

EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. Find the term independent of x , $x \neq 0$ in the expansion of

$$\left(\frac{3x^2}{2} - \frac{1}{3x} \right)^{15}$$

Sol. General Term $T_{r+1} = {}^n C_r x^{n-r} y^r$

$$\begin{aligned} &= {}^{15} C_r \left(\frac{3x^2}{2} \right)^{15-r} \left(-\frac{1}{3x} \right)^r = {}^{15} C_r \left(\frac{3}{2} \right)^{15-r} \cdot (x)^{30-2r} \cdot \left(-\frac{1}{3} \right)^r \cdot \frac{1}{x^r} \\ &= {}^{15} C_r \left(\frac{3}{2} \right)^{15-r} \cdot (x)^{30-2r-r} (-1)^r \cdot \frac{1}{(3)^r} \\ &= {}^{15} C_r \left(\frac{3}{2} \right)^{15-r} \cdot x^{30-3r} (-1)^r \cdot \frac{1}{(3)^r} \end{aligned}$$

for getting the term independent of x ,

$$30 - 3r = 0 \Rightarrow r = 10$$

On putting the value of r in the above expression, we get

$$\begin{aligned} &= {}^{15} C_{10} \left(\frac{3}{2} \right)^{15-10} (-1)^{10} \cdot \frac{1}{(3)^{10}} = {}^{15} C_{10} \frac{(3)^5}{(2)^5} \cdot \frac{1}{(3)^{10}} \\ &= {}^{15} C_{10} \cdot \frac{1}{(2)^5 \cdot (3)^5} = {}^{15} C_{10} \left(\frac{1}{6} \right)^5 \end{aligned}$$

$$\text{Hence, the required term} = {}^{15} C_{10} \left(\frac{1}{6} \right)^5$$

Q2. If the term free from x in the expansion of $\left(\sqrt{x} - \frac{K}{x^2} \right)^{10}$ is 405, find the value of K .

Sol. The given expression is $\left(\sqrt{x} - \frac{K}{x^2} \right)^{10}$

General term $T_{r+1} = {}^n C_r x^{n-r} y^r$

$$= {}^{10} C_r (\sqrt{x})^{10-r} \left(\frac{-K}{x^2} \right)^r = {}^{10} C_r (x)^{\frac{10-r}{2}} (-K)^r \left(\frac{1}{x^{2r}} \right)$$

$$\begin{aligned}
 &= {}^{10}C_r(x)^{\frac{10-r}{2}-2r}(-K)^r = {}^{10}C_r(x)^{\frac{10-r-4r}{2}}(-K)^r \\
 &= {}^{10}C_r(x)^{\frac{10-5r}{2}}(-K)^r
 \end{aligned}$$

For getting term free from x , $\frac{10-5r}{2} = 0$
 $\Rightarrow r = 2$

On putting the value of r in the above expression,
 we get ${}^{10}C_2(-K)^2$

According to the condition of the question, we have

$$\begin{aligned}
 {}^{10}C_2 K^2 = 405 &\Rightarrow \frac{10 \cdot 9}{2 \cdot 1} K^2 = 405 \\
 \Rightarrow 45K^2 = 405 &\Rightarrow K^2 = \frac{405}{45} = 9
 \end{aligned}$$

$$\therefore K = \pm 3$$

Hence, the value of $K = \pm 3$

Q3. Find the coefficient of x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$

Sol. The given expression is $(1 - 3x + 7x^2)(1 - x)^{16}$
 $= (1 - 3x + 7x^2)[{}^{16}C_0(1)^{16}(-x)^0 + {}^{16}C_1(1)^{15}(-x)$
 $+ {}^{16}C_2(1)^{14}(-x)^2 + \dots]$
 $= (1 - 3x + 7x^2)(1 - 16x + 120x^2 \dots)$

Collecting the term containing x , we get $-16x - 3x = -19x$

Hence, the coefficient of $x = -19$

Q4. Find the term independent of x in the expansion of $\left(3x - \frac{2}{x^2}\right)^{15}$

Sol. Given expression is $\left(3x - \frac{2}{x^2}\right)^{15}$

General term $T_{r+1} = {}^nC_r x^{n-r} y^r$

$$\begin{aligned}
 &= {}^{15}C_r(3x)^{15-r} \left(-\frac{2}{x^2}\right)^r = {}^{15}C_r(3)^{15-r} \cdot x^{15-r}(-2)^r \cdot \frac{1}{x^{2r}} \\
 &= {}^{15}C_r(3)^{15-r} \cdot x^{15-r-2r} \cdot (-2)^r = {}^{15}C_r(3)^{15-r} \cdot x^{15-3r}(-2)^r
 \end{aligned}$$

For getting a term independent of x , put $15 - 3r = 0 \Rightarrow r = 5$

\therefore The required term is ${}^{15}C_5(3)^{15-5}(-2)^5$

$$\begin{aligned}
 &= -{}^{15}C_5(3)^{10}(2)^5 = -\frac{15 \times 14 \times 13 \times 12 \times 11}{5 \times 4 \times 3 \times 2 \times 1} \cdot (3)^{10}(2)^5 \\
 &= -7 \times 13 \times 3 \times 11 \cdot (3)^{10}(2)^5 = -3003(3)^{10}(2)^5
 \end{aligned}$$

Hence, the required term = $-3003(3)^{10}(2)^5$

Q5. Find the middle term (Terms) in the expansion of

$$(i) \left(\frac{x}{a} - \frac{a}{x}\right)^{10} \quad (ii) \left(3x - \frac{x^3}{6}\right)^9$$

Sol. (i) Given expression is $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$

Number of terms = $10 + 1 = 11$ (odd)

$$\therefore \text{Middle term} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} = \frac{11+1}{2} = \frac{12}{2} = 6^{\text{th}} \text{ term}$$

General Term $T_{r+1} = {}^n C_r x^{n-r} y^r$

$$\begin{aligned} \Rightarrow T_{5+1} &= {}^{10} C_5 \left(\frac{x}{a}\right)^{10-5} \left(-\frac{a}{x}\right)^5 = -{}^{10} C_5 \frac{x^5}{a^5} \cdot \frac{a^5}{x^5} = -{}^{10} C_5 \\ &= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = -9 \times 7 \times 4 = -252 \end{aligned}$$

