EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. Find the equation of the circle which touches the both axes in first quadrant and whose radius is a.

Sol. Clearly centre of the circle = (a, a)
and radius = a
Equation of circle with radius r and centre (h, k) is

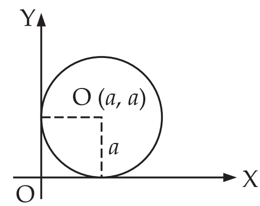
$$(x-h)^2 + (y-k)^2 = r^2$$

So, the equation of the required circle

$$\Rightarrow \quad (x-a)^2 + (y-a)^2 = a^2$$
$$\Rightarrow \quad x^2 - 2ax + a^2 + y^2 - 2ay + a^2 = a^2$$
$$\Rightarrow \quad x^2 + y^2 - 2ax - 2ay + a^2 = 0$$

Hence, the required equation is

$$x^2 + y^2 - 2ax - 2ay + a^2 = 0$$



Q2. Show that the point (x, y) given by $x = \dfrac{2at}{1+t^2}$ and $y = \dfrac{a(1-t^2)}{1+t^2}$ lies on a circle.

Sol. Given $\quad x = \dfrac{2at}{1+t^2}$ and $y = \dfrac{a(1-t^2)}{1+t^2}$

$$\Rightarrow \quad x^2 + y^2 = \left(\frac{2at}{1+t^2}\right)^2 + \left(\frac{a(1-t^2)}{1+t^2}\right)^2$$

$$= \frac{4a^2t^2}{(1+t^2)^2} + \frac{a^2(1-t^2)^2}{(1+t^2)^2} = \frac{4a^2t^2 + a^2(1+t^4-2t^2)}{(1+t^2)^2}$$

$$= \frac{4a^2t^2 + a^2 + a^2t^4 - 2a^2t^2}{(1+t^2)^2} = \frac{a^2 + a^2t^4 + 2a^2t^2}{(1+t^2)^2}$$

$$= \frac{a^2(1+t^4+2t^2)}{(1+t^2)^2} = \frac{a^2(1+t^2)^2}{(1+t^2)^2}$$

$$= a^2$$

$\therefore \quad x^2 + y^2 = a^2$ which is the equation of a circle.

Hence, the given points lie on a circle.

Q3. If a circle passes through the points (0, 0), $(a, 0)$ and $(0, b)$, then find the coordinates of its centre.

Sol. Given points are (0, 0), $(a, 0)$ and $(0, b)$

General equation of the circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

where the centre is $(-g, -f)$ and radius $= \sqrt{g^2 + f^2 - c}$

If it passes through (0, 0)

$\therefore\ c = 0$

If it passes through $(a, 0)$ and $(0, b)$ then

$$a^2 + 2ga + c = 0 \quad \Rightarrow \quad a^2 + 2ga = 0 \qquad [\because\ c = 0]$$

$$\therefore \qquad g = -\frac{a}{2}$$

and $\quad 0 + b^2 + 0 + 2fb + c = 0 \quad \Rightarrow \quad b^2 + 2fb = 0 \qquad [\because\ c = 0]$

$$\Rightarrow \qquad f = -\frac{b}{2}$$

Hence, the coordinates of centre of the circle are $(-g, -f)$

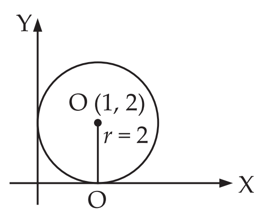
$$= \left(\frac{a}{2}, \frac{b}{2}\right)$$

Q4. Find the equation of the circle which touches X-axis and whose centre is (1, 2).

Sol. Since the circle whose centre is (1, 2) touch x-axis

$\therefore\ r = 2$

So, the equation of the circle is



$$(x - h)^2 + (y - k)^2 = r^2$$

$$\Rightarrow \quad (x - 1)^2 + (y - 2)^2 = (2)^2$$

$$\Rightarrow \quad x^2 - 2x + 1 + y^2 - 4y + 4 = 4$$

$$\Rightarrow \quad x^2 + y^2 - 2x - 4y + 1 = 0$$

Hence, the required equation is

$$x^2 + y^2 - 2x - 4y + 1 = 0.$$

Q5. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then find the radius of the circle.

Sol. Given equation are $3x - 4y + 4 = 0$

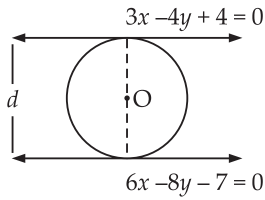
and $\quad 6x - 8y - 7 = 0 \quad \Rightarrow \quad 3x - 4y - \dfrac{7}{2} = 0$

Since $\dfrac{3}{6} = \dfrac{-4}{-8} = \dfrac{1}{2}$ then the lines are parallel.

So, the distance between the parallel lines

$$= \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right| = \left|\frac{4 + \frac{7}{2}}{\sqrt{(3)^2 + (-4)^2}}\right|$$

$$= \left|\frac{\frac{15}{2}}{5}\right| = \frac{3}{2}$$



$$\text{Diameter} = \frac{3}{2}$$

$$\therefore \quad \text{Radius} = \frac{3}{4}.$$

Hence, the required radius $= \frac{3}{4}$.

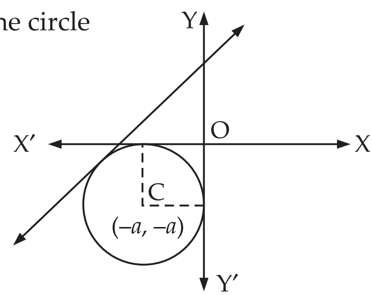
Q6. Find the equation of a circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant.

Sol. Let a be the radius of the circle.

Centre of the circle $= (-a, -a)$

Distance of the line $3x - 4y + 8 = 0$

From the centre = Radius of the circle



$$\left|\frac{-3a + 4a + 8}{\sqrt{(3)^2 + (-4)^2}}\right| = a$$

$$\Rightarrow \quad \left|\frac{a+8}{5}\right| = a$$

$$\Rightarrow \quad \pm\left(\frac{a+8}{5}\right) = a$$

$$\Rightarrow \quad \frac{a+8}{5} = a \text{ and } -\left(\frac{a+8}{5}\right) = a$$

$$\Rightarrow \quad a = 5a - 8$$

$$\Rightarrow \quad 5a - a = 8$$

$$\Rightarrow \quad 4a = 8 \Rightarrow a = 2$$

and $\frac{a+8}{5} = -a \quad \Rightarrow \quad a + 8 = -5a$

$$\Rightarrow \quad 6a = -8 \Rightarrow a = -\frac{4}{3}$$

$\therefore a = 2$ and $a \neq -\frac{4}{3}$

$\therefore$ The equation of the circle is

$$(x+2)^2 + (y+2)^2 = (2)^2$$

$$\Rightarrow \quad x^2 + 4x + 4 + y^2 + 4y + 4 = 4$$

$$\Rightarrow \quad x^2 + y^2 + 4x + 4y + 4 = 0$$

Hence, the required equation of the circle

$$x^2 + y^2 + 4x + 4y + 4 = 0.$$

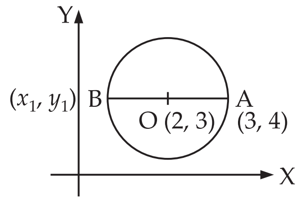
Q7. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the coordinates of the other end of the diameter.

Sol. Let the other end of the diameter is (x_1, y_1).

Equation of given circle is

$$x^2 + y^2 - 4x - 6y + 11 = 0$$

Centre = $(-g, -f) = (2, 3)$



$\therefore \quad \dfrac{x_1 + 3}{2} = 2 \quad \Rightarrow \quad x_1 + 3 = 4$

$\Rightarrow \quad x_1 = 1$

and $\quad \dfrac{y_1 + 4}{2} = 3 \quad \Rightarrow \quad y_1 + 4 = 6$

$\Rightarrow \quad y_1 = 2$

Hence, the required coordinates are (1, 2).

Q8. Find the equation of the circle having (1, – 2) as its centre and passing through $3x + y = 14$ and $2x + 5y = 18$.

Sol. Given equations are

$$3x + y = 14 \quad \text{...}(i)$$

and $$2x + 5y = 18 \quad \text{...}(ii)$$

From eq. (*i*) we get $\quad y = 14 - 3x \quad \text{...}(iii)$

Putting the value of y in eq. (*ii*) we get

$\Rightarrow \quad 2x + 5(14 - 3x) = 18$

$\Rightarrow \quad 2x + 70 - 15x = 18$

$\Rightarrow \quad -13x = -70 + 18$

$\Rightarrow \quad -13x = -52$

$\therefore \quad x = 4$

From eq. (*iii*) we get, $\quad y = 14 - 3 \times 4 = 2$

$\therefore$ Point of intersection is (4, 2)

Now, $\quad$ radius $r = \sqrt{(4-1)^2 + (2+2)^2}$

$$= \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = 5$$

So, the equation of circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

$\Rightarrow \quad (x - 1)^2 + (y + 2)^2 = (5)^2$

$\Rightarrow \quad x^2 - 2x + 1 + y^2 + 4y + 4 = 25$

$\Rightarrow \quad x^2 + y^2 - 2x + 4y - 20 = 0$

Hence, the required equation is $x^2 + y^2 - 2x + 4y - 20 = 0$

Q9. If the line $y = \sqrt{3}x + k$ touches the circle $x^2 + y^2 = 16$, then find the value of k.

