta + 2 = 4$$

$$\Rightarrow \quad \sin^2\theta + \operatorname{cosec}^2\theta = 2$$

Hence, the correct option is (*c*).

**Q31.** If $f(x) = \cos^2 x + \sec^2 x$ then

(*a*) $f(x) < 1$ (*b*) $f(x) = 1$

(*c*) $2 < f(x) < 1$ (*d*) $f(x) \geq 2$

**Sol.** Given that: $f(x) = \cos^2 x + \sec^2 x$

We know that AM ≥ GM

$$\Rightarrow \quad \frac{\cos^2 x + \sec^2 x}{2} \geq \sqrt{\cos^2 x . \sec^2 x}$$

$$\Rightarrow \quad \frac{\cos^2 x + \sec^2 x}{2} \geq 1 \Rightarrow \cos^2 x + \sec^2 x \geq 2$$

$$\Rightarrow \quad f(x) \geq 2$$

Hence, the correct option is (*d*).

**Q32.** If $\tan \theta = \frac{1}{2}$ and $\tan \phi = \frac{1}{3}$, then the value of $\theta + \phi$ is

(*a*) $\frac{\pi}{6}$ (*b*) $\pi$ (*c*) 0 (*d*) $\frac{\pi}{4}$

**Sol.** We know that

$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\Rightarrow \quad \tan(\theta + \phi) = \tan\frac{\pi}{4}$$

$\therefore \quad \theta + \phi = \frac{\pi}{4}$. Hence the correct option is (*d*).

**Q33.** Which of the following is not correct?

(*a*) $\sin\theta = -\frac{1}{5}$ (*b*) $\cos\theta = 1$

(*c*) $\sec\theta = \frac{1}{2}$ (*d*) $\tan\theta = 20$

**Sol.** $\sin\theta = -\frac{1}{5}$ is correct. $\because -1 \leq \sin\theta \leq 1$

So (*a*) is correct.

$\cos\theta = 1$ is correct. $\because \cos 0° = 1$

So (*b*) is correct.

$\sec\theta = \frac{1}{2} \Rightarrow \cos\theta = 2$ is not correct. $\because -1 \leq \cos\theta \leq 1$

Hence, (*c*) is not correct.

**Q34.** The value of tan 1° tan 2° tan 3° .... tan 89° is

(*a*) 0 (*b*) 1 (*c*) $\frac{1}{2}$ (*d*) Not defined

**Sol.** Given that: tan 1° tan 2° tan 3° .... tan 89°

= tan 1° tan 2° tan 3° .... tan 45°.tan (90 – 44°).tan (90 – 43°) ...tan (90 – 1°)

= tan 1° cot 1°.tan 2°.cot 2°.tan 3°.cot 3° ... tan 89°.cot 89°

= 1.1.1.1 ... 1.1 = 1

Hence, the correct option is (*b*).

**Q35.** The value of $\dfrac{1-\tan^2 15°}{1+\tan^2 15°}$ is

(*a*) 1 (*b*) $\sqrt{3}$ (*c*) $\dfrac{\sqrt{3}}{2}$ (*d*) 2

**Sol.** Given that: $\dfrac{1-\tan^2 15°}{1+\tan^2 15°}$

Let $\theta = 15°$ $\therefore$ $2\theta = 30°$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\Rightarrow \quad \cos 30° = \frac{1-\tan^2 15°}{1+\tan^2 15°} \Rightarrow \frac{\sqrt{3}}{2} = \frac{1-\tan^2 15°}{1+\tan^2 15°}$$

Hence, the correct option is (*c*).

**Q36.** The value of cos 1°.cos 2°.cos 3° ... cos 179° is

(*a*) $\dfrac{1}{\sqrt{2}}$ (*b*) 0 (*c*) 1 (*d*) – 1

**Sol.** Given expression is cos 1°.cos 2°.cos 3° ... cos 179°

$\Rightarrow$ cos 1°.cos 2°.cos 3° ... cos 90°.cos 91° ... cos 179°

$\Rightarrow 0$ $[\because \cos 90° = 0]$

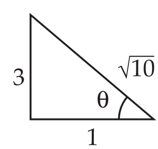
Hence, the correct option is (*b*).

**Q37.** If tan θ = 3 and θ lies in third quadrant, then the value of sin θ is

(*a*) $\dfrac{1}{\sqrt{10}}$ (*b*) $-\dfrac{1}{\sqrt{10}}$ (*c*) $-\dfrac{3}{\sqrt{10}}$ (*d*) $\dfrac{3}{\sqrt{10}}$

**Sol.** tan θ = 3, θ lies in third quadrant

$\therefore \sin\theta = \dfrac{-3}{\sqrt{10}}$ where θ lies in third quadrant




Hence the correct option is (*c*).

**Q38.** The value of tan 75° – cot 75° is equal to

(*a*) $2\sqrt{3}$ (*b*) $2+\sqrt{3}$ (*c*) $2-\sqrt{3}$ (*d*) 1

**Sol.** The given expression is tan 75° – cot 75°

$$\tan 75° - \cot 75° = \tan 75° - \cot(90 - 15°)$$

$$= \tan 75° - \tan 15° = \frac{\sin 75°}{\cos 75°} - \frac{\sin 15°}{\cos 15°}$$

$$= \frac{\sin 75° \cos 15° - \cos 75° \sin 15°}{\cos 75° \cos 15°}$$

$$= \frac{\sin(75° - 15°)}{\frac{1}{2} \times 2\cos 75° \cos 15°}$$

$$= \frac{2\sin 60^\circ}{\cos(75^\circ + 15^\circ) + \cos(75^\circ - 15^\circ)}$$

$$= \frac{2 \times \frac{\sqrt{3}}{2}}{\cos 90^\circ + \cos 60^\circ} = \frac{\sqrt{3}}{0 + \frac{1}{2}} = 2\sqrt{3}$$

Hence, the correct option is (*a*).