Hence, the required middle term = -252

(ii) Given expression is $\left(3x - \frac{x^3}{6}\right)^9$

Number of terms = $9 + 1 = 10$ (even)

$$\begin{aligned} \therefore \text{Middle terms are } \frac{n}{2}^{\text{th}} \text{ term and } \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ term} \\ = \frac{10}{2}^{\text{th}} = 5^{\text{th}} \text{ term and } (5 + 1) = 6^{\text{th}} \text{ term} \end{aligned}$$

General Term $T_{r+1} = {}^n C_r x^{n-r} y^r$

$$\begin{aligned} \therefore T_5 = T_{4+1} &= {}^9 C_4 (3x)^{9-4} \left(-\frac{x^3}{6}\right)^4 \\ &= {}^9 C_4 (3)^5 \cdot x^5 \left(-\frac{1}{6}\right)^4 \cdot x^{12} \\ &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times \frac{3 \times 3 \times 3 \times 3 \times 3}{6 \times 6 \times 6 \times 6} x^{17} \\ &= \frac{189}{8} x^{17} \end{aligned}$$

$$\begin{aligned} \text{Now, } T_6 = T_{5+1} &= {}^9 C_5 (3x)^{9-5} \left(-\frac{x^3}{6}\right)^5 \\ &= {}^9 C_5 (3)^4 x^4 \left(-\frac{1}{6}\right)^5 \cdot x^{15} \end{aligned}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5}{5 \times 4 \times 3 \times 2 \times 1} (3)^4 \left(-\frac{1}{6}\right)^5 \cdot x^{19} = -\frac{21}{16} x^{19}$$

Hence, the required middle terms are $\frac{189}{8} x^{17}$ and $-\frac{21}{16} x^{19}$

Q6. Find the coefficient of x^{15} in the expansion of $(x - x^2)^{10}$.

Sol. The given expression is $(x - x^2)^{10}$

General Term $T_{r+1} = {}^n C_r x^{n-r} y^r$

$$= {}^{10} C_r (x)^{10-r} (-x^2)^r = {}^{10} C_r (x)^{10-r} (-1)^r \cdot (x^2)^r$$

$$= (-1)^r \cdot {}^{10} C_r (x)^{10-r+2r} = (-1)^r \cdot {}^{10} C_r (x)^{10+r}$$

To find the coefficient of x^{15} , Put $10 + r = 15 \Rightarrow r = 5$

$$\therefore \text{Coefficient of } x^{15} = (-1)^5 {}^{10} C_5 = -{}^{10} C_5 = -252$$

Hence, the required coefficient = -252

Q7. Find the coefficient of $\frac{1}{x^{17}}$ in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$

Sol. The given expression is $\left(x^4 - \frac{1}{x^3}\right)^{15}$

General Term $T_{r+1} = {}^n C_r x^{n-r} y^r = {}^{15} C_r (x^4)^{15-r} \left(-\frac{1}{x^3}\right)^r$

$$= {}^{15} C_r (x)^{60-4r} (-1)^r \cdot \frac{1}{x^{3r}} = {}^{15} C_r (-1)^r \cdot \frac{1}{x^{3r-60+4r}}$$

$$= {}^{15} C_r (-1)^r \cdot \frac{1}{x^{7r-60}}$$

To find the coefficient of $\frac{1}{x^{17}}$, Put $7r - 60 = 17$

$$\Rightarrow 7r = 60 + 17 \Rightarrow 7r = 77$$

$$\therefore r = 11$$

Putting the value of r in the above expression, we get

$$= {}^{15} C_{11} (-1)^{11} \cdot \frac{1}{x^{17}} = -{}^{15} C_4 \cdot \frac{1}{x^{17}}$$

$$= -\frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} \cdot \frac{1}{x^{17}} = -1365 \cdot \frac{1}{x^{17}}$$

Hence, the coefficient of $\frac{1}{x^{17}}$ = -1365

Q8. Find the sixth term of the expansion $(y^{1/2} + x^{1/3})^n$, if the binomial coefficient of the third term from the end is 45.

Sol. The given expression is $(y^{1/2} + x^{1/3})^n$, since the binomial coefficient of third term from the end = Binomial coefficient of third term from the beginning = ${}^n C_2$

$$\begin{aligned} \therefore & \quad {}^n C_2 = 45 \\ \Rightarrow & \quad \frac{n(n-1)}{2} = 45 \Rightarrow n^2 - n = 90 \\ \Rightarrow & \quad n^2 - n - 90 = 0 \Rightarrow n^2 - 10n + 9n - 90 = 0 \\ \Rightarrow & \quad n(n-10) + 9(n-10) = 0 \Rightarrow (n-10)(n+9) = 0 \\ \Rightarrow & \quad n = 10, n = -9 \Rightarrow n = 10, n \neq -9 \end{aligned}$$

So, the given expression becomes $(y^{1/2} + x^{1/3})^{10}$

Sixth term is this expression

$$\begin{aligned} T_6 &= T_{5+1} = {}^{10} C_5 (y^{1/2})^{10-5} (x^{1/3})^5 = {}^{10} C_5 y^{5/2} \cdot x^{5/3} \\ &= 252 y^{5/2} x^{5/3} \end{aligned}$$

Hence, the required term = $252 y^{5/2} \cdot x^{5/3}$

Q9. Find the value of r if the coefficients of $(2r+4)^{\text{th}}$ and $(r-2)^{\text{th}}$ terms in the expansion of $(1+x)^{18}$ are equal

Sol. General Term $T_{r+1} = {}^n C_r x^{n-r} y^r$

For coefficient of $(2r+4)^{\text{th}}$ term, we have

$$T_{2r+4} = T_{2r+3+1} = {}^{18} C_{2r+3} (1)^{18-2r-3} \cdot x^{2r+3}$$

$$\therefore \text{Coefficient of } (2r+4)^{\text{th}} \text{ term} = {}^{18} C_{2r+3}$$