Sol. Given circle is $\quad x^2 + y^2 = 16$

Centre = (0, 0)

radius $r = 4$

Perpendicular from the origin to the given line $y = \sqrt{3}x + k$ is equal to the radius.

$$\therefore \qquad 4 = \left| \frac{0 - 0 - k}{\sqrt{(1)^2 + (\sqrt{3})^2}} \right| = \left| \frac{-k}{\sqrt{4}} \right|$$

$$\Rightarrow \qquad 4 = \pm \frac{k}{2} \quad \Rightarrow \quad k = \pm 8.$$

Hence, the required values of k are ± 8.

Q10. Find the equation of a circle concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and has double of its area.

Sol. Given equation of the circle is

$$x^2 + y^2 - 6x + 12y + 15 = 0 \qquad ...(i)$$

$$\text{Centre} = (-g, -f) = (3, -6) \quad \begin{bmatrix} \because & 2g = -6 \Rightarrow g = -3 \\ & 2f = 12 \Rightarrow f = 6 \end{bmatrix}$$

Since the circle is concentric with the given circle

$$\therefore \qquad \text{Centre} = (3, -6)$$

Now let the radius of the circle is r

$$\therefore \qquad r = \sqrt{g^2 + f^2 - c} = \sqrt{9 + 36 - 15} = \sqrt{30}$$

Area of the given circle (i) = $\pi r^2 = 30\pi$ sq unit

Area of the required circle = $2 \times 30\pi = 60\pi$ sq. unit

If r_1 be the radius of the required circle

$$\pi r_1^2 = 60\pi \quad \Rightarrow \quad r_1^2 = 60$$

So, the required equations of the circle is

$$(x - 3)^2 + (y + 6)^2 = 60$$

$$\Rightarrow \qquad x^2 + 9 - 6x + y^2 + 36 + 12y - 60 = 0$$

$$\Rightarrow \qquad x^2 + y^2 - 6x + 12y - 15 = 0$$

Hence, the required equation is $x^2 + y^2 - 6x + 12y - 15 = 0$.

Q11. If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentricity.

Sol. Let the equation of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Length of major axis = $2a$

Length of minor axis = $2b$.

and the length of latus rectum = $\dfrac{2b^2}{a}$

According to the question, we have

$$\frac{2b^2}{a} = \frac{2b}{2} \quad \Rightarrow \quad b = \frac{a}{2}$$

Now $\quad b^2 = a^2(1 - e^2)$, where e is the eccentricity

$\Rightarrow \quad b^2 = 4b^2(1 - e^2)$

$\Rightarrow \quad 1 = 4(1 - e^2)$

$$\Rightarrow \quad 1 - e^2 = \frac{1}{4} \quad \Rightarrow \quad e^2 = 1 - \frac{1}{4}$$

$$\Rightarrow \quad e^2 = \frac{3}{4} \quad \therefore e = \pm\frac{\sqrt{3}}{2}$$

$$\text{So, } e = \frac{\sqrt{3}}{2} \qquad [\because e \text{ is not } (-)]$$

Hence, the required value of eccentricity is $\frac{\sqrt{3}}{2}$.

Q12. If the ellipse with equation $9x^2 + 25y^2 = 225$, then find the eccentricity and foci.

Sol. Given equation of ellipse is

$$9x^2 + 25y^2 = 225$$

$$\Rightarrow \quad \frac{9}{225}x^2 + \frac{25}{225}y^2 = 1$$

$$\Rightarrow \quad \frac{x^2}{25} + \frac{y^2}{9} = 1$$

Here $a = 5$ and $b = 3$

$$b^2 = a^2(1 - e^2)$$

$$\Rightarrow \quad 9 = 25(1 - e^2)$$

$$\Rightarrow \quad 1 - e^2 = \frac{9}{25} \quad \Rightarrow \quad e^2 = 1 - \frac{9}{25} = \frac{16}{25} \quad \Rightarrow \quad e = \frac{4}{5}$$

Now foci $= (\pm\, ae, 0) = \left(\pm\, 5 \times \frac{4}{5}, 0\right) = (\pm\, 4, 0)$

Hence, eccentricity $= \frac{4}{5}$, foci $= (\pm\, 4, 0)$.

Q13. If the eccentricity of an ellipse is $\frac{5}{8}$ and the distance between its foci is 10, then find latus rectum of the ellipse.

Sol. Equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Eccentricity, $e = \frac{5}{8}$, foci $= (\pm\, ae, 0)$

Distance between its foci $= ae + ae = 2ae$

$\therefore \quad 2ae = 10$

$$\Rightarrow \quad ae = 5 \quad \Rightarrow \quad a \times \frac{5}{8} = 5 \quad \Rightarrow \quad a = 8$$

Now $\quad b^2 = a^2(1 - e^2)$

$\Rightarrow \quad b^2 = 64\left(1 - \frac{25}{64}\right)$

$\Rightarrow \quad b^2 = 64 \times \frac{39}{64} \quad \Rightarrow \quad b^2 = 39$

So, the length of the latus rectum = $\frac{2b^2}{a} = \frac{2 \times 39}{8} = \frac{39}{4}$

Hence, the length of the latus rectum = $\frac{39}{4}$.

Q14. Find the equation of an ellipse whose eccentricity is $\frac{2}{3}$, latus rectum is 5 and the centre is (0, 0).

Sol. Equations of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(*i*)

Given that, $\quad e = \frac{2}{3}$

and $\quad$ latus rectum $\frac{2b^2}{a} = 5$

$\Rightarrow \quad b^2 = \frac{5}{2}a$ ...(*ii*)

We know that $\quad b^2 = a^2(1 - e^2)$

$\Rightarrow \quad \frac{5a}{2} = a^2\left(1 - \frac{4}{9}\right)$

$\Rightarrow \quad \frac{5}{2} = a \times \frac{5}{9} \quad \Rightarrow \quad a = \frac{9}{2} \quad \Rightarrow \quad a^2 = \frac{81}{4}$

and $\quad b^2 = \frac{5}{2} \times \frac{9}{2} = \frac{45}{4}$

Hence, the required equation of ellipse is

$$\frac{x^2}{81/4} + \frac{y^2}{45/4} = 1 \quad \Rightarrow \quad \frac{4}{81}x^2 + \frac{4}{45}y^2 = 1.$$

Q15. Find the distance between the directrices of the ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$.

Sol. Given equation of ellipse is

$$\frac{x^2}{36} + \frac{y^2}{20} = 1$$

Here $\quad a^2 = 36 \quad \Rightarrow \quad a = 6$

$$b^2 = 20 \quad \Rightarrow \quad b = 2\sqrt{5}$$

We know that $b^2 = a^2(1 - e^2)$

$\Rightarrow \quad 20 = 36(1 - e^2)$

$\Rightarrow \quad 1 - e^2 = \dfrac{20}{36}$

$\Rightarrow \quad e^2 = 1 - \dfrac{20}{36} = \dfrac{16}{36}$

$\Rightarrow \quad e = \dfrac{4}{6} = \dfrac{2}{3}$

Now distance between the directrices is

$$\frac{a}{e} - \left(-\frac{a}{e}\right) = \frac{a}{e} + \frac{a}{e} = \frac{2a}{e}$$

$$= 2 \times \frac{6}{2/3} = 2 \times 6 \times \frac{3}{2} = 18$$

Hence, the required distance = 18.

Q16. Find the coordinates of a point on a parabola $y^2 = 8x$ whose focal distance is 4.

Sol. Given parabola is $y^2 = 8x$ ...(i)

Comparing with the equation of parabola $y^2 = 4ax$

$$4a = 8 \quad \Rightarrow \quad a = 2$$

Now focal distance $= |x + a|$

$\Rightarrow \quad |x + a| = 4$

$\Rightarrow \quad (x + a) = \pm 4$

$\Rightarrow \quad x + 2 = \pm 4$

$\Rightarrow \quad x = 4 - 2 = 2 \quad$ and $\quad x = -6$

But $x \neq -6 \quad \therefore x = 2$

Put $x = 2$ in equation (i) we get

$$y^2 = 8 \times 2 = 16$$

$\therefore \quad y = \pm 4$

So, the coordinates of the point are (2, 4), (2, – 4).

Hence, the required coordinates are (2, 4) and (2, – 4).

Q17. Find the length of the line-segment joining the vertex of the parabola $y^2 = 4ax$ and a point on the parabola where line segment makes an angle θ to the x-axis.