**Q39.** Which of the following is correct?

(*a*) $\sin 1^\circ > \sin 1$ (*b*) $\sin 1^\circ < \sin 1$

(*c*) $\sin 1^\circ = \sin 1$ (*d*) $\sin 1^\circ = \frac{\pi}{180^\circ}\sin 1$

**Sol.** We know that if $\theta$ increases then the value of $\sin\theta$ also increases

So $\sin 1^\circ < \sin 1$ $\left[\because 1 \text{ radian} = \frac{\pi}{180}\sin 1\right]$

Hence the correct option is (*b*).

**Q40.** If $\tan\alpha = \frac{m}{m+1}$, $\tan\beta = \frac{1}{2m+1}$, then $\alpha + \beta$ is equal to

(*a*) $\frac{\pi}{2}$ (*b*) $\frac{\pi}{3}$ (*c*) $\frac{\pi}{6}$ (*d*) $\frac{\pi}{4}$

**Sol.** Given that $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{m+1} \times \frac{1}{2m+1}}$$

$$= \frac{\frac{2m^2 + m + m + 1}{(m+1)(2m+1)}}{\frac{(m+1)(2m+1) - m}{(m+1)(2m+1)}} = \frac{2m^2 + 2m + 1}{2m^2 + 2m + m + 1 - m}$$

$$= \frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} = 1$$

$$\Rightarrow \tan(\alpha+\beta) = \tan\frac{\pi}{4} \quad \therefore \alpha + \beta = \frac{\pi}{4}$$

Hence, the correct option is (*d*).

**Q41.** The minimum value of $3\cos x + 4\sin x + 8$ is

(*a*) 5 (*b*) 9 (*c*) 7 (*d*) 3

**Sol.** The given expression is $3\cos x + 4\sin x + 8$

Let $y = 3\cos x + 4\sin x + 8$

$\Rightarrow \quad y - 8 = 3\cos x + 4\sin x$

Minimum value of $y - 8 = -\sqrt{(3)^2 + (4)^2}$

$\Rightarrow \quad y - 8 = -\sqrt{9 + 16} = -5$

$\Rightarrow \quad y = 8 - 5 = 3$

So, the minimum value of the given expression is 3.

Hence, the correct option is (*d*).

**Q42.** The value of tan 3A – tan 2A – tan A is equal to

(*a*) tan 3A tan 2A tan A

(*b*) – tan 3A tan 2A tan A

(*c*) tan A tan 2A – tan 2A tan 3A – tan 3A tan A

(*d*) None of these

**Sol.** The given expression is tan 3A – tan 2A – tan A

$$\tan 3A = \tan(2A + A)$$

$$\Rightarrow \quad \tan 3A = \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$$

$$\Rightarrow \quad \tan 3A(1 - \tan 2A \tan A) = \tan 2A + \tan A$$

$$\Rightarrow \tan 3A - \tan 3A \tan 2A \tan A = \tan 2A + \tan A$$

$$\Rightarrow \quad \tan 3A - \tan 2A - \tan A = \tan 3A \tan 2A \tan A$$

Hence, the correct option is (*a*).

**Q43.** The value of sin (45° + θ) – cos (45° – θ) is

(*a*) 2 cos θ (*b*) 2 sin θ (*c*) 1 (*d*) 0

**Sol.** Given expression is sin (45° + θ) – cos (45° – θ)

$$\sin(45^\circ + \theta) = \sin 45^\circ \cos\theta + \cos 45^\circ \sin\theta$$

$$= \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta$$

$$\cos(45^\circ - \theta) = \cos 45^\circ \cos\theta + \sin 45^\circ \sin\theta$$

$$= \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta$$

$\sin(45^\circ + \theta) - \cos(45^\circ - \theta)$

$$= \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta$$

= 0. Hence, the correct option is (*d*).

**Q44.** The value of $\cot\left(\frac{\pi}{4} + \theta\right).\cot\left(\frac{\pi}{4} - \theta\right)$ is

(*a*) – 1 (*b*) 0 (*c*) 1 (*d*) Not defined

**Sol.** $$\cot\left(\frac{\pi}{4} + \theta\right).\cot\left(\frac{\pi}{4} - \theta\right) = \frac{\cot\frac{\pi}{4}\cot\theta - 1}{\cot\theta + \cot\frac{\pi}{4}} \times \frac{\cot\frac{\pi}{4}\cot\theta + 1}{\cot\theta - \cot\frac{\pi}{4}}$$

$$= \frac{1.\cot\theta - 1}{\cot\theta + 1} \times \frac{1.\cot\theta + 1}{\cot\theta - 1}$$

$$= \frac{\cot\theta - 1}{\cot\theta + 1} \times \frac{\cot\theta + 1}{\cot\theta - 1} = 1$$

Hence, the correct option is (*c*).