Similarly, $T_{r-2} = T_{r-3+1} = {}^{18} C_{r-3} (1)^{18-r+3} \cdot x^{r-3}$

$$\therefore \text{Coefficient of } (r-2)^{\text{th}} \text{ term} = {}^{18} C_{r-3}$$

As per the condition of the questions, we have ${}^{18} C_{2r+3} = {}^{18} C_{r-3}$

$$\Rightarrow 2r+3+r-3 = 18 \Rightarrow 3r = 18 \Rightarrow r = 6$$

Q10. If the coefficient of second, third and fourth terms in the expansion of $(1+x)^{2n}$ are in A.P., show that $2n^2 - 9n + 7 = 0$

Sol. Given expression = $(1+x)^{2n}$

Coefficient of second term = ${}^{2n} C_1$

Coefficient of third term = ${}^{2n} C_2$

and coefficient of fourth term = ${}^{2n} C_3$

As the given condition, ${}^{2n} C_1$, ${}^{2n} C_2$ and ${}^{2n} C_3$ are in A.P.

$$\therefore {}^{2n} C_2 - {}^{2n} C_1 = {}^{2n} C_3 - {}^{2n} C_2$$

$$\Rightarrow 2 \cdot {}^{2n} C_2 = {}^{2n} C_1 + {}^{2n} C_3$$

$$\Rightarrow 2 \cdot \frac{2n!}{2!(2n-2)!} = \frac{2n!}{(2n-1)!} + \frac{2n!}{3!(2n-3)!}$$

$$\Rightarrow 2 \left[\frac{2n(2n-1)(2n-2)!}{2 \times 1 \times (2n-2)!} \right] = \frac{2n(2n-1)!}{(2n-1)!} + \frac{2n(2n-1)(2n-2)(2n-3)!}{3 \times 2 \times 1 \times (2n-3)!}$$

$$\Rightarrow n(2n-1) = n + \frac{n(2n-1)(2n-2)}{6}$$

$$\Rightarrow 2n-1 = 1 + \frac{(2n-1)(2n-2)}{6}$$

$$\begin{aligned} \Rightarrow 12n - 6 &= 6 + 4n^2 - 4n - 2n + 2 \\ \Rightarrow 12n - 12 &= 4n^2 - 6n + 2 \\ \Rightarrow 4n^2 - 6n - 12n + 2 + 12 &= 0 \\ \Rightarrow 4n^2 - 18n + 14 &= 0 \\ \Rightarrow 2n^2 - 9n + 7 &= 0 \end{aligned}$$

Hence proved.

Q11. Find the coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^{11}$.

Sol. Given expression is $(1 + x + x^2 + x^3)^{11}$
 $= [(1 + x) + x^2(1 + x)]^{11} = [(1 + x)(1 + x^2)]^{11}$
 $= (1 + x)^{11} \cdot (1 + x^2)^{11}$

Expanding the above expression, we get

$$({}^{11}C_0 + {}^{11}C_1x + {}^{11}C_2x^2 + {}^{11}C_3x^3 + {}^{11}C_4x^4 + \dots)$$

$$({}^{11}C_0 + {}^{11}C_1x^2 + {}^{11}C_2x^4 + \dots)$$

$$= (1 + 11x + 55x^2 + 165x^3 + 330x^4 \dots) \cdot (1 + 11x^2 + 55x^4 + \dots)$$

Collecting the terms containing x^4 , we get

$$(55 + 605 + 330)x^4 = 990x^4$$

Hence, the coefficient of $x^4 = 990$

LONG ANSWER TYPE QUESTIONS

Q12. If P is a real number and if the middle term in the expansion

of $\left(\frac{P}{2} + 2\right)^8$ is 1120, find P.

Sol. Given expression is $\left(\frac{P}{2} + 2\right)^8$

Number of terms = $8 + 1 = 9$ (odd)

$$\therefore \text{Middle term} = \frac{9+1}{2} \text{th term} = 5 \text{th term}$$

$$\therefore T_5 = T_{4+1} = {}^8C_4 \left(\frac{P}{2}\right)^{8-4} (2)^4$$

$$= {}^8C_4 \frac{P^4}{2^4} \times 2^4 = {}^8C_4 P^4$$

$$\text{Now } {}^8C_4 P^4 = 1120 \Rightarrow \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} P^4 = 1120$$

$$\Rightarrow 70P^4 = 1120$$

$$\Rightarrow P^4 = \frac{1120}{70} = 16 \Rightarrow P^4 = 2^4 \Rightarrow P = \pm 2$$

Hence, the required value of $P = \pm 2$

Q13. Show that the middle term in the expansion of $\left(x - \frac{1}{x}\right)^{2n}$ is

$$\frac{1 \times 3 \times 5 \times \dots (2n-1)}{n!} \times (-2)^n$$

Sol. Given expression is $\left(x - \frac{1}{x}\right)^{2n}$

Number of terms = $2n + 1$ (odd)

\therefore Middle term = $\frac{2n+1+1}{2}$ th term i.e., $(n+1)$ th term

General Term $T_{r+1} = {}^nC_r (x)^{n-r} (y)^r$

$$\begin{aligned} \therefore T_{n+1} &= {}^{2n}C_n (x)^{2n-n} \left(-\frac{1}{x}\right)^n = {}^{2n}C_n (x)^n (-1)^n \cdot \frac{1}{x^n} \\ &= (-1)^n \cdot {}^{2n}C_n = (-1)^n \cdot \frac{2n!}{n!(2n-n)!} \\ &= (-1)^n \cdot \frac{2n!}{n!n!} = (-1)^n \cdot \frac{2n(2n-1)(2n-2)(2n-3)\cdots 1}{n!n(n-1)(n-2)(n-3)\cdots 1} \\ &= (-1)^n \frac{2n \cdot (2n-1) \cdot 2(n-1)(2n-3)\cdots 1}{n!n(n-1)(n-2)(n-3)\cdots 1} \\ &= \frac{(-1)^n \cdot 2^n \cdot [n(n-1)(n-2)\cdots] \cdot [(2n-1) \cdot (2n-3)\cdots 5 \cdot 3 \cdot 1]}{n! \cdot n(n-1)(n-2)\cdots 1} \\ &= \frac{(-2)^n [(2n-1)(2n-3)\cdots 5 \cdot 3 \cdot 1]}{n!} \\ &= \frac{1 \times 3 \times 5 \times \cdots (2n-1)}{n!} \times (-2)^n \end{aligned}$$

Hence, the middle term = $\frac{1 \times 3 \times 5 \times \cdots (2n-1)}{n!} \times (-2)^n$

Q14. Find n in the binomial $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$ if the ratio of 7th term from the beginning to the 7th term from the end is $1/6$.