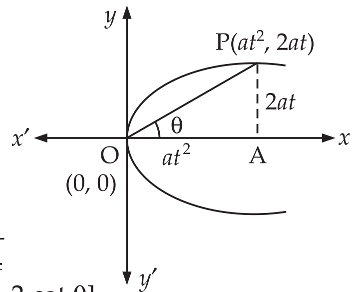
Sol. Equation of parabola is $y^2 = 4ax$

Let P(at^2, $2at$) be any point on the parabola.

In ΔPOA, we have

$$\tan\theta = \frac{2at}{at^2} = \frac{2}{t} \quad \Rightarrow \quad t = \frac{2}{\tan\theta}$$

$\Rightarrow \quad t = 2 \cot \theta \quad ...(i)$



$$OP = \sqrt{(at^2 - 0)^2 + (2at - 0)^2}$$

$$= \sqrt{a^2t^4 + 4a^2t^2}$$

$$= at\sqrt{t^2 + 4}$$

$$= a \times 2 \cot \theta\sqrt{4 \cot^2 \theta + 4}$$

$[\because \ t = 2 \cot \theta]$

$$= 2a \cot \theta . 2\sqrt{\cot^2 \theta + 1} = 4a \cot \theta . \text{cosec } \theta$$

$$= 4a . \frac{\cos \theta}{\sin \theta} . \frac{1}{\sin \theta} = \frac{4a \cos \theta}{\sin^2 \theta}$$

Hence, the required length $= \dfrac{4a \cos \theta}{\sin^2 \theta}$.

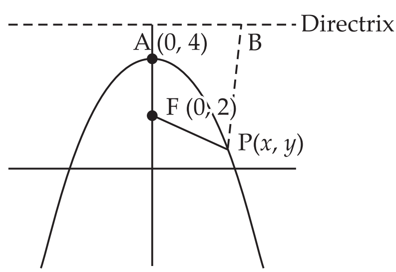
Q18. If the points (0, 4) and (0, 2) are respectively the vertex and focus of a parabola, then find the equation of the parabola.

Sol. Given that:

Vertex = (0, 4)

and Focus = (0, 2)

Let P(x, y) be any point on the parabola. PB is perpendicular to the directrix.



According to the definition of parabola, we have

$$PF = PB$$

$$\Rightarrow \quad \sqrt{(x-0)^2 + (y-2)^2} = \left| \frac{0 + y - 6}{\sqrt{0+1}} \right|$$

[Equation of directrix is $y = 6$]

$$\Rightarrow \quad \sqrt{x^2 + (y-2)^2} = (y - 6)$$

Squaring both sides, we have

$$x^2 + (y-2)^2 = (y-6)^2$$

$$\Rightarrow \quad x^2 + y^2 + 4 - 4y = y^2 + 36 - 12y$$

$$\Rightarrow \quad x^2 - 4y + 12y - 32 = 0$$

$$\Rightarrow \quad x^2 + 8y - 32 = 0$$

Hence, the required equation is $x^2 + 8y = 32$.

Q19. If the line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$ then find the value of m.

Sol. Given that $y^2 = 4x$ ...(*i*)

and $y = mx + 1$ ...(*ii*)

From eq. (*i*) and (*ii*) we get

$$(mx+1)^2 = 4x$$

$$\Rightarrow \quad m^2x^2 + 1 + 2mx - 4x = 0$$

$$\Rightarrow \quad m^2x^2 + (2m-4)x + 1 = 0$$

Applying condition of tangency, we have

$$(2m-4)^2 - 4m^2 \times 1 = 0$$

$$\Rightarrow \quad 4m^2 + 16 - 16m - 4m^2 = 0$$

$$\Rightarrow \quad -16m = -16$$

$$\Rightarrow \quad m = 1$$

Hence, the required value of m is 1.

Q20. If the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then obtain the equation of the hyperbola.

Sol. Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Distance between the foci = $2ae$

$$2ae = 16 \quad \Rightarrow \quad ae = 8$$

$$\Rightarrow \quad a \times \sqrt{2} = 8$$

$$\Rightarrow \quad a = \frac{8}{\sqrt{2}} = 4\sqrt{2} \qquad \left[\because e = \sqrt{2}\right]$$

Now, $b^2 = a^2(e^2 - 1)$ [for hyperbola]

$$\Rightarrow \quad b^2 = (4\sqrt{2})^2\,(2-1)$$

$$\Rightarrow \quad b^2 = 32$$

$a = 4\sqrt{2} \quad \Rightarrow \quad a^2 = 32$

Hence, the required equation is $\frac{x^2}{32} - \frac{y^2}{32} = 1$

$$\Rightarrow \quad x^2 - y^2 = 32$$

Q21. Find the eccentricity of the hyperbola $9y^2 - 4x^2 = 36$.

Sol. Given equation is $9y^2 - 4x^2 = 36$

$$\Rightarrow \quad \frac{y^2}{4} - \frac{x^2}{9} = 1$$

Clearly it is a vertical hyperbola .

Where $a = 3$ and $b = 2$

We know that $b^2 = a^2(e^2 - 1)$

$$\Rightarrow \quad 4 = 9(e^2 - 1)$$

$$\Rightarrow \quad e^2 - 1 = \frac{4}{9}$$

$$\Rightarrow \quad e^2 = 1 + \frac{4}{9} = \frac{13}{9}$$

$\therefore \qquad e = \frac{\sqrt{13}}{3}$

Hence, the required value of e is $\frac{\sqrt{13}}{3}$.

Q22. Find the equation of the hyperbola with eccentricity $\frac{3}{2}$ and foci at $(\pm 2, 0)$.

Sol. Given that $e = \frac{3}{2}$ and foci $= (\pm 2, 0)$

We know that foci $= (\pm ae, 0)$

$\therefore \qquad ae = 2$

$\Rightarrow \qquad a \times \frac{3}{2} = 2$

$\Rightarrow \qquad a = \frac{4}{3} \Rightarrow a^2 = \frac{16}{9}$

We know that $\qquad b^2 = a^2(e^2 - 1)$

$\Rightarrow \qquad b^2 = \frac{16}{9}\left(\frac{9}{4} - 1\right) = \frac{16}{9} \times \frac{5}{4} = \frac{20}{9}$

So, the equation of the hyperbola is

$$\frac{x^2}{16/9} - \frac{y^2}{20/9} = 1$$

$$\Rightarrow \qquad \frac{9x^2}{16} - \frac{9y^2}{20} = 1$$

$$\Rightarrow \qquad \frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$$

Hence, the required equation is $\frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$.

LONG ANSWER TYPE QUESTIONS

Q23. If the lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

Sol. We know that the intersection point of the diameter gives the centre of the circle.

Given equations of diameters are

$$2x - 3y = 5 \qquad ...(i)$$

$$3x - 4y = 7 \qquad ...(ii)$$

From eq. (*i*) we have $\qquad x = \frac{5 + 3y}{2} \qquad ...(iii)$

Putting the value of x in eq. (ii) we have

$$3\left(\frac{5+3y}{2}\right) - 4y = 7$$

$$\Rightarrow \quad 15 + 9y - 8y = 14$$

$$\Rightarrow \quad y = 14 - 15 \quad \Rightarrow \quad y = -1$$

Now from eq. (iii) we have

$$x = \frac{5+3(-1)}{2} \quad \Rightarrow \quad x = \frac{5-3}{2} \quad \Rightarrow \quad x = 1$$

So, the centre of the circle = (1, – 1)

Given that area of the circle = 154

$$\Rightarrow \quad \pi r^2 = 154$$

$$\Rightarrow \quad \frac{22}{7} \times r^2 = 154 \quad \Rightarrow \quad r^2 = 154 \times \frac{7}{22}$$

$$\Rightarrow \quad r^2 = 7 \times 7$$

$$\Rightarrow \quad r = 7$$

So, the equation of the circle is

$$(x-1)^2 + (y+1)^2 = (7)^2$$

$$\Rightarrow \quad x^2 + 1 - 2x + y^2 + 1 + 2y = 49$$

$$\Rightarrow \quad x^2 + y^2 - 2x + 2y = 47$$

Hence, the required equation of the circle is

$$x^2 + y^2 - 2x + 2y = 47$$

Q24. Find the equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line $y - 4x + 3 = 0$.