**Q45.** $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$ is equal to

(*a*) $\sin 2(\theta + \phi)$ (*b*) $\cos 2(\theta + \phi)$

(*c*) $\sin 2(\theta - \phi)$ (*d*) $\cos 2(\theta - \phi)$

**Sol.** Given that: $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$

$\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$

$= \cos 2\theta \cos 2\phi + \sin(\theta - \phi + \theta + \phi).\sin(\theta - \phi - \theta - \phi)$

$[\because \sin^2 A - \sin^2 B = \sin(A + B).\sin(A - B)]$

$= \cos 2\theta \cos 2\phi + \sin 2\theta.\sin(-2\phi)$

$= \cos 2\theta \cos 2\phi - \sin 2\theta \sin 2\phi$ $[\because \sin(-\theta) = -\sin\theta]$

$= \cos(2\theta + 2\phi) = \cos 2(\theta + \phi)$

Hence, the correct option is (*b*).

**Q46.** The value of $\cos 12° + \cos 84° + \cos 156° + \cos 132°$ is

(*a*) $\frac{1}{2}$ (*b*) 1 (*c*) $-\frac{1}{2}$ (*d*) $\frac{1}{8}$

**Sol.** The given expression is $\cos 12° + \cos 84° + \cos 156° + \cos 132°$

$(\cos 132° + \cos 12°) + (\cos 156° + \cos 84°)$

$$= \left(2\cos\frac{132° + 12°}{2}.\cos\frac{132° - 12°}{2}\right) + \left(2\cos\frac{156° + 84°}{2}.\cos\frac{156° - 84°}{2}\right)$$

$= 2\cos 72°.\cos 60° + 2\cos 120°.\cos 36°$

$$= 2\cos 72° \times \frac{1}{2} + 2 \times \left(-\frac{1}{2}\right)\cos 36° = \cos 72° - \cos 36°$$

$= \cos(90° - 18°) - \cos 36° = \sin 18° - \cos 36°$

$$= \frac{\sqrt{5} - 1}{4} - \frac{\sqrt{5} + 1}{4} \qquad \left[\because \sin 18° = \frac{\sqrt{5} - 1}{4}, \cos 36° = \frac{\sqrt{5} + 1}{4}\right]$$

$$= \frac{\sqrt{5} - 1 - \sqrt{5} - 1}{4} = \frac{-1}{2}$$

Hence, the correct option is (*c*).

**Q47.** If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, then $\tan(2A + B)$ is equal to

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

**Sol.** Given that: $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

So, $\quad \tan 2A = \frac{4}{3}$ and $\tan B = \frac{1}{3}$

$$\tan(2A+B) = \frac{\tan 2A + \tan B}{1-\tan 2A.\tan B} = \frac{\frac{4}{3}+\frac{1}{3}}{1-\frac{4}{3}\times\frac{1}{3}}$$

$$= \frac{\frac{5}{3}}{\frac{9-4}{9}} = \frac{5}{3}\times\frac{9}{5} = 3$$

Hence, the correct option is (*c*).

**Q48.** The value of $\sin\frac{\pi}{10}.\sin\frac{13\pi}{10}$ is

(*a*) $\frac{1}{2}$ (*b*) $-\frac{1}{2}$ (*c*) $-\frac{1}{4}$ (*d*) 1

**Sol.** $\sin\frac{\pi}{10}.\sin\frac{13\pi}{10} = \sin\frac{\pi}{10}.\sin\left(\pi+\frac{3\pi}{10}\right)$

$$= \sin\frac{\pi}{10}.\left(-\sin\frac{3\pi}{10}\right) = -\sin 18°.\sin 54°$$

$$= -\sin 18°.\sin(90°-36°) = -\sin 18°.\cos 36°$$

$$= -\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)$$

$$\left[\because \sin 18° = \frac{\sqrt{5}-1}{4}, \cos 36° = \frac{\sqrt{5}+1}{4}\right]$$

$$= -\left(\frac{5-1}{16}\right) = -\frac{1}{4}$$

Hence, the correct option is (*c*).

**Q49.** The value of sin 50° – sin 70° + sin 10° is equal to

(*a*) 1 (*b*) 0 (*c*) $\frac{1}{2}$ (*d*) 2

**Sol.** Given expression is sin 50° – sin 70° + sin 10°

$$(\sin 50° - \sin 70°) + \sin 10° = 2\cos\frac{50°+70°}{2}.\sin\frac{50°-70°}{2} + \sin 10°$$
$$= 2\cos 60°.(-\sin 10°) + \sin 10°$$
$$= -2 \times \frac{1}{2}\sin 10° + \sin 10°$$
$$= -\sin 10° + \sin 10°$$
$$= 0$$

Hence, the correct option is (*b*).

**Q50.** If $\sin\theta + \cos\theta = 1$, then the value of $\sin 2\theta$ is equal to

(*a*) 1 (*b*) $\frac{1}{2}$ (*c*) 0 (*d*) $-1$

**Sol.** Given that: $\sin\theta + \cos\theta = 1$

$\Rightarrow \quad (\sin\theta + \cos\theta)^2 = (1)^2$

$\Rightarrow \quad \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$

$\Rightarrow \quad 1 + \sin 2\theta = 1 \quad \Rightarrow \quad \sin 2\theta = 1 - 1 = 0$

Hence, the correct option is (*c*).