Sol. The given expression is $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$

$$= \left(2^{1/3} + \frac{1}{3^{1/3}}\right)^n$$

General Term $T_{r+1} = {}^nC_r x^{n-r} y^r$

$$\begin{aligned} T_7 = T_{6+1} &= {}^nC_6 (2^{1/3})^{n-6} \left(\frac{1}{3^{1/3}}\right)^6 \\ &= {}^nC_6 (2)^{n-6/3} \cdot \left(\frac{1}{3^2}\right) = {}^nC_6 (2)^{n-6/3} \cdot (3)^{-2} \end{aligned}$$

7th term from the end = $(n-7+2)$ th term from the beginning

= $(n - 5)^{\text{th}}$ term from the beginning

$$\begin{aligned} \text{So, } T_{n-6+1} &= {}^n C_{n-6} (2^{1/3})^{n-n+6} \left(\frac{1}{3^{1/3}} \right)^{n-6} \\ &= {}^n C_{n-6} (2)^2 \cdot \left(\frac{1}{3^{n-6/3}} \right) = {}^n C_{n-6} (2)^2 (3)^{6-n/3} \end{aligned}$$

According to the question, we get

$$\begin{aligned} \frac{{}^n C_{n-6} (2)^2 (3)^{6-n/3}}{{}^n C_{n-6} (2)^2 (3)^{6-n/3}} &= \frac{1}{6} \\ \Rightarrow \frac{{}^n C_{n-6} (2)^2 (3)^{6-n/3}}{{}^n C_{n-6} (2)^2 (3)^{6-n/3}} &= \frac{1}{6} \Rightarrow (2)^{\frac{n-6}{3}-2} \cdot (3)^{-2-\frac{6-n}{3}} = \frac{1}{6} \\ \Rightarrow (2)^{\frac{n-6-6}{3}} \cdot (3)^{\frac{-6-6+n}{3}} &= \frac{1}{6} \Rightarrow (2)^{\frac{n-12}{3}} \cdot (3)^{\frac{n-12}{3}} = (6)^{-1} \\ \Rightarrow (6)^{\frac{n-12}{3}} &= (6)^{-1} \\ \Rightarrow \frac{n-12}{3} &= -1 \Rightarrow n-12 = -3 \Rightarrow n = 12-3 = 9 \end{aligned}$$

Hence, the required value of n is 9.

- Q15.** In the expansion of $(x + a)^n$ if the sum of odd terms is denoted by O and the sum of even terms by E then prove that
(i) $O^2 - E^2 = (x^2 - a^2)^n$ (ii) $4OE = (x + a)^{2n} - (x - a)^{2n}$

Sol. Given expression is $(x + a)^n$

$$(x + a)^n = {}^n C_0 x^n a^0 + {}^n C_1 x^{n-1} a + {}^n C_2 x^{n-2} a^2 + {}^n C_3 x^{n-3} a^3 + \dots + {}^n C_n a^n$$

Sum of odd terms,

$$O = {}^n C_0 x^n + {}^n C_2 x^{n-2} a^2 + {}^n C_4 x^{n-4} a^4 + \dots$$

and the sum of even terms,

$$E = {}^n C_1 x^{n-1} \cdot a + {}^n C_3 x^{n-3} a^3 + {}^n C_5 x^{n-5} a^5 + \dots$$

$$\text{Now } (x + a)^n = O + E \quad \dots(i)$$

$$\text{Similarly } (x - a)^n = O - E \quad \dots(ii)$$

Multiplying eq. (i) and eq. (ii), we get,

$$(x + a)^n (x - a)^n = (O + E)(O - E)$$

$$\Rightarrow (x^2 - a^2)^n = O^2 - E^2$$

$$\text{Hence } O^2 - E^2 = (x^2 - a^2)^n$$

$$\begin{aligned} \text{(ii) } 4OE &= (O + E)^2 - (O - E)^2 \\ &= [(x + a)^n]^2 - [(x - a)^n]^2 \\ &= [x + a]^{2n} - [x - a]^{2n} \end{aligned}$$

$$\text{Hence, } 4OE = (x + a)^{2n} - (x - a)^{2n}$$

Q16. If x^p occurs in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2n}$, then prove that its coefficient is $\frac{2n!}{\left(\frac{4n-p}{3}\right)! \left(\frac{2n+p}{3}\right)!}$.

Sol. Given expression is $\left(x^2 + \frac{1}{x}\right)^{2n}$

$$\begin{aligned} \text{General terms, } T_{r+1} &= {}^{2n}C_r x^{2n-r} y^r \\ &= {}^{2n}C_r (x^2)^{2n-r} \cdot \left(\frac{1}{x}\right)^r = {}^{2n}C_r (x)^{4n-2r} \cdot \frac{1}{x^r} \\ &= {}^{2n}C_r (x)^{4n-2r-r} = {}^{2n}C_r (x)^{4n-3r} \end{aligned}$$

If x^p occurs in $\left(x^2 + \frac{1}{x}\right)^{2n}$

$$\text{then } 4n - 3r = p \Rightarrow 3r = 4n - p$$

$$\Rightarrow r = \frac{4n - p}{3}$$

$$\therefore \text{Coefficient of } x^p = {}^{2n}C_r = {}^{2n}C_{\frac{4n-p}{3}}$$

$$\begin{aligned} &= \frac{(2n)!}{\left(\frac{4n-p}{3}\right)! \left(2n - \frac{4n-p}{3}\right)!} = \frac{(2n)!}{\left(\frac{4n-p}{3}\right)! \left(\frac{6n-4n+p}{3}\right)!} \\ &= \frac{(2n)!}{\left(\frac{4n-p}{3}\right)! \left(\frac{2n+p}{3}\right)!} \end{aligned}$$

$$\text{Hence, the coefficient of } x^p = \frac{(2n)!}{\left(\frac{4n-p}{3}\right)! \left(\frac{2n+p}{3}\right)!}$$