Sol. Let the equation of the circle be

$$(x-h)^2 + (y-k)^2 = r^2 \qquad \ldots(i)$$

If the circle passes through (2, 3) and (4, 5) then

$$(2-h)^2 + (3-k)^2 = r^2 \qquad \ldots(ii)$$

and $$(4-h)^2 + (5-k)^2 = r^2 \qquad \ldots(iii)$$

Subtracting eq. (iii) from eq. (ii) we have

$$(2-h)^2 - (4-h)^2 + (3-k)^2 - (5-k)^2 = 0$$

$$\Rightarrow 4 + h^2 - 4h - 16 - h^2 + 8h + 9 + k^2 - 6k - 25 - k^2 + 10k = 0$$

$$\Rightarrow \quad 4h + 4k - 28 = 0$$

$$\Rightarrow \quad h + k = 7 \qquad \ldots(iv)$$

Since, the centre (h, k) lies on the line $y - 4x + 3 = 0$

then $$k - 4h + 3 = 0$$

$$\Rightarrow \quad k = 4h - 3$$

Putting the value of k in eq. (iv) we get

$$h + 4h - 3 = 7$$

$$\Rightarrow \quad 5h = 10 \quad \Rightarrow \quad h = 2$$

From (*iv*) we get $k = 5$

Putting the value of h and k in eq. (*ii*) we have

$$(2-2)^2 + (3-5)^2 = r^2$$

$$\Rightarrow \quad r^2 = 4$$

So, the equation of the circle is

$$(x-2)^2 + (y-5)^2 = 4$$

$$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 25 - 10y = 4$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 10y + 25 = 0$$

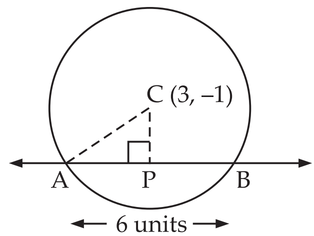
Hence, the required equation is $x^2 + y^2 - 4x - 10y + 25 = 0$.

Q25. Find the equation of a circle whose centre is (3, – 1) and which cuts off a chord of length 6 units on the line $2x - 5y + 18 = 0$.

Sol. Given that:

Centre of the circle = (3, – 1)

Length of chord AB = 6 units



$$CP = \left|\frac{2\times 3 - 5\times -1 + 18}{\sqrt{(2)^2 + (-5)^2}}\right|$$

$$= \left|\frac{6+5+18}{\sqrt{29}}\right| = \sqrt{29}$$

Now AB = 6 units.

$$\therefore \quad AP = \frac{1}{2}AB = \frac{1}{2}\times 6 = 3 \text{ units}$$

In ΔCPA, $AC^2 = CP^2 + AP^2$

$$= (\sqrt{29})^2 + (3)^2 = 29 + 9 = 38$$

$$\therefore \quad AC = \sqrt{38}$$

So, the radius of the circle, $r = \sqrt{38}$

$\therefore$ Equation of the circle is

$$(x-3)^2 + (y+1)^2 = (\sqrt{38})^2$$

$$\Rightarrow \quad (x-3)^2 + (y+1)^2 = 38$$

$$\Rightarrow \quad x^2 + 9 - 6x + y^2 + 1 + 2y = 38$$

$$\Rightarrow \quad x^2 + y^2 - 6x + 2y = 28$$

Hence, the required equation is $x^2 + y^2 - 6x + 2y = 28$.

Q26. Find the equation of a circle of radius 5 which is touching another circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at (5, 5).

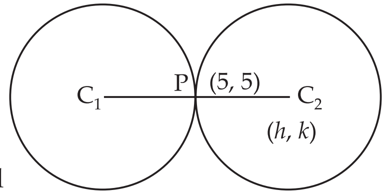
Sol. Given circle is

$$x^2 + y^2 - 2x - 4y - 20 = 0$$

$$2g = -2 \quad \Rightarrow \quad g = -1$$

$$2f = -4 \quad \Rightarrow \quad f = -2$$

$\therefore$ Centre $C_1 = (1, 2)$



and $\quad r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{1+4+20} = 5$

Let the centre of the required circle be (h, k).

Clearly, P is the mid-point of C_1C_2

$\therefore \quad 5 = \dfrac{1+h}{2} \quad \Rightarrow \quad h = 9$

and $\quad 5 = \dfrac{2+k}{2} \quad \Rightarrow \quad k = 8$

Radius of the required circle = 5

$\therefore$ Eq. of the circle is $(x-9)^2 + (y-8)^2 = (5)^2$

$\Rightarrow \quad x^2 + 81 - 18x + y^2 + 64 - 16y = 25$

$\Rightarrow \quad x^2 + y^2 - 18x - 16y + 145 - 25 = 0$

$\Rightarrow \quad x^2 + y^2 - 18x - 16y + 120 = 0$

Hence, the required equation is $x^2 + y^2 - 18x - 16y + 120 = 0$.

Q27. Find the equation of a circle passing through the point (7, 3) having radius 3 units and whose centre lies on the line $y = x - 1$.

Sol. Let the equation of the circle be

$$(x-h)^2 + (y-k)^2 = r^2$$

If it passes through (7, 3) then

$$(7-h)^2 + (3-k)^2 = (3)^2 \qquad [\because r = 3]$$

$\Rightarrow \quad 49 + h^2 - 14h + 9 + k^2 - 6k = 9$

$\Rightarrow \quad h^2 + k^2 - 14h - 6k + 49 = 0 \qquad ...(i)$

If centre (h, k) lies on the line $y = x - 1$ then

$$k = h - 1 \qquad ...(ii)$$

Putting the value of k in eq. (i) we get

$$h^2 + (h-1)^2 - 14h - 6(h-1) + 49 = 0$$

$\Rightarrow \quad h^2 + h^2 + 1 - 2h - 14h - 6h + 6 + 49 = 0$

$\Rightarrow \quad 2h^2 - 22h + 56 = 0$

$\Rightarrow \quad h^2 - 11h + 28 = 0$

$\Rightarrow \quad h^2 - 7h - 4h + 28 = 0$

$\Rightarrow \quad h(h-7) - 4(h-7) = 0$

$\Rightarrow \quad (h-4)(h-7) = 0$

$\therefore \quad h = 4, h = 7$

From eq. (ii) we get $k = 4 - 1 = 3$ and $k = 7 - 1 = 6$.

So, the centres are (4, 3) and (7, 6).

$\therefore$ Equation of the circle is

Taking centre (4, 3),

$$(x-4)^2 + (y-3)^2 = 9$$
$$x^2 + 16 - 8x + y^2 + 9 - 6y = 9$$
$$\Rightarrow \quad x^2 + y^2 - 8x - 6y + 16 = 0$$

Taking centre (7, 6)

$$(x-7)^2 + (y-6)^2 = 9$$
$$\Rightarrow \quad x^2 + 49 - 14x + y^2 + 36 - 12y = 9$$
$$\Rightarrow \quad x^2 + y^2 - 14x - 12y + 76 = 0$$

Hence, the required equations are

$$x^2 + y^2 - 8x - 6y + 16 = 0$$
and $$x^2 + y^2 - 14x - 12y + 76 = 0.$$

Q28. Find the equation of each of the parabolas

(*i*) directrix = 0 and focus at (6, 0)

(*ii*) vertex at (0, 4), focus at (0, 2)

(*iii*) focus at (– 1, –2), directrix $x - 2y + 3 = 0$

Sol. (*i*) Given that directrix = 0 and focus (6, 0)

$\therefore$ The equation of the parabola is

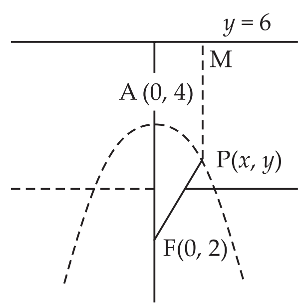
$$(x-6)^2 + y^2 = x^2$$
$$\Rightarrow \quad x^2 + 36 - 12x + y^2 = x^2$$
$$\Rightarrow \quad y^2 - 12x + 36 = 0$$

Hence, the required equations is $y^2 - 12x + 36 = 0$

(*ii*) Given that vertex at (0, 4) and focus at (0, 2).

So, the equation of directrix is $y - 6 = 0$

According to the definition of the parabola



$$PF = PM.$$
$$\sqrt{(x-0)^2 + (y-2)^2} = |y-6|$$
$$\Rightarrow \quad \sqrt{x^2 + y^2 + 4 - 4y} = |y-6|$$

Squaring both the sides, we get

$$x^2 + y^2 + 4 - 4y = y^2 + 36 - 12y$$
$$\Rightarrow \quad x^2 + 4 - 4y = 36 - 12y$$
$$\Rightarrow \quad x^2 + 8y - 32 = 0$$
$$\Rightarrow \quad x^2 = 32 - 8y$$

Hence, the required equation is $x^2 = 32 - 8y$.

(*iii*) Given that focus at (– 1, – 2) and directrix $x - 2y + 3 = 0$

Let (x, y) be any point on the parabola.

According to the definition of the parabola, we have

$$PF = PM$$

$$\sqrt{(x+1)^2+(y+2)^2} = \left|\frac{x-2y+3}{\sqrt{(1)^2+(-2)^2}}\right|$$

$$\Rightarrow \quad \sqrt{x^2+1+2x+y^2+4+4y} = \left|\frac{x-2y+3}{\sqrt{5}}\right|$$

Squaring both sides, we get

$$x^2 + 1 + 2x + y^2 + 4 + 4y = \frac{x^2+4y^2+9-4xy-12y+6x}{5}$$

$\Rightarrow 5x^2 + 5 + 10x + 5y^2 + 20 + 20y = x^2 + 4y^2 + 9 - 4xy - 12y + 6x$

$\Rightarrow 4x^2 + y^2 + 4xy + 4x + 32y + 16 = 0$

Hence, the required equation is

$4x^2 + 4xy + y^2 + 4x + 32y + 16 = 0$

Q29. Find the equation of the set of all points the sum of whose distances from the points (3, 0), (9, 0) is 12.