**Q51.** If $\alpha + \beta = \frac{\pi}{4}$, then the value of $(1 + \tan\alpha)(1 + \tan\beta)$ is

(*a*) 1 (*b*) 2 (*c*) $-2$ (*d*) Not defined

**Sol.** Given that: $\alpha + \beta = \frac{\pi}{4}$

$\Rightarrow \quad \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 1$

$\Rightarrow \quad \tan\alpha + \tan\beta = 1 - \tan\alpha\tan\beta$

$\Rightarrow \quad \tan\alpha + \tan\beta + \tan\alpha\tan\beta = 1$

$\Rightarrow \quad 1 + \tan\alpha + \tan\beta + \tan\alpha\tan\beta = 1 + 1$

$\Rightarrow \quad 1(1 + \tan\alpha) + \tan\beta(1 + \tan\alpha) = 2$

$\Rightarrow \quad (1 + \tan\alpha)(1 + \tan\beta) = 2$

Hence, the correct option is (*b*).

**Q52.** If $\sin\theta = -\frac{4}{5}$ and $\theta$ lies in third quadrant then the value of $\cos\frac{\theta}{2}$ is

(*a*) $\frac{1}{5}$ (*b*) $-\frac{1}{\sqrt{10}}$ (*c*) $-\frac{1}{\sqrt{15}}$ (*d*) $\frac{1}{\sqrt{10}}$

**Sol.** Given that: $\sin\theta = -\frac{4}{5}$, $\theta$ lies in third quadrant

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(-\frac{4}{5}\right)^2}$$

$$= \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{+3}{-5}$$

$\therefore \quad \cos\theta = -\frac{3}{5}$, $\theta$ lies in third quadrant

$$\cos\theta = 2\cos^2\frac{\theta}{2} - 1 \left[\because \pi < \theta < \frac{3\pi}{2}, \therefore \frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}\right]$$

$$\Rightarrow \quad \frac{-3}{5} = 2\cos^2\frac{\theta}{2} - 1$$

$$\Rightarrow \quad 2\cos^2\frac{\theta}{2} = 1 - \frac{3}{5} = \frac{2}{5} \Rightarrow \cos^2\frac{\theta}{2} = \frac{2}{5\times 2} = \frac{1}{5}$$

$$\Rightarrow \quad \cos\frac{\theta}{2} = \pm\frac{1}{\sqrt{5}}$$

$$\Rightarrow \quad \cos\frac{\theta}{2} = -\frac{1}{\sqrt{5}} \qquad \left[\because \frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}\right]$$

Hence, the correct option is (*c*).

**Q53.** Number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

**Sol.** Given equation is $\tan x + \sec x = 2\cos x$

$$\Rightarrow \quad \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$$

$$\Rightarrow \quad 1 + \sin x = 2\cos^2 x \Rightarrow 2\cos^2 x - \sin x - 1 = 0$$

$$\Rightarrow 2(1-\sin^2 x) - \sin x - 1 = 0 \Rightarrow 2 - 2\sin^2 x - \sin x - 1 = 0$$

$$\Rightarrow \quad -2\sin^2 x - \sin x + 1 = 0 \Rightarrow 2\sin^2 x + \sin x - 1 = 0$$

Since, the equation is a quadratic equation in $\sin x$. So it will have 2 solutions.

Hence, the correct option is (*c*).

**Q54.** The value of $\sin\frac{\pi}{18} + \sin\frac{\pi}{9} + \sin\frac{2\pi}{9} + \sin\frac{5\pi}{18}$ is

(*a*) $\sin\frac{7\pi}{18} + \sin\frac{4\pi}{9}$ (*b*) 1

(*c*) $\cos\frac{\pi}{6} + \cos\frac{3\pi}{7}$ (*d*) $\cos\frac{\pi}{9} + \sin\frac{\pi}{9}$

**Sol.** The given expression is $\sin\frac{\pi}{18} + \sin\frac{\pi}{9} + \sin\frac{2\pi}{9} + \sin\frac{5\pi}{18}$

$$= \left(\sin\frac{5\pi}{18} + \sin\frac{\pi}{18}\right) + \left(\sin\frac{2\pi}{9} + \sin\frac{\pi}{9}\right)$$

$$= 2\sin\left(\frac{\frac{5\pi}{18}+\frac{\pi}{18}}{2}\right).\cos\left(\frac{\frac{5\pi}{18}-\frac{\pi}{18}}{2}\right)+2\sin\left(\frac{\frac{2\pi}{9}+\frac{\pi}{9}}{2}\right).\cos\left(\frac{\frac{2\pi}{9}-\frac{\pi}{9}}{2}\right)$$

$$= 2\sin\frac{\pi}{6}.\cos\frac{\pi}{9}+2\sin\frac{\pi}{6}.\cos\frac{\pi}{18}$$

$$= 2\times\frac{1}{2}\cos\frac{\pi}{9}+2\times\frac{1}{2}\cos\frac{\pi}{18} = \cos\frac{\pi}{9}+\cos\frac{\pi}{18}$$

$$= \sin\left(\frac{\pi}{2}-\frac{\pi}{9}\right)+\sin\left(\frac{\pi}{2}-\frac{\pi}{18}\right) = \sin\frac{7\pi}{18}+\sin\frac{8\pi}{18}$$

$= \sin\frac{7\pi}{18}+\sin\frac{4\pi}{9}$. Hence, the correct option is (*a*).