Q17. Find the term independent of x in the expansion of

$$\left(1 + x + 2x^3\right) \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$$

Sol. Given expression is $\left(1 + x + 2x^3\right) \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

Let us consider $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

General Term $T_{r+1} = {}^9C_r x^{9-r} y^r$

$$\begin{aligned}
 T_{r+1} &= {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(-\frac{1}{3x}\right)^r \\
 &= {}^9C_r \left(\frac{3}{2}\right)^{9-r} (x)^{18-2r} \cdot \left(-\frac{1}{3}\right)^r \cdot \frac{1}{(x)^r} \\
 &= {}^9C_r \left(\frac{3}{2}\right)^{9-r} (x)^{18-2r-r} \cdot \left(-\frac{1}{3}\right)^r \\
 &= {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r \cdot x^{18-3r}
 \end{aligned}$$

So, the general term in the expansion of

$$\begin{aligned}
 &(1+x+2x^3) \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9 \\
 &= {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r \cdot (x)^{18-3r} + {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r \cdot (x)^{19-3r} \\
 &\quad + 2 \cdot {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r \cdot (x)^{21-3r}
 \end{aligned}$$

For getting the term independent of x ,

Put $18 - 3r = 0$, $19 - 3r = 0$ and $21 - 3r = 0$, we get

$$r = 6, r = \frac{19}{3} \text{ and } r = 7$$

The possible value of r are 6 and 7

$$\left(\because r \neq \frac{19}{3}\right)$$

\therefore The term independent of x is

$$\begin{aligned}
 &= {}^9C_6 \left(\frac{3}{2}\right)^{9-6} \left(-\frac{1}{3}\right)^6 + 2 \cdot {}^9C_7 \left(\frac{3}{2}\right)^{9-7} \left(-\frac{1}{3}\right)^7 \\
 &= \frac{9 \times 8 \times 7 \times 6!}{3 \times 2 \times 1 \times 6!} \cdot \frac{3^3}{2^3} \cdot \frac{1}{3^6} - 2 \cdot \frac{9 \times 8 \times 7!}{7!2 \times 1} \cdot \frac{3^2}{2^2} \cdot \frac{1}{3^7} \\
 &= \frac{84}{8} \cdot \frac{1}{3^3} - \frac{36}{4} \cdot \frac{2}{3^5} = \frac{7}{18} - \frac{2}{27} = \frac{21-4}{54} = \frac{17}{54}
 \end{aligned}$$

Hence, the required term = $\frac{17}{54}$

OBJECTIVE TYPE QUESTIONS

Q18. The total number of terms in the expansion of $(x+a)^{100} + (x-a)^{100}$ after simplification is

- (a) 50 (b) 202
(c) 51 (d) None of these

Sol. Number of terms in the expansion of $(x+a)^{100} = 101$
Number of terms in the expansion of $(x-a)^{100} = 101$

Now 50 terms of expansion will cancel out with negative 50 terms of $(x - a)^{100}$

So, the remaining 51 terms of first expansion will be added to 51 terms of other.

Therefore, the number of terms = 51

Hence, the correct option is (c).

Q19. If the integers $r > 1$, $n > 2$ and coefficients of $(3r)^{\text{th}}$ and $(r + 2)^{\text{th}}$ terms in the Binomial expansion of $(1 + x)^{2n}$ are equal, then

(a) $n = 2r$

(b) $n = 3r$

(c) $n = 2r + 1$

(d) none of these

Sol. Given that $r > 1$ and $n > 2$

$$\text{then } T_{3r} = T_{3r-1+1} = {}^{2n}C_{3r-1} \cdot x^{3r-1}$$

$$\text{and } T_{r+2} = T_{r+1+1} = {}^{2n}C_{r+1} x^{r+1}$$

As per the question, we have

$${}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$$

$$\Rightarrow 3r - 1 + r + 1 = 2n \quad [\because {}^nC_p = {}^nC_q \Rightarrow n = p + q]$$

$$\Rightarrow 4r = 2n$$

$$n = 2r$$

Hence, the correct option is (a).

Q20. The two successive terms in the expansion of $(1 + x)^{24}$ whose coefficients are in the ratio 1 : 4 are

(a) 3rd and 4th

(b) 4th and 5th

(c) 5th and 6th

(d) 6th and 7th

Sol. Let r^{th} and $(r + 1)^{\text{th}}$ be two successive terms in the expansion $(1 + x)^{24}$

$$\therefore T_{r+1} = {}^{24}C_r \cdot x^r$$

$$T_{r+2} = T_{r+1+1} = {}^{24}C_{r+1} x^{r+1}$$

$$\text{As per the question, we have } \frac{{}^{24}C_r}{{}^{24}C_{r+1}} = \frac{1}{4}$$

$$\Rightarrow \frac{r!(24-r)!}{24!} = \frac{1}{4}$$

$$\Rightarrow \frac{24!}{r!(24-r)!} \times \frac{(r+1)!(24-r-1)!}{24!} = \frac{1}{4}$$

$$\Rightarrow \frac{(r+1) \cdot r!(24-r-1)!}{r!(24-r)(24-r-1)!} = \frac{1}{4}$$

$$\Rightarrow \frac{r+1}{24-r} = \frac{1}{4}$$

$$\Rightarrow 4r + 4 = 24 - r$$

$$\Rightarrow 5r = 20 \Rightarrow r = 4$$

$$\therefore T_{4+1} = T_5 \quad \text{and} \quad T_{4+2} = T_6$$

Hence, the correct option is (c).