Sol. Let (x, y) be any point.

Given points are (3, 0) and (9, 0)

According to the question, we have

$$\sqrt{(x-3)^2+(y-0)^2} + \sqrt{(x-9)^2+(y-0)^2} = 12$$

$$\Rightarrow \sqrt{x^2+9-6x+y^2} + \sqrt{x^2+81-18x+y^2} = 12$$

Putting $x^2 + 9 - 6x + y^2 = k$

$$\Rightarrow \quad \sqrt{k} + \sqrt{72-12x+k} = 12$$

$$\Rightarrow \quad \sqrt{72-12x+k} = 12 - \sqrt{k}$$

Squaring both sides, we have

$$\Rightarrow \quad 72 - 12x + k = 144 + k - 24\sqrt{k}$$

$$\Rightarrow \quad 24\sqrt{k} = 144 - 72 + 12x$$

$$\Rightarrow \quad 24\sqrt{k} = 72 + 12x$$

$$\Rightarrow \quad 2\sqrt{k} = 6 + x$$

Again squaring both sides, we get

$$4k = 36 + x^2 + 12x$$

Putting the value of k, we have

$$4(x^2 + 9 - 6x + y^2) = 36 + x^2 + 12x$$

$$\Rightarrow \quad 4x^2 + 36 - 24x + 4y^2 = 36 + x^2 + 12x$$

$$\Rightarrow \quad 3x^2 + 4y^2 - 36x = 0$$

Hence, the required equation is $3x^2 + 4y^2 - 36x = 0$

Q30. Find the equation of the set of all points whose distance from (0, 4) are $\frac{2}{3}$ of their distance from the line $y = 9$.

Sol. Let P(x, y) be a point.

According to question, we have

$$\sqrt{(x-0)^2+(y-4)^2} = \frac{2}{3}\left|\frac{y-9}{1}\right|$$

Squaring both sides, we have

$$x^2 + (y-4)^2 = \frac{4}{9}(y^2+81-18y)$$

$$\Rightarrow \quad 9x^2 + 9(y-4)^2 = 4y^2 + 324 - 72y$$

$$\Rightarrow \quad 9x^2 + 9y^2 + 144 - 72y = 4y^2 + 324 - 72y$$

$$\Rightarrow \quad 9x^2 + 5y^2 + 144 - 324 = 0$$

$$\Rightarrow \quad 9x^2 + 5y^2 - 180 = 0$$

Hence, the required equation is $9x^2 + 5y^2 - 180 = 0$.

Q31. Show that the set of all points such that the difference of their distances from (4, 0) and (– 4, 0) is always equal to 2 represents a hyperbola.

Sol. Let P(x, y) be any point.

According to the question, we have

$$\sqrt{(x-4)^2+(y-0)^2} - \sqrt{(x+4)^2+(y-0)^2} = 2$$

$$\Rightarrow \sqrt{x^2+16-8x+y^2} - \sqrt{x^2+16+8x+y^2} = 2$$

Putting the $\quad x^2 + y^2 + 16 = z \quad$ ...(i)

$$\Rightarrow \quad \sqrt{z-8x} - \sqrt{z+8x} = 2$$

Squaring both sides, we get

$$\Rightarrow \quad z - 8x + z + 8x - 2\sqrt{(z-8x)(z+8x)} = 4$$

$$\Rightarrow \quad 2z - 2\sqrt{z^2-64x^2} = 4$$

$$\Rightarrow \quad z - \sqrt{z^2-64x^2} = 2$$

$$\Rightarrow \quad (z-2) = \sqrt{z^2-64x^2}$$

Again squaring both sides, we have

$$z^2 + 4 - 4z = z^2 - 64x^2$$

$$\Rightarrow \quad 4 - 4z + 64x^2 = 0$$

Putting the value of z, we have

$$\Rightarrow \quad 4 - 4(x^2 + y^2 + 16) + 64x^2 = 0$$

$$\Rightarrow \quad 4 - 4x^2 - 4y^2 - 64 + 64x^2 = 0$$

$$\Rightarrow \quad 60x^2 - 4y^2 - 60 = 0$$

$$\Rightarrow \quad 60x^2 - 4y^2 = 60$$

$$\Rightarrow \quad \frac{60x^2}{60} - \frac{4y^2}{60} = 1$$

$$\Rightarrow \quad \frac{x^2}{1} - \frac{y^2}{15} = 1$$

Which represent a hyperbola. Hence proved.

Q32. Find the equation of the hyperbola with

(*i*) vertices (± 5, 0), foci (± 7, 0)

(*ii*) vertices (0, ± 7), $e = \frac{4}{3}$

(*iii*) foci $(0, \pm\sqrt{10})$ passing through (2, 3)

Sol. (*i*) Given that vertices (± 5, 0), foci (± 7, 0)

Vertex of hyperbola = (± a, 0) and foci (± ae, 0)

$\therefore\ a = 5$ and $ae = 7 \quad \Rightarrow \quad 5 \times e = 7 \quad \Rightarrow \quad e = \frac{7}{5}$

Now $\quad b^2 = a^2 (e^2 - 1)$

$$\Rightarrow \quad b^2 = 25\left(\frac{49}{25} - 1\right) \quad \Rightarrow \quad b^2 = 25 \times \frac{24}{25} \quad \Rightarrow \quad b^2 = 24$$

The equation of the hyperbola is

$$\frac{x^2}{25} - \frac{y^2}{24} = 1$$

(*ii*) Given that vertices (0, ± 7), $e = \frac{4}{3}$

Clearly, the hyperbola is vertical.

$$\text{Vertices} = (\pm 0, a)$$

$\therefore\ a = 7$ and $e = \frac{4}{3}$

We know that $\quad b^2 = a^2(e^2 - 1)$

$$\Rightarrow \quad b^2 = 49\left(\frac{16}{9} - 1\right)$$

$$\Rightarrow \quad b^2 = 49 \times \frac{7}{9}$$

$$\Rightarrow \quad b^2 = \frac{343}{9}$$

Hence, the equation of the hyperbola is

$$\frac{y^2}{49} - \frac{9x^2}{343} = 1$$

$\Rightarrow \quad 9x^2 - 7y^2 + 343 = 0$

(*iii*) Given that: foci $= (0, \pm\sqrt{10})$

$\therefore \quad ae = \sqrt{10} \quad \Rightarrow \quad a^2e^2 = 10$

We know that $b^2 = a^2(e^2 - 1)$

$\Rightarrow \quad b^2 = a^2e^2 - a^2$

$\Rightarrow \quad b^2 = 10 - a^2$

Equation of hyperbola is

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\Rightarrow \quad \frac{y^2}{a^2} - \frac{x^2}{10 - a^2} = 1$$

If it passes through the point (2, 3) then

$$\frac{9}{a^2} - \frac{4}{10 - a^2} = 1$$

$$\Rightarrow \quad \frac{90 - 9a^2 - 4a^2}{a^2(10 - a^2)} = 1$$

$\Rightarrow \quad 90 - 13a^2 = a^2(10 - a^2)$

$\Rightarrow \quad 90 - 13a^2 = 10a^2 - a^4$

$\Rightarrow \quad a^4 - 23a^2 + 90 = 0$

$\Rightarrow \quad a^4 - 18a^2 - 5a^2 + 90 = 0$

$\Rightarrow \quad a^2(a^2 - 18) - 5(a^2 - 18) = 0$

$\Rightarrow \quad (a^2 - 18)(a^2 - 5) = 0$

$\Rightarrow \quad a^2 = 18, a^2 = 5$

$\therefore \quad b^2 = 10 - 18 = -8$ and $b^2 = 10 - 5 = 5$

$b \neq -8 \qquad \therefore \quad b^2 = 5$

Here, the required equation is $\frac{y^2}{5} - \frac{x^2}{5} = 1$ or $y^2 - x^2 = 5$.

State True or False Statements:

Q33. The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 + 6x + 2y = 0$

Sol. Given equation of the circle is

$$x^2 + y^2 + 6x + 2y = 0$$

Centre is $(-3, -1)$

If $x + 3y = 0$ is the equation of diameter, then the centre $(-3, -1)$ will lie on $x + 3y = 0$

$$-3 + 3(-1) = 0$$

$$\Rightarrow \quad -6 \neq 0$$

So, $x + 3y = 0$ is not the diameter of the circle.

Hence, the given statement is **False**.

Q34. The shortest distance from the point (2, – 7) to the circle $x^2 + y^2 - 14x - 10y - 151 = 0$ is equal to 5.