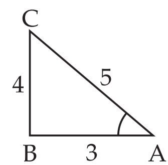
**Q55.** If A lies in the second quadrant and 3 tan A + 4 = 0, then the value of 2 cot A – 5 cos A + sin A is equal to

(*a*) $\frac{-53}{10}$ (*b*) $\frac{23}{10}$ (*c*) $\frac{37}{10}$ (*d*) $\frac{7}{10}$

**Sol.** Given that: 3 tan A + 4 = 0, A lies in second quadrant

$\therefore$ $\tan A = \frac{-4}{3}$

$\cos A = \frac{-3}{5}$




[A lies in second quadrant]

and $\sin A = \frac{4}{5}$ and $\cot A = \frac{-3}{4}$

$$\therefore\ 2\cot A - 5\cos A + \sin A = 2\left(\frac{-3}{4}\right)-5\left(\frac{-3}{5}\right)+\frac{4}{5}$$

$$= \frac{-3}{2}+3+\frac{4}{5} = \frac{-15+30+8}{10} = \frac{23}{10}$$

Hence, the correct option is (*b*).

**Q56.** The value of $\cos^2 48° - \sin^2 12°$ is

(*a*) $\frac{\sqrt{5}+1}{8}$ (*b*) $\frac{\sqrt{5}-1}{8}$ (*c*) $\frac{\sqrt{5}+1}{5}$ (*d*) $\frac{\sqrt{5}+1}{2\sqrt{2}}$

**Sol.** Given expression is $\cos^2 48° - \sin^2 12°$

$\cos^2 48° - \sin^2 12° = \cos(48° + 12°).\cos(48° - 12°)$

$[\because \cos^2 A - \sin^2 B = \cos(A+B).\cos(A-B)]$

$$= \cos 60°.\cos 36° = \frac{1}{2}\times\frac{\sqrt{5}+1}{4} = \frac{\sqrt{5}+1}{8}$$

Hence, the correct option is (*a*).

**Q57.** If $\tan \alpha = \frac{1}{7}$, $\tan \beta = \frac{1}{3}$ then $\cos 2\alpha$ is equal to

(*a*) $\sin 2\beta$ (*b*) $\sin 4\beta$ (*c*) $\sin 3\beta$ (*d*) $\cos 3\beta$

**Sol.** Given that: $\tan \alpha = \frac{1}{7}$ and $\tan \beta = \frac{1}{3}$

$$\cos 2\alpha = \frac{1-\tan^2 \alpha}{1+\tan^2 \alpha} = \frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2} = \frac{1-\frac{1}{49}}{1+\frac{1}{49}}$$

$$= \frac{48}{50} = \frac{24}{25}$$

Now $$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\times\frac{1}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{2}{3}\times\frac{9}{8} = \frac{3}{4}$$

$\therefore$ $$\tan 2\beta = \frac{3}{4}$$

$$\sin 4\beta = \frac{2\tan 2\beta}{1+\tan^2 2\beta}$$

$$= \frac{2\times\frac{3}{4}}{1+\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1+\frac{9}{16}} = \frac{3}{2}\times\frac{16}{25} = \frac{24}{25}$$

$$\cos 2\alpha = \sin 4\beta = \frac{24}{25}$$

Hence, the correct option is (*b*).

**Q58.** If $\tan \theta = \frac{a}{b}$, then $b \cos 2\theta + a \sin 2\theta$ is equal to

(*a*) $a$ (*b*) $b$ (*c*) $\frac{a}{b}$ (*d*) None

**Sol.** Given that: $\tan \theta = \frac{a}{b}$

$$b\cos 2\theta + a\sin 2\theta = b\left[\frac{1-\tan^2\theta}{1+\tan^2\theta}\right] + a\left[\frac{2\tan\theta}{1+\tan^2\theta}\right]$$

$$= b\left[\frac{1-\frac{a^2}{b^2}}{1+\frac{a^2}{b^2}}\right] + a\left[\frac{\frac{2a}{b}}{1+\frac{a^2}{b^2}}\right]$$

$$= b\left[\frac{b^2-a^2}{b^2+a^2}\right]+\left[\frac{\frac{2a^2}{b}}{\frac{b^2+a^2}{b^2}}\right]$$

$$= \frac{b^3-a^2b}{b^2+a^2}+\frac{2a^2b}{b^2+a^2} = \frac{b^3-a^2b+2a^2b}{b^2+a^2}$$

$$= \frac{b^3+a^2b}{b^2+a^2} = \frac{b(b^2+a^2)}{b^2+a^2} = b$$

Hence, the correct option is (*b*).

**Q59.** If for real value of $x$, $\cos\theta = x+\frac{1}{x}$ then

(*a*) $\theta$ is an acute angle (*b*) $\theta$ is right angle

(*c*) $\theta$ is an obtuse angle (*d*) No value of $\theta$ is possible

**Sol.** Given that: $\cos\theta = x+\frac{1}{x} \Rightarrow \cos\theta = \frac{x^2+1}{x}$

$\Rightarrow \quad x^2+1 = x\cos\theta \Rightarrow x^2 - x\cos\theta + 1 = 0$

For real value of $x$, $b^2 - 4ac \geq 0$

$\Rightarrow \quad (-\cos\theta)^2 - 4\times1\times1 \geq 0$

$\Rightarrow \quad \cos^2\theta - 4 \geq 0 \Rightarrow \cos^2\theta \geq 4$

$\Rightarrow \quad \cos\theta \geq \pm 2 \qquad [-1 \leq \cos\theta \leq 1]$

So, the value of $\theta$ is not possible.

Hence, the correct option is (*d*).

**Fill in the Blanks in Each of the Exercises 60 to 67.**

**Q60.** The value of $\frac{\sin 50°}{\sin 130°}$ is ................

**Sol.** $$\frac{\sin 50°}{\sin 130°} = \frac{\sin 50°}{\sin(180° - 50°)} = \frac{\sin 50°}{\sin 50°} = 1$$

Hence, the value of filler is **1**.