Q21. The coefficient of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ are in the ratio.

$$(a) 1 : 2 \quad (b) 1 : 3 \quad (c) 3 : 1 \quad (d) 2 : 1$$

Sol. General Term $T_{r+1} = {}^n C_r x^{n-r} y^r$

In the expansion of $(1+x)^{2n}$, we get $T_{r+1} = {}^{2n} C_r x^r$

To get the coefficient of x^n , put $r = n$

$$\therefore \text{Coefficient of } x^n = {}^{2n} C_n$$

In the expansion of $(1+x)^{2n-1}$, we get $T_{r+1} = {}^{2n-1} C_r x^r$

$$\therefore \text{Coefficient of } x^n \text{ is } {}^{2n-1} C_{n-1}$$

$$\text{The required ratio is } \frac{{}^{2n} C_n}{{}^{2n-1} C_{n-1}}$$

$$\begin{aligned} &= \frac{\frac{2n!}{n!(n!)}}{(2n-1)!} = \frac{\frac{2n!}{n! \cdot n!}}{(2n-1)!} \\ &= \frac{(n-1)!(2n-1-n+1)!}{(n-1)!(n!)} \\ &= \frac{2n!}{n!n!} \times \frac{(n-1)! \cdot n!}{(2n-1)!} = \frac{2n(2n-1)!}{n!n(n-1)!} \times \frac{(n-1)! \cdot n!}{(2n-1)!} \\ &= \frac{2}{1} = 2 : 1 \end{aligned}$$

Hence, the correct option is (d).

Q22. If the coefficients of 2nd, 3rd and the 4th terms in the expansion of $(1+x)^n$ are in A.P. Then value of n is

$$(a) 2 \quad (b) 7 \quad (c) 1 \quad (d) 14$$

Sol. Given expression is $(1+x)^n$

$$(1+x)^n = {}^n C_0 + {}^n C_1 x + {}^n C_2 x^2 + {}^n C_3 x^3 + \dots + {}^n C_n x^n$$

Here, coefficient of 2nd term = ${}^n C_1$

Coefficient of 3rd term = ${}^n C_2$

and coefficient of 4th term = ${}^n C_3$

Given that ${}^n C_1$, ${}^n C_2$ and ${}^n C_3$ are in A.P.

$$\therefore 2 \cdot {}^n C_2 = {}^n C_1 + {}^n C_3$$

$$\Rightarrow 2 \cdot \frac{n(n-1)}{2} = n + \frac{n(n-1)(n-2)}{3 \cdot 2 \cdot 1}$$

$$\Rightarrow n(n-1) = n + \frac{n(n-1)(n-2)}{6}$$

$$\Rightarrow n-1 = 1 + \frac{(n-1)(n-2)}{6}$$

$$\begin{aligned} \Rightarrow & 6n - 6 = 6 + n^2 - 3n + 2 \\ \Rightarrow & n^2 - 3n - 6n + 14 = 0 \Rightarrow n^2 - 9n + 14 = 0 \\ \Rightarrow & n^2 - 7n - 2n + 14 = 0 \Rightarrow n(n-7) - 2(n-7) = 0 \\ \Rightarrow & (n-2)(n-7) = 0 \Rightarrow n = 2, 7 \Rightarrow n = 7 \end{aligned}$$

whereas $n = 2$ is not possible

Hence, the correct option is (b).

Q23. If A and B are coefficient of x^n in the expansions of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then A/B equals

- (a) 1 (b) 2 (c) 1/2 (d) 1/n

Sol. Given expression is $(1+x)^{2n}$

$$T_{r+1} = {}^{2n}C_r x^r$$

\therefore Coefficient of $x^n = {}^{2n}C_n = A$ (Given)

In the expression $(1+x)^{2n-1}$

$$T_{r+1} = {}^{2n-1}C_r x^r$$

\therefore Coefficient of $x^n = {}^{2n-1}C_n = B$ (Given)

$$\text{So, } \frac{A}{B} = \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{2}{1} \quad [\text{from Q. no. 21}]$$

Hence, the correct option is (b).

Q24. If the middle term of $\left(\frac{1}{x} + x \sin x\right)^{10}$ is equal to $7\frac{7}{8}$, then value of x is

- (a) $2n\pi + \frac{\pi}{6}$ (b) $n\pi + \frac{\pi}{6}$
(c) $n\pi + (-1)^n \frac{\pi}{6}$ (d) $n\pi + (-1)^n \frac{\pi}{3}$

Sol. Given expression is $\left(\frac{1}{x} + x \sin x\right)^{10}$

Number of terms = $10 + 1 = 11$ odd

\therefore Middle term = $\frac{11+1}{2}$ th term = 6th term

$$T_6 = T_{5+1} = {}^{10}C_5 \left(\frac{1}{x}\right)^{10-5} (x \sin x)^5$$

$$\therefore {}^{10}C_5 \left(\frac{1}{x}\right)^5 \cdot x^5 \cdot \sin^5 x = 7\frac{7}{8} \Rightarrow {}^{10}C_5 \cdot \sin^5 x = \frac{63}{8}$$

$$\Rightarrow \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \sin^5 x = \frac{63}{8} \Rightarrow 252 \cdot \sin^5 x = \frac{63}{8}$$

$$\Rightarrow \sin^5 x = \frac{63}{8 \times 252} \Rightarrow \sin^5 x = \frac{1}{32}$$

$$\Rightarrow \sin^5 x = \left(\frac{1}{2}\right)^5 \Rightarrow \sin x = \frac{1}{2}$$

$$\Rightarrow \sin x = \sin \frac{\pi}{6}$$

$$\therefore x = n\pi + (-1)^n \cdot \frac{\pi}{6}$$

Hence, the correct option is (c).

FILL IN THE BLANKS

Q25. The largest coefficient in the expansion of $(1 + x)^{30}$ is _____.

Sol. Here $n = 30$ which is even

$$\therefore \text{the largest coefficient in } (1 + x)^n = {}^n C_{n/2}$$

$$\text{So, the largest coefficient in } (1 + x)^{30} = {}^{30} C_{15}$$

Hence, the value of the filler is ${}^{30} C_{15}$.

Q26. The number of terms in the expansion of $(x + y + z)^n$ is _____.