Sol. Given equation of circle is $x^2 + y^2 - 14x - 10y - 151 = 0$
Shortest distance = distance between the point (2, – 7) and the centre – radius of the circle
Centre of the given circle is

$$2g = -14 \Rightarrow g = -7$$
$$2f = -10 \Rightarrow f = -5$$

$\therefore$ Centre $= (-g, -f) = (7, 5)$

and $r = \sqrt{(-7)^2 + (-5)^2 + 151} = \sqrt{49+25+151}$
$= \sqrt{225} = 15$

$\therefore$ Shortest distance $= \sqrt{(7-2)^2 + (5+7)^2} - 15$
$= \sqrt{25+144} - 15$
$= 13 - 15 = |-2| = 2$

Hence, the given statement is **False**.

Q35. If the line $lx + my = 1$ is a tangent to the circle $x^2 + y^2 = a^2$ then the point (l, m) lies on a circle.

Sol. Given equation of circle is $x^2 + y^2 = a^2$
and the tangent is $lx + my = 1$
Here centre is (0, 0) and radius $= a$
If (l, m) lies on the circle

$\therefore$ $\sqrt{(l-0)^2 + (m-0)^2} = a$

$\Rightarrow$ $\sqrt{l^2 + m^2} = a$

$\Rightarrow$ $l^2 + m^2 = a^2$ (which is a circle)

So, the point (l, m) lies on the circle.
Hence, the given statement is **True**.

Q36. The point (1, 2) lies inside the circle $x^2 + y^2 - 2x + 6y + 1 = 0$.

Sol. Given equation of circle is $x^2 + y^2 - 2x + 6y + 1 = 0$
Here

$$2g = -2 \Rightarrow g = -1$$
$$2f = 6 \Rightarrow f = 3$$

$\therefore$ Centre $= (-g, -f) = (1, -3)$

and $r = \sqrt{g^2 + f^2 - c} = \sqrt{1+9-1} = 3$

$\therefore$ Distance between the point (1, 2) and the centre (1, – 3)
$= \sqrt{(1-1)^2 + (2+3)^2} = 5$

Here 5 > 3, so the point lies out side the circle.
Hence, the given statement is **False**.

Q37. The line $lx + my + n = 0$ will touch the parabola $y^2 = 4ax$ if $ln = am^2$.

Sol. Given equation of parabola is $y^2 = 4ax$ ...(*i*)

and the equation of line is $lx + my + n = 0$ ...(*ii*)

From eq. (*ii*), we have

$$y = \frac{-lx - n}{m}$$

Putting the value of y in eq. (*i*) we get

$$\left(\frac{-lx-n}{m}\right)^2 = 4ax$$

$$\Rightarrow \quad l^2x^2 + n^2 + 2lnx - 4am^2x = 0$$

$$\Rightarrow \quad l^2x^2 + (2ln - 4am^2)x + n^2 = 0$$

If the line is the tangent to the circle, then

$$b^2 - 4ac = 0$$

$$(2ln - 4am^2)^2 - 4l^2n^2 = 0$$

$$\Rightarrow \quad 4l^2n^2 + 16a^2m^4 - 16lnm^2a - 4l^2n^2 = 0$$

$$\Rightarrow \quad 16a^2m^4 - 16lnm^2a = 0$$

$$\Rightarrow \quad 16am^2(am^2 - ln) = 0$$

$$\Rightarrow \quad am^2(am^2 - ln) = 0$$

$$\Rightarrow am^2 \neq 0 \quad \therefore \quad am^2 - ln = 0$$

$$\therefore \quad ln = am^2$$

Hence, the given statement is **True**.

Q38. If P is a point on the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ whose foci are S and S′, then PS + PS′ = 8.

Sol. Let $P(x_1, y_1)$ be a point on the ellipse.

foci = $(\pm ae, 0)$

Here $a^2 = 25 \Rightarrow a = 5$

$$b^2 = 16 \Rightarrow b = 4$$

$$b^2 = a^2(1 - e^2)$$

$$16 = 25(1 - e^2)$$

$$\Rightarrow \quad \frac{16}{25} = 1 - e^2$$

$$\Rightarrow \quad e^2 = 1 - \frac{16}{25} \Rightarrow e^2 = \frac{9}{25} \quad \therefore \quad e = \frac{3}{5}$$

$$\therefore \quad ae = 5 \times \frac{3}{5} = 3$$

So, the foci are S(3, 0) and S′(– 3, 0).

Since PS + PS′ = $2a = 2 \times 5 = 10$.

Hence, the given statement is **False**.

Q39. The line $2x + 3y = 12$ touches the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 2$ at the point (3, 2).

Sol. If line $2x + 3y = 12$ touches the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 2$, then the point (3, 2) satisfies both line and ellipse.

$\therefore$ For line

$$2x + 3y = 12$$
$$2(3) + 3(2) = 12$$
$$6 + 6 = 12$$
$$12 = 12 \text{ True}$$

For ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 2$$
$$\frac{(3)^2}{9} + \frac{(2)^2}{4} = 2$$
$$\frac{9}{9} + \frac{4}{4} = 2$$
$$1 + 1 = 2$$
$$2 = 2 \text{ True}$$

Hence, the given statement is **True**.

Q40. The locus of the point of intersection of lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different value of k is a hyperbola whose eccentricity is 2.

Sol. The given equations are

$$\sqrt{3}x - y - 4\sqrt{3}k = 0 \qquad ...(i)$$

and

$$\sqrt{3}kx + ky - 4\sqrt{3} = 0 \qquad ...(ii)$$

From eq. (i) we get

$$4\sqrt{3}k = \sqrt{3}x - y$$

$$\therefore \qquad k = \frac{\sqrt{3}x - y}{4\sqrt{3}}$$

Putting the value of k in eq. (ii), we get

$$\sqrt{3}\left[\frac{\sqrt{3}x - y}{4\sqrt{3}}\right]x + \left[\frac{\sqrt{3}x - y}{4\sqrt{3}}\right]y - 4\sqrt{3} = 0$$

$$\Rightarrow \qquad \left(\frac{\sqrt{3}x - y}{4}\right)x + \left(\frac{\sqrt{3}x - y}{4\sqrt{3}}\right)y - 4\sqrt{3} = 0$$

$$\Rightarrow \qquad \frac{(3x - \sqrt{3}y)x + (\sqrt{3}x - y)y - 48}{4\sqrt{3}} = 0$$

$$\Rightarrow \quad 3x^2 - \sqrt{3}xy + \sqrt{3}xy - y^2 - 48 = 0$$
$$\Rightarrow \quad 3x^2 - y^2 = 48$$
$$\Rightarrow \quad \frac{x^2}{16} - \frac{y^2}{48} = 1 \text{ which is a hyperbola.}$$

Here $a^2 = 16$, $b^2 = 48$

We know that $b^2 = a^2(e^2 - 1)$

$$\Rightarrow \quad 48 = 16(e^2 - 1)$$
$$\Rightarrow \quad 3 = e^2 - 1$$
$$\Rightarrow \quad e^2 = 4 \quad \Rightarrow \quad e = 2$$

Hence, the given statement is **True**.

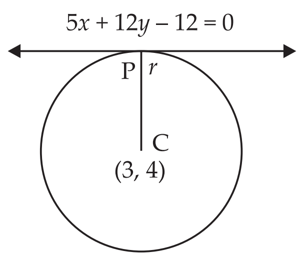
Fill in the Blanks.

Q41. The equation of the circle having centre at (3, – 4) and touching the line $5x + 12y - 12 = 0$ is ....

Sol. Given equation of the line is $5x + 12y - 12 = 0$ and the centre is (3, – 4)

CP = radius of the circle

$$\left| \frac{5 \times 3 + 12 \times (-4) - 12}{\sqrt{(5)^2 + (12)^2}} \right| = r$$



$$\Rightarrow \quad \left| \frac{15 - 48 - 12}{13} \right| = r$$
$$\Rightarrow \quad \left| \frac{-45}{13} \right| = r$$
$$\Rightarrow \quad r^2 = \frac{2025}{169}$$

So, the equation of the circle is

$$(x - 3)^2 + (y + 4)^2 = \left(\frac{45}{13}\right)^2.$$

Hence, the value of the filler is $\mathbf{(x - 3)^2 + (y + 4)^2 = \left(\frac{45}{13}\right)^2}$.