**Q61.** If $k = \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$, then the numerical value of $k$ is ................

**Sol.** Given that: $k = \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$

$\Rightarrow k = \sin 10°.\ \sin 50°.\ \sin 70°$

$\Rightarrow k = \sin 10°\ \sin(90° - 40°)\ \sin(90° - 20°)$

$\Rightarrow k = \sin 10°\ \cos 40°\ \cos 20°$

$\Rightarrow k = \sin 10°.\frac{1}{2}[2\cos 40°\ \cos 20°]$

$$\Rightarrow k = \sin 10° . \frac{1}{2}[\cos (40° + 20°) + \cos (40° - 20°)]$$

$$\Rightarrow k = \frac{1}{2} \sin 10° [\cos 60° + \cos 20°]$$

$$\Rightarrow k = \frac{1}{2} \sin 10° \left(\frac{1}{2} + \cos 20°\right)$$

$$\Rightarrow k = \frac{1}{4} \sin 10° + \frac{1}{2} \sin 10° . \cos 20°$$

$$\Rightarrow k = \frac{1}{4} \sin 10° + \frac{1}{4}(2 \sin 10° \cos 20°)$$

$$\Rightarrow k = \frac{1}{4} \sin 10° + \frac{1}{4}[\sin (10° + 20°) + \sin (10° - 20°)]$$

$$\Rightarrow k = \frac{1}{4} \sin 10° + \frac{1}{4}[\sin 30° + \sin (-10°)]$$

$$\Rightarrow k = \frac{1}{4} \sin 10° + \frac{1}{4} \sin 30° - \frac{1}{4} \sin 10°$$

$$\Rightarrow k = \frac{1}{4} \sin 30° = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

Hence, the value of the filler is $\mathbf{\frac{1}{8}}$.

**Q62.** If $\tan A = \dfrac{1 - \cos B}{\sin B}$, then $\tan 2A =$ ...............

**Sol.** Given that: $\tan A = \dfrac{1 - \cos B}{\sin B}$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2\left(\frac{1 - \cos B}{\sin B}\right)}{1 - \left(\frac{1 - \cos B}{\sin B}\right)^2}$$

$$= \frac{2\left(\frac{2 \sin^2 B/2}{2 \sin B/2 \cos B/2}\right)}{1 - \left(\frac{2 \sin^2 B/2}{2 \sin B/2 \cos B/2}\right)^2} \qquad \left[\begin{array}{l} \because 1 - \cos B = 2 \sin^2 B/2 \\ \quad \sin B = 2 \sin B/2 \cos B/2 \end{array}\right]$$

$$= \frac{2\left(\frac{\sin B/2}{\cos B/2}\right)}{1 - \left(\frac{\sin B/2}{\cos B/2}\right)^2} = \frac{2 \tan B/2}{1 - \tan^2 B/2} = \tan B$$

So, tan 2A = tan B

Hence, the value of the filler is **tan B**.

**Q63.** If $\sin x + \cos x = a$ then,

($i$) $\sin^6 x + \cos^6 x =$ ....... ($ii$) $|\sin x - \cos x|$

**Sol.** Given that: $\sin x + \cos x = a$

$$(\sin x + \cos x)^2 = a^2$$

$$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x \cos x = a^2$$

$$\Rightarrow 1 + 2\sin x \cos x = a^2$$

$$\Rightarrow \sin x \cos x = \frac{a^2 - 1}{2} \qquad ...(i)$$

($i$) $\sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$

$$= (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x(\sin^2 x + \cos^2 x)$$

$$= (1)^3 - 3\left(\frac{a^2 - 1}{2}\right)^2 .1 = 1 - \frac{3(a^2 - 1)^2}{4}$$

$$= \frac{1}{4}[4 - 3(a^2 - 1)^2]$$

Hence, the value of the filler is $\mathbf{\frac{1}{4}[4 - 3(a^2 - 1)^2]}$

($ii$) $|\sin x - \cos x|^2 = \sin^2 x + \cos^2 x - 2\sin x \cos x$

$$= 1 - 2\left(\frac{a^2 - 1}{2}\right) = 1 - (a^2 - 1) = 1 - a^2 + 1$$

$$= 2 - a^2$$

$$\therefore \quad |\sin x - \cos x| = \sqrt{2 - a^2} \qquad [\because |\sin x - \cos x| > 0]$$

Hence, the value of the filler is $\mathbf{\sqrt{2 - a^2}}$.

**Q64.** In a ΔABC with ∠C = 90°, the equation whose roots are tan A and tan B is ...............

**Sol.** Given a ΔABC with ∠C = 90°

So, the equation whose roots are tan A and tan B is

$x^2 - (\tan A + \tan B)x + \tan A.\tan B = 0$

$$A + B = 90° \qquad [\because \angle C = 90°]$$

$$\Rightarrow \tan(A + B) = \tan 90°$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{1}{0}$$

$$\Rightarrow 1 - \tan A \tan B = 0$$

$$\Rightarrow \tan A \tan B = 1 \qquad ...(i)$$

Now $\tan A + \tan B = \frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}$

$$= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}$$

$$= \frac{\sin(A+B)}{\cos A \cos B} = \frac{\sin 90^\circ}{\cos A . \cos (90^\circ - A)}$$

$$= \frac{1}{\cos A \sin A}$$

$$\therefore \quad \tan A + \tan B = \frac{2}{2 \sin A \cos A} = \frac{2}{\sin 2A} \quad ...(ii)$$

From (*i*) and (*ii*) we get

$$x^2 - \left(\frac{2}{\sin 2A}\right)x + 1 = 0$$

Hence, the value of the filler is $\boldsymbol{x^2 - \left(\frac{2}{\sin 2A}\right)x + 1 = 0}$.