Sol. The expression $(x + y + z)^n$ can be written as $[x + (y + z)]^n$

$$\therefore [x + (y + z)]^n = {}^n C_0 x^n (y + z)^0 + {}^n C_1 x^{n-1} (y + z) + {}^n C_2 x^{n-2} (y + z)^2 + \dots + {}^n C_n (y + z)^n$$

$$\therefore \text{Number of terms } 1 + 2 + 3 + 4 + \dots + (n + 1)$$

$$= \frac{(n + 1)(n + 2)}{2}$$

Hence, the value of the filler is $\frac{(n + 1)(n + 2)}{2}$

Q27. In the expansion of $\left(x^2 - \frac{1}{x^2}\right)^{16}$, the value of constant term is _____.

Sol. Let T_{r+1} be the constant term in the expansion of $\left(x^2 - \frac{1}{x^2}\right)^{16}$

$$\therefore T_{r+1} = {}^{16} C_r (x^2)^{16-r} \left(\frac{-1}{x^2}\right)^r = {}^{16} C_r (x)^{32-2r} (-1)^r \cdot \frac{1}{x^{2r}}$$

$$= (-1)^r \cdot {}^{16} C_r (x)^{32-2r-2r} \Rightarrow (-1)^r \cdot {}^{16} C_r (x)^{32-4r}$$

For getting constant term, $32 - 4r = 0$

$$\Rightarrow r = 8$$

$$\therefore T_{r+1} = (-1)^8 \cdot {}^{16} C_8 = {}^{16} C_8$$

Hence, the value of the filler is ${}^{16} C_8$.

Q28. If the seventh term from the beginning and the end in the

expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$ are equal, then n equals _____.

Sol. The given expansion is $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$

$$\therefore T_7 = T_{6+1} = {}^n C_6 (2^{1/3})^{n-6} \cdot \frac{1}{(3^{1/3})^6} = {}^n C_6 (2)^{\frac{n-6}{3}} \cdot \frac{1}{(3)^2}$$

Now the T_7 from the end = T_7 from the beginning in

$$\left(\frac{1}{\sqrt[3]{3}} + \sqrt[3]{2}\right)^n.$$

$$\therefore T_7 = T_{6+1} = {}^n C_6 \left(\frac{1}{3^{1/3}}\right)^{n-6} \cdot (2^{1/3})^6$$

As per the questions, we get

$${}^n C_6 (2)^{\frac{n-6}{3}} \cdot \left(\frac{1}{3^2}\right) = {}^n C_6 \frac{1}{3^{\frac{n-6}{3}}} \cdot (2)^2$$

$$\Rightarrow (2)^{\frac{n-6}{3}} \cdot (3)^{-2} = (3)^{-\left(\frac{n-6}{3}\right)} \cdot (2)^2$$

$$\Rightarrow (2)^{\frac{n-6}{3}-2} \cdot (3)^{-2+\frac{n-6}{3}} = 1$$

$$\Rightarrow 2^{\frac{n-12}{3}} \cdot (3)^{\frac{n-12}{3}} = 1$$

$$\Rightarrow (6)^{\frac{n-12}{3}} = (6)^0$$

$$\Rightarrow \frac{n-12}{3} = 0 \Rightarrow n = 12$$

Hence, the value of the filler is 12.

Q29. The coefficient of $a^{-6}b^4$ in the expansion of $\left(\frac{1}{a} - \frac{2b}{3}\right)^{10}$ is _____.

Sol. The given expansion is $\left(\frac{1}{a} - \frac{2b}{3}\right)^{10}$

from $a^{-6}b^4$, we can take $r = 4$

$$\begin{aligned} \therefore T_5 = T_{4+1} &= {}^{10} C_4 \left(\frac{1}{a}\right)^{10-4} \left(-\frac{2b}{3}\right)^4 = {}^{10} C_4 \left(\frac{1}{a}\right)^6 \left(\frac{-2}{3}\right)^4 \cdot b^4 \\ &= \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} \times \frac{16}{81} \cdot a^{-6}b^4 = 210 \times \frac{16}{81} a^{-6}b^4 \\ &= \frac{1120}{27} a^{-6}b^4 \end{aligned}$$

Hence, the value of the filler = $\frac{1120}{27}$

Q30. Middle term in the expansion of $(a^3 + ba)^{28}$ is _____.

Sol. Number of term in the expansion $(a^3 + ba)^{28} = 28 + 1 = 29$ (odd)

$$\therefore \text{Middle term} = \frac{29 + 1}{2} = 15\text{th term}$$

$$\therefore T_{15} = T_{14+1} = {}^{28}C_{14} (a^3)^{28-14} \cdot (ba)^{14} = {}^{28}C_{14} (a)^{42} \cdot b^{14} \cdot a^{14} \\ = {}^{28}C_{14} a^{56} b^{14}$$

Hence, the value of the filler is ${}^{28}C_{14} a^{56} b^{14}$.

Q31. The ratio of the coefficient of x^p and x^q in the expansion of $(1 + x)^{p+q}$ is _____.

Sol. Given expansion is $(1 + x)^{p+q}$

$$T_{r+1} = {}^{p+q}C_r x^r$$

$$\text{Put } r = p \quad = {}^{p+q}C_p x^p$$

$$\therefore \text{the coefficient of } x^p = {}^{p+q}C_p$$

$$\text{Similarly, coefficient of } x^q = {}^{p+q}C_q$$

$${}^{p+q}C_p = \frac{(p+q)!}{p!(p+q-p)!} = \frac{(p+q)!}{p!q!}$$

$$\text{and} \quad {}^{p+q}C_q = \frac{(p+q)!}{q!(p+q-q)!} = \frac{(p+q)!}{p!q!}$$

So, the ratio is 1 : 1.