Q42. The equation of the circle circumscribing the triangle whose sides are the lines $y = x + 2$, $3y = 4x$, $2y = 3x$ is

Sol. Let AB represents $2y = 3x$ ...(*i*)

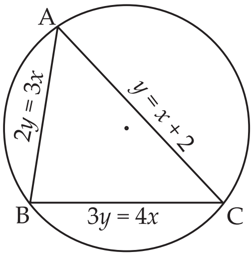
BC represents $3y = 4x$ ...(*ii*)

and AC represents $y = x + 2$ ...(*iii*)

From eq. (*i*) and (*ii*)

$$2y = 3x \quad \Rightarrow \quad y = \frac{3x}{2}$$

Putting the value of y in eq. (*ii*) we get



$$3\left(\frac{3x}{2}\right) = 4x$$

$\Rightarrow \quad 9x = 8x$

$\Rightarrow \quad x = 0$ and $y = 0$

$\therefore$ Coordinates of B = (0, 0)

From eq. (*i*) and (*iii*) we get

$$y = x + 2$$

Putting $y = x + 2$ in eq. (*i*) we get

$$2(x + 2) = 3x$$

$\Rightarrow \quad 2x + 4 = 3x \quad \Rightarrow \quad x = 4$ and $y = 6$

$\therefore$ Coordinates of A = (4, 6)

Solving eq. (*ii*) and (*iii*) we get

$$y = x + 2$$

Putting the value of y in eq. (*ii*) we get

$$3(x + 2) = 4x \quad \Rightarrow \quad 3x + 6 = 4x \quad \Rightarrow \quad x = 6 \text{ and } y = 8$$

$\therefore$ Coordinates of C = (6, 8)

It implies that the circle is passing through (0, 0), (4, 6) and (6, 8).

We know that the general equation of the circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad ...(i)$$

Since the points (0, 0), (4, 6) and (6, 8) lie on the circle then

$$0 + 0 + 0 + 0 + c = 0 \quad \Rightarrow \quad c = 0$$

$$16 + 36 + 8g + 12f + c = 0$$

$\Rightarrow \quad 8g + 12f + 0 = -52$

$\Rightarrow \quad 2g + 3f = -13 \qquad ...(ii)$

and $\quad 36 + 64 + 12g + 16f + c = 0$

$\Rightarrow \quad 12g + 16f + 0 = -100$

$\Rightarrow \quad 3g + 4f = -25 \qquad ...(iii)$

Solving eq. (*ii*) and (*iii*) we get

$$2g + 3f = -13$$
$$3g + 4f = -25$$

$$\Rightarrow \quad \begin{array}{rrr} 6g & +9f & = -39 \\ 6g & +8f & = -50 \\ (-) & (-) & (+) \\ \hline & f & = \quad 11 \end{array}$$

Putting the value of f in eq. (*ii*) we get

$$2g + 3 \times 11 = -13$$

$\Rightarrow \quad 2g + 33 = -13$

$\Rightarrow \quad 2g = -46 \quad \Rightarrow \quad g = -23$

Putting the values of g, f and c in eq. (i) we get

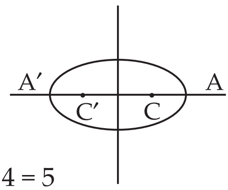
$$x^2 + y^2 + 2(-23)x + 2(11)y + 0 = 0$$

$$\Rightarrow \quad x^2 + y^2 - 46x + 22y = 0$$

Hence, the value of the filler is $\mathbf{x^2 + y^2 - 46x + 22y = 0}$.

Q43. An ellipse is described by using an endless string which is passed over two pins. If the axes are 6 cm and 4 cm, the length of the string and distance between the pins are ....

Sol. Let equation of ellipse is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$



Here $\quad 2a = 6 \quad \Rightarrow \quad a = 3$

and $\quad 2b = 4 \quad \Rightarrow \quad b = 2$

We know that $\quad c^2 = a^2 - b^2$

$$= (3)^2 - (2)^2 = 9 - 4 = 5$$

$\therefore \quad c = \sqrt{5}$, we have $e = \dfrac{c}{a} \Rightarrow e = \dfrac{\sqrt{5}}{3}$

$$\text{Length of string} = 2a + 2ae = 2a(1 + e)$$

$$= 6\left(1 + \frac{\sqrt{5}}{3}\right) = \frac{6(3 + \sqrt{5})}{3} = 6 + 2\sqrt{5}$$

Distance between the pins = CC′ = $2ae = 2 \times 3 \times \dfrac{\sqrt{5}}{3} = 2\sqrt{5}$

Hence, the value of the filler are $\mathbf{6 + 2\sqrt{5}}$ **cm** and $\mathbf{2\sqrt{5}}$ **cm.**

Q44. The equation of the ellipse having foci (0, 1), (0, – 1) and minor axis of length 1 is ....

Sol. We know that the foci of the ellipse are $(0, \pm ae)$ and given foci are $(0, \pm 1)$, so $ae = 1$

Length of minor axis = $2b = 1 \quad \Rightarrow \quad b = \dfrac{1}{2}$

We know that $\quad b^2 = a^2(1 - e^2)$

$$\left(\frac{1}{2}\right)^2 = a^2 - a^2e^2$$

$$\Rightarrow \quad \frac{1}{4} = a^2 - 1 \quad \Rightarrow \quad a^2 = 1 + \frac{1}{4} = \frac{5}{4}$$

$\therefore$ Equation of ellipse is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\Rightarrow \quad \frac{x^2}{1/4} + \frac{y^2}{5/4} = 1$$

$$\Rightarrow \quad \frac{4x^2}{1} + \frac{4y^2}{5} = 1$$

Hence, the value of the filler is $\mathbf{\frac{4x^2}{1} + \frac{4y^2}{5} = 1.}$

Q45. The equation of the parabola having focus at (– 1, – 2) and directrix is $x - 2y + 3 = 0$ is ....

Sol. Let (x_1, y_1) be any point on the parabola.

According to the definition of the parabola

$$\sqrt{(x_1+1)^2 + (y_1+2)^2} = \left|\frac{x_1 - 2y_1 + 3}{\sqrt{(1)^2 + (-2)^2}}\right|$$

Squaring both sides, we get

$$x_1^2 + 1 + 2x_1 + y_1^2 + 4 + 4y_1 = \frac{x_1^2 + 4y_1^2 + 9 - 4x_1y_1 - 12y_1 + 6x_1}{5}$$

$$\Rightarrow \quad x_1^2 + y_1^2 + 2x_1 + 4y_1 + 5 = \frac{x_1^2 + 4y_1^2 - 4x_1y_1 - 12y_1 + 6x_1 + 9}{5}$$

$$\Rightarrow 5x_1^2 + 5y_1^2 + 10x_1 + 20y_1 + 25$$

$$= x_1^2 + 4y_1^2 - 4x_1y_1 - 12y_1 + 6x_1 + 9$$

$$\Rightarrow 4x_1^2 + y_1^2 + 4x_1 + 32y_1 + 4x_1y_1 + 16 = 0$$

Hence, the value of the filler is $\mathbf{4x^2 + 4xy + y^2 + 4x + 32y + 16 = 0}$.

Q46. The equation of the hyperbola with vertices at (0, ± 6) and eccentricity $\frac{5}{3}$ is .... and its foci are ....

Sol. Let equation of the hyperbola is $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Vertices are $(0, \pm b)$ $\quad\therefore\quad b = 6$ and $e = \frac{5}{3}$

We know that $\quad e = \sqrt{1 + \frac{a^2}{b^2}}$

$$\Rightarrow \quad \frac{5}{3} = \sqrt{1 + \frac{a^2}{36}} \quad \Rightarrow \quad \frac{25}{9} = 1 + \frac{a^2}{36}$$

$$\Rightarrow \quad \frac{a^2}{36} = \frac{25}{9} - 1 = \frac{16}{9} \quad \Rightarrow \quad a^2 = \frac{16}{9} \times 36$$

$$\Rightarrow \quad a^2 = 64$$

So the equation of the hyperbola is

$$\frac{-x^2}{64} + \frac{y^2}{36} = 1 \quad \Rightarrow \quad \frac{y^2}{36} - \frac{x^2}{64} = 1$$

and foci = $(0, \pm be) = \left(0, \pm 6 \times \frac{5}{3}\right) = (0, \pm 10)$

Hence, the value of the filler is $\mathbf{\frac{y^2}{36} - \frac{x^2}{64} = 1}$ and **$(0, \pm 10)$**.

OBJECTIVE TYPE QUESTIONS

47. The area of the circle centred at (1, 2) and passing through (4, 6) is

(*a*) 5π (*b*) 10π

(*c*) 25π (*d*) None of these

Sol. Given that the centre of the circle is (1, 2)

$$\text{Radius of the circle} = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{9+16} = 5$$

So, the area of the circle $= \pi r^2 = \pi \times (5)^2 = 25\pi$

Hence, the correct option is (*c*).