**Q65.** $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$ ...........

**Sol.** Given expression is

$$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$$

$$= 3[\sin^2 x + \cos^2 x - 2 \sin x \cos x]^2 + 6(\sin^2 x + \cos^2 x + 2 \sin x \cos x) + 4 [(\sin^2 x)^3 + (\cos^2 x)^3]$$

$$= 3[1 - 2 \sin x \cos x]^2 + 6(1 + 2 \sin x \cos x) + 4[(\sin^2 x + \cos^2 x)^3 - 3 \sin^2 x \cos^2 x (\sin^2 x + \cos^2 x)]$$

$$= 3[1 + 4 \sin^2 x \cos^2 x - 4 \sin x \cos x] + 6(1 + 2 \sin x \cos x) + 4[1 - 3 \sin^2 x \cos^2 x]$$

$$= 3 + 12 \sin^2 x \cos^2 x - 12 \sin x \cos x + 6 + 12 \sin x \cos x + 4 - 12 \sin^2 x \cos^2 x$$

$$= 3 + 6 + 4 = 13$$

Hence, the value of the filler is **13**.

**Q66.** Given $x > 0$, the values of $f(x) = -3 \cos \sqrt{3 + x + x^2}$ lie in the interval ................

**Sol.** Given that: $f(x) = -3 \cos \sqrt{3 + x + x^2}$

Put $\sqrt{3 + x + x^2} = y$

$\therefore \quad f(x) = -3 \cos y$

$\because \quad -1 \le \cos y \le 1$

$\quad 3 \ge -3 \cos y \ge -3$

$\Rightarrow \quad -3 \le -3 \cos y \le 3$

$\therefore \quad -3 \le -3 \cos \sqrt{3 + x + x^2} \le 3, x > 0$

Hence, the value of the filler is **[– 3, 3]**.

**Q67.** The maximum distance of a point on the graph of the function $y = \sqrt{3} \sin x + \cos x$ from $x$-axis is ...............

**Sol.** Given that $y = \sqrt{3}\sin x + \cos x$ ...(*i*)

$\therefore$ The maximum distance from a point on the graph of eqn. (*i*) from $x$-axis

$$= \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = 2.$$

Hence, the value of the filler is **2**.

**State True or False for the Statements in Each of the Exercises 68 to 76.**

**Q68.** If $\tan A = \dfrac{1-\cos B}{\sin B}$, then $\tan 2A = \tan B$.

**Sol.** Given that: $\tan A = \dfrac{1-\cos B}{\sin B} = \dfrac{2\sin^2 B/2}{2\sin B/2 \cos B/2} = \tan\dfrac{B}{2}$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\tan B/2}{1-\tan^2 B/2}$$

$\therefore$ $\tan 2A = \tan B$

Hence, the statement is 'True'.

**Q69.** The equality $\sin A + \sin 2A + \sin 3A = 3$ holds for some real value of A.

**Sol.** Given that: $\sin A + \sin 2A + \sin 3A = 3$

Since the maximum value of sin A is 1 but for sin 2A and sin 3A it is not equal to 1. So it is not possible.

Hence, the given statement is 'False'.

**Q70.** Sin 10° is greater than cos 10°

**Sol.** If $\sin 10° > \cos 10°$

$\Rightarrow$ $\sin 10° > \cos(90° - 80°)$

$\Rightarrow$ $\sin 10° > \sin 80°$ which is not possible.

Hence, the statement is 'False'.

**Q71.** $\cos\dfrac{2\pi}{15}.\cos\dfrac{4\pi}{15}.\cos\dfrac{8\pi}{15}.\cos\dfrac{16\pi}{15} = \dfrac{1}{16}$

**Sol.** L.H.S. $\cos\dfrac{2\pi}{15}.\cos\dfrac{4\pi}{15}.\cos\dfrac{8\pi}{15}.\cos\dfrac{16\pi}{15}$

$= \cos 24°.\cos 48°.\cos 96°.\cos 192°$

$$= \frac{1}{16\sin 24°}[(2\sin 24°\cos 24°)(2\cos 48°)(2\cos 96°)(2\cos 192°)]$$

$$= \frac{1}{16\sin 24°}[\sin 48°.2\cos 48°\,(2\cos 96°)(2\cos 192°)]$$

$$= \frac{1}{16\sin 24°}[2\sin 48°\cos 48°\,(2\cos 96°)(2\cos 192°)]$$

$$= \frac{1}{16\sin 24°}[\sin 96°(2\cos 96°)(2\cos 192°)]$$

$$= \frac{1}{16 \sin 24°}[2 \sin 96°.\cos 96° (2 \cos 192°)]$$

$$= \frac{1}{16 \sin 24°}[\sin 192°.(2 \cos 192°)]$$

$$= \frac{1}{16 \sin 24°} 2 \sin 192° \cos 192°$$

$$= \frac{1}{16 \sin 24°} \sin 384° = \frac{1}{16 \sin 24°} \sin(360° + 24°)$$

$$= \frac{1}{16 \sin 24°} \times \sin 24° \qquad [\because \sin (360° + \theta) = \sin \theta]$$

$$= \frac{1}{16} \text{ R.H.S.}$$

Hence, the given statement is 'True'.