Q32. The position of the term independent of x in the expansion of

$$\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2} \right)^{10} \text{ is _____.}$$

Sol. The given expansion is $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2} \right)^{10}$

$$T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}} \right)^{10-r} \left(\frac{3}{2x^2} \right)^r = {}^{10}C_r \left(\frac{x}{3} \right)^{\frac{10-r}{2}} \left(\frac{3}{2} \right)^r \cdot \frac{1}{x^{2r}} \\ = {}^{10}C_r \left(\frac{1}{3} \right)^{\frac{10-r}{2}} \cdot x^{\frac{10-r}{2}} \left(\frac{3}{2} \right)^r \cdot \frac{1}{x^{2r}} \\ = {}^{10}C_r \left(\frac{1}{3} \right)^{\frac{10-r}{2}} \cdot x^{\frac{10-r}{2}-2r} \cdot \left(\frac{3}{2} \right)^r \\ = {}^{10}C_r \left(\frac{1}{3} \right)^{\frac{10-r}{2}} \cdot x^{\frac{10-r-4r}{2}} \left(\frac{3}{2} \right)^r$$

For independent of x , we get

$$\begin{aligned}\frac{10 - r - 4r}{2} &= 0 \\ 10 - 5r &= 0 \\ r &= 2\end{aligned}$$

So, the position of the term independent of x is 3rd term.

Hence, the value of the filler is Third term

Q33. If 25^{15} is divided by 13, the remainder is _____.

$$\begin{aligned}\text{Sol. Let } 25^{15} &= (26-1)^{15} \\ &= {}^{15}C_0(26)^{15}(-1)^0 + {}^{15}C_1(26)^{14}(-1)^1 \\ &\quad + {}^{15}C_2(26)^{13}(-1)^2 + \dots + {}^{15}C_{15}(-1)^{15} \\ &= 26^{15} - 15(26)^{14} + \dots - 1 - 13 + 13 \\ &= 26^{15} - 15 \cdot (26)^{14} + \dots - 13 + 12 \\ &= 13\lambda + 12\end{aligned}$$

\therefore The remainder = 12

Hence, the value of the filler is 12.

TRUE OR FALSE

Q34. The sum of the series $\sum_{r=0}^{10} {}^{20}C_r$ is $2^{19} + \frac{{}^{20}C_{10}}{2}$

$$\begin{aligned}\text{Sol. } \sum_{r=0}^{10} {}^{20}C_r &= {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + {}^{20}C_3 + \dots + {}^{20}C_{10} \\ &= {}^{20}C_0 + {}^{20}C_1 + \dots + {}^{20}C_{10} + {}^{20}C_{11} + \dots + {}^{20}C_{20} \\ &\quad - ({}^{20}C_{11} + \dots + {}^{20}C_{20}) \\ &= 2^{20} - ({}^{20}C_{11} + \dots + {}^{20}C_{20})\end{aligned}$$

Hence, the given statement is **False**.

Q35. The expression $7^9 + 9^7$ is divisible by 64.

$$\begin{aligned}\text{Sol. } 7^9 + 9^7 &= (1+8)^7 - (1-8)^9 \\ &= [{}^7C_0 + {}^7C_1 \cdot 8 + {}^7C_2(8)^2 + {}^7C_3(8)^3 + \dots + {}^7C_7(8)^7] \\ &\quad - [{}^9C_0 - {}^9C_1 8 + {}^9C_2(8)^2 - {}^9C_3(8)^3 + \dots - {}^9C_9(8)^9] \\ &= (7 \times 8 + 9 \times 8) + (21 \times 8^2 - 36 \times 8^2) + \dots \\ &= (56 + 72) + (21 - 36)8^2 + \dots = 128 + 64(21 - 36) + \dots \\ &= 64[2 + (21 - 36) + \dots]\end{aligned}$$

which is divisible by 64

Hence, the given statement is **True**.

Q36. The number of terms in the expansion of $[(2x + 3y)^4]^7$ is 8.

$$\text{Sol. Given expression is } [(2x + 3y)^4]^7 = (2x + 3y)^{28}$$

So, the number of terms = $28 + 1 = 29$

Hence, the given statement is **False**.

Q37. The sum of coefficients of the two middle terms in the expansion of $(1+x)^{2n-1}$ is equal to ${}^{2n-1}C_n$.

Sol. The given expression is $(1+x)^{2n-1}$

Number of terms = $2n - 1 + 1 = 2n$ (even)

\therefore Middle terms are $\frac{2n}{2}$ th term and $\left(\frac{2n}{2} + 1\right)$ th terms

= n th terms and $(n+1)$ th terms

Coefficient of n th term = ${}^{2n-1}C_{n-1}$

and the coefficient of $(n+1)$ th term = ${}^{2n-1}C_n$

Sum of the coefficients = ${}^{2n-1}C_{n-1} + {}^{2n-1}C_n$
 $= {}^{2n-1}C_{n-1} + {}^{2n-1}C_n = {}^{2n-1}C_{n-1} + {}^{2n-1}C_n = {}^{2n}C_n$

Hence, the statement [$\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$] is **False**.

Q38. The last two digits of the numbers 3^{400} are 01.

Sol. Given that $3^{400} = (9)^{200} = (10-1)^{200}$

$$\begin{aligned} \therefore (10-1)^{200} &= {}^{200}C_0(10)^{200} - {}^{200}C_1(10)^{199} \\ &+ \dots - {}^{200}C_{199}(10)^1 + {}^{200}C_{200}(1)^{200} \\ &= 10^{200} - 200 \times 10^{199} + \dots - 10 \times 200 + 1 \end{aligned}$$

So, it is clear that last two digits are 01.

Hence, the given statement is **True**.

Q39. If the expansion of $\left(x - \frac{1}{x^2}\right)^{2n}$ contains a term independent of x , then n is a multiple of 2.

Sol. The given expression is $\left(x - \frac{1}{x^2}\right)^{2n}$

$$\begin{aligned} T_{r+1} &= {}^{2n}C_r(x)^{2n-r} \left(-\frac{1}{x^2}\right)^r = {}^{2n}C_r(x)^{2n-r}(-1)^r \cdot \frac{1}{x^{2r}} \\ &= {}^{2n}C_r(x)^{2n-r-2r}(-1)^r = {}^{2n}C_r(x)^{2n-3r}(-1)^r \end{aligned}$$

For the term independent of x , $2n - 3r = 0$

$\therefore r = \frac{2n}{3}$ which not an integer and the expression is not possible to be true

Hence, the given statement is **False**.

Q40. The number of terms in the expansion of $(a+b)^n$ where $n \in N$ is one less than the power n .

Sol. Since, the number of terms in the given expression $(a+b)^n$ is 1 more than n i.e., $n+1$

Hence, the given statement is **False**.