**Q72.** One value of $\theta$ which satisfies the equation $\sin^4 \theta - 2 \sin^2 \theta - 1 = 0$ lies between 0 and $2\pi$.

**Sol.** Given equation is

$\sin^4 \theta - 2 \sin^2 \theta - 1 = 0$

$$\sin^2 \theta = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times -1}}{2 \times 1}$$

$$= \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$$

$$= 1 \pm \sqrt{2}$$

$$\therefore \quad \sin^2 \theta = \left(1 + \sqrt{2}\right) \text{ or } \left(1 - \sqrt{2}\right) \Rightarrow -1 \le \sin \theta \le 1$$

$$\Rightarrow \quad \sin^2 \theta \le 1 \text{ but } \sin^2 \theta = \left(1 + \sqrt{2}\right) \text{ or } \left(1 - \sqrt{2}\right)$$

Which is not possible.

Hence, the given statement is 'False'.

**Q73.** If cosec $x = 1 + \cot x$, then $x = 2n\pi,\ 2n\pi + \frac{\pi}{2}$.

**Sol.** Given that: $\quad \operatorname{cosec} x = 1 + \cot x$

$$\Rightarrow \quad \frac{1}{\sin x} = 1 + \frac{\cos x}{\sin x}$$

$$\Rightarrow \quad \frac{1}{\sin x} = \frac{\sin x + \cos x}{\sin x}$$

$$\Rightarrow \quad \sin x + \cos x = 1$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$$x = 2n\pi + \frac{\pi}{4} + \frac{\pi}{4} \Rightarrow x = 2n\pi + \frac{\pi}{2}$$

or $$x = 2n\pi + \frac{\pi}{4} - \frac{\pi}{4} \Rightarrow x = 2n\pi$$

Hence, the given statement is 'True'.

**Q74.** If $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$ then $\theta = \frac{n\pi}{3} + \frac{\pi}{9}$.

**Sol.** Given that: $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

$$\Rightarrow \quad \tan\theta + \tan 2\theta = \sqrt{3} - \sqrt{3}\tan\theta\tan 2\theta$$

$$\Rightarrow \quad \tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$$

$$\Rightarrow \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3}$$

$$\Rightarrow \quad \tan(\theta + 2\theta) = \sqrt{3}$$

$$\Rightarrow \quad \tan 3\theta = \tan\frac{\pi}{3} \quad \therefore 3\theta = n\pi + \frac{\pi}{3}$$

So $$\theta = \frac{n\pi}{3} + \frac{\pi}{9}$$

Hence, the given statement is 'True'.

**Q75.** If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then $\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$.

**Sol.** Given that: $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$

$$\Rightarrow \quad \tan(\pi\cos\theta) = \tan\left(\frac{\pi}{2} - \pi\sin\theta\right)$$

$$\Rightarrow \quad \pi\cos\theta = \frac{\pi}{2} - \pi\sin\theta$$

$$\Rightarrow \quad \pi\cos\theta + \pi\sin\theta = \frac{\pi}{2}$$

$$\Rightarrow \quad \cos\theta + \sin\theta = \frac{1}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \cos\frac{\pi}{4}\cos\theta + \sin\frac{\pi}{4}\sin\theta = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \qquad \cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}} \quad \left[\because \cos\left(\theta - \frac{\pi}{4}\right) \text{or} \cos\left(\frac{\pi}{4} - \theta\right)\right]$$

Hence, the given statement is 'True'.

**Q76.** In the following matches each of the item given under the column $C_1$ to its correct answer given under the column $C_2$.

| | | | |
|---|---|---|---|
| (*a*) | $\sin(x+y)\sin(x-y)$ | (*i*) | $\cos^2 x - \cos^2 y$ |
| (*b*) | $\cos(x+y)\cos(x-y)$ | (*ii*) | $\dfrac{1-\tan\theta}{1+\tan\theta}$ |
| (*c*) | $\cot\left(\dfrac{\pi}{4}+\theta\right)$ | (*iii*) | $\dfrac{1+\tan\theta}{1-\tan\theta}$ |
| (*d*) | $\tan\left(\dfrac{\pi}{4}+\theta\right)$ | (*iv*) | $\sin^2 x - \sin^2 y$ |

**Sol.** (*a*) $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$

$\therefore$ (*a*) $\leftrightarrow$ (*iv*)

(*b*) $\cos(x+y)\cos(x-y) = \cos^2 x - \cos^2 y$

$\therefore$ (*b*) $\leftrightarrow$ (*i*)

(*c*)
$$\cot\left(\frac{\pi}{4}+\theta\right) = \frac{\cot\frac{\pi}{4}\cot\theta - 1}{\cot\theta + \cot\frac{\pi}{4}}$$
$$= \frac{\cot\theta - 1}{\cot\theta + 1} = \frac{1-\tan\theta}{1+\tan\theta}$$

$\therefore$ (*c*) $\leftrightarrow$ (*ii*)

(*d*)
$$\tan\left(\frac{\pi}{4}+\theta\right) = \frac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\tan\theta} = \frac{1+\tan\theta}{1-\tan\theta}$$

$\therefore$ (*d*) $\leftrightarrow$ (*iii*)

Hence, (*a*) $\leftrightarrow$ (*iv*), (*b*) $\leftrightarrow$ (*i*), (*c*) $\leftrightarrow$ (*ii*) and (*d*) $\leftrightarrow$ (*iii*).