Q48. Equation of a circle when passes through (3, 6) and touches the axes is

(*a*) $x^2 + y^2 + 6x + 6y + 3 = 0$

(*b*) $x^2 + y^2 - 6x - 6y - 9 = 0$

(*c*) $x^2 + y^2 - 6x - 6y + 9 = 0$

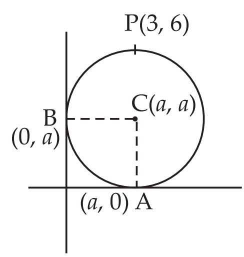
(*d*) None of these

Sol. Let the required circle touch the axes at $(a, 0)$ and $(0, a)$

$\therefore$ Centre is (a, a) and $r = a$

So the equation of the circle is

$$(x-a)^2 + (y-a)^2 = a^2$$



If it passes through a point P(3, 6) then

$$(3-a)^2 + (6-a)^2 = a^2$$

$\Rightarrow$ $9 + a^2 - 6a + 36 + a^2 - 12a = a^2$

$\Rightarrow$ $a^2 - 18a + 45 = 0$

$\Rightarrow$ $a^2 - 15a - 3a + 45 = 0$

$\Rightarrow$ $a(a-15) - 3(a-15) = 0$

$\Rightarrow$ $(a-3)(a-15) = 0$

$\Rightarrow$ $a = 3$ and $a = 15$ which is not possible

$\therefore$ $a = 3$

So, the required equation of the circle is

$$(x-3)^2 + (y-3)^2 = 9$$

$\Rightarrow$ $x^2 + 9 - 6x + y^2 + 9 - 6y = 9$

$\Rightarrow$ $x^2 + y^2 - 6x - 6y + 9 = 0$

Hence, the correct option is (*c*).

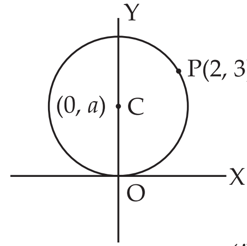
Q49. Equation of the circle with centre on the y-axis and passing through the origin and (2, 3) is

(a) $x^2 + y^2 + 13y = 0$ (b) $3x^2 + 3y^2 + 13y + 3 = 0$

(c) $6x^2 + 6y^2 - 13x = 0$ (d) $x^2 + y^2 + 13x + 3 = 0$

Sol. Let the equation of the circle be

$$(x - h)^2 + (y - k)^2 = r^2$$



Let the centre be $(0, a)$

$\therefore$ Radius $r = a$

So, the equation of the circle is

$$(x - 0)^2 + (y - a)^2 = a^2$$

$\Rightarrow$ $x^2 + (y - a)^2 = a^2$

$\Rightarrow$ $x^2 + y^2 + a^2 - 2ay = a^2$

$\Rightarrow$ $x^2 + y^2 - 2ay = 0$ ...(i)

Now CP = r

$\Rightarrow$ $\sqrt{(2-0)^2 + (3-a)^2} = a$

$\Rightarrow$ $\sqrt{4 + 9 + a^2 - 6a} = a$

$\Rightarrow$ $\sqrt{13 + a^2 - 6a} = a$

$\Rightarrow$ $13 + a^2 - 6a = a^2$

$\Rightarrow$ $13 - 6a = 0$

$\therefore$ $a = \dfrac{13}{6}$

Putting the value of a in eq. (i) we get

$$x^2 + y^2 - 2\left(\frac{13}{6}\right)y = 0$$

$\Rightarrow$ $3x^2 + 3y^2 - 13y = 0$

(Note: (a) option is correct and it should be $3x^2 + 3y^2 - 13y = 0$)

Hence, the correct option is (a).

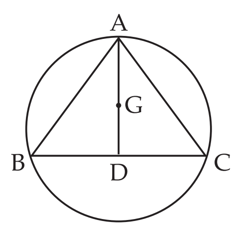
Q50. The equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length $3a$ is

(a) $x^2 + y^2 = 9a^2$ (b) $x^2 + y^2 = 16a^2$

(c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 = a^2$

Sol. Let ABC be an equilateral triangle in which median AD = $3a$.

Centre of the circle is same as the centroid of the triangle i.e., (0, 0)



$$AG : GD = 2 : 1$$

So, $AG = \dfrac{2}{3}AD = \dfrac{2}{3} \times 3a = 2a$

$\therefore$ The equation of the circle is

$$(x-0)^2 + (y-0)^2 = (2a)^2$$

$$\Rightarrow \quad x^2 + y^2 = 4a^2$$

Hence, the correct option is (*c*).

Q51. If the focus of a parabola is (0, – 3) and its directrix is $y = 3$, then its equation is

(*a*) $x^2 = -12y$ (*b*) $x^2 = 12y$

(*c*) $y^2 = -12x$ (*d*) $y^2 = 12x$

Sol. According to the definition of parabola

$$\sqrt{(x-0)^2 + (y+3)^2} = \left|\frac{y-3}{\sqrt{(0)^2 + (1)^2}}\right|$$

$$\Rightarrow \quad \sqrt{x^2 + y^2 + 9 + 6y} = |y-3|$$

Squaring both sides, we have

$$x^2 + y^2 + 9 + 6y = y^2 + 9 - 6y$$

$$\Rightarrow \quad x^2 + 9 + 6y = 9 - 6y$$

$$\Rightarrow \quad x^2 = -12y$$

Hence, the correct option is (*a*).

Q52. If the parabola $y^2 = 4ax$ passes through the point (3, 2) then the length of its latus rectum is

(*a*) $\frac{2}{3}$ (*b*) $\frac{4}{3}$

(*c*) $\frac{1}{3}$ (*d*) 4

Sol. Given parabola is $y^2 = 4ax$

If the parabola is passing through (3, 2)

then $(2)^2 = 4a \times 3$

$$\Rightarrow \quad 4 = 12a \quad \Rightarrow \quad a = \frac{1}{3}$$

Now length of the latus rectum $= 4a = 4 \times \frac{1}{3} = \frac{4}{3}$

Hence, the correct option is (*b*).

Q53. If the vertex of the parabola is the point (– 3, 0) and the directrix is the line $x + 5 = 0$, then the equation is

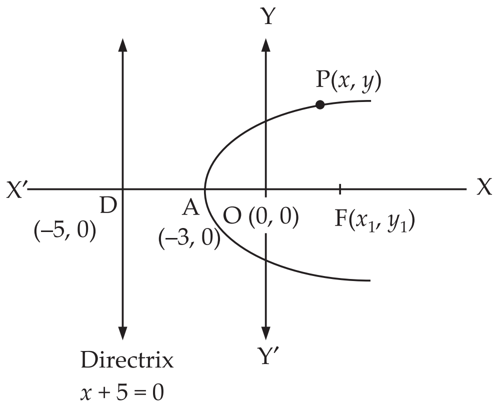
(*a*) $y^2 = 8(x+3)$ (*b*) $x^2 = 8(y+3)$

(*c*) $y^2 = -8(x+3)$ (*d*) $y^2 = 8(x+5)$

Sol. Given that vertex = (– 3, 0)

$\therefore a = -3$

and directrix is $x + 5 = 0$



According to the definition of the parabola, we get

AF = AD i.e., A is the mid-point of DF

$\therefore \qquad -3 = \dfrac{x_1 - 5}{2} \quad \Rightarrow \quad x_1 = -6 + 5 = -1$

and $\qquad 0 = \dfrac{0 + y_1}{2} \quad \Rightarrow \quad y_1 = 0$

$\therefore \qquad$ Focus F $= (-1, 0)$

Now $\sqrt{(x+1)^2 + (y-0)^2} = \left| \dfrac{x+5}{\sqrt{1^2 + 0^2}} \right|$

Squaring both sides, we get

$$(x+1)^2 + y^2 = (x+5)^2$$

$$\Rightarrow \quad x^2 + 1 + 2x + y^2 = x^2 + 25 + 10x$$

$$\Rightarrow \quad y^2 = 10x - 2x + 24 \Rightarrow y^2 = 8x + 24$$

$$\Rightarrow \quad y^2 = 8(x+3)$$

Hence, the correct option is (*a*).

Q54. The equation of the ellipse, whose focus is (1, – 1), the directrix the line $x - y - 3 = 0$ and eccentricity $\dfrac{1}{2}$, is

(*a*) $7x^2 + 2xy + 7y^2 - 10x + 10y + 7 = 0$

(*b*) $7x^2 + 2xy + 7y^2 + 7 = 0$

(*c*) $7x^2 + 2xy + 7y^2 + 10x - 10y - 7 = 0$

(*d*) None of the above

Sol. Given that focus of the ellipse is (1, – 1) and the equation of the directrix is $x - y - 3 = 0$ and $e = \dfrac{1}{2}$.

Let P(x, y) by any point on the parabola

$$\therefore \quad \frac{\text{PF}}{\text{Distance of the point P from the directrix}} = e$$

$$= \frac{\sqrt{(x-1)^2 + (y+1)^2}}{\left|\dfrac{x-y-3}{\sqrt{(1)^2 + (-1)^2}}\right|} = \frac{1}{2}$$

$$\Rightarrow 2\sqrt{x^2 + 1 - 2x + y^2 + 1 + 2y} = \left|\frac{x-y-3}{\sqrt{2}}\right|$$

Squaring both sides, we have

$$\Rightarrow 4(x^2 + y^2 - 2x + 2y + 2) = \frac{x^2 + y^2 + 9 - 2xy + 6y - 6x}{2}$$

$$\Rightarrow 8x^2 + 8y^2 - 16x + 16y + 16 = x^2 + y^2 - 2xy + 6y - 6x + 9$$

$$\Rightarrow 7x^2 + 7y^2 + 2xy - 10x + 10y + 7 = 0$$

Hence, the correct option is (*a*).

Q55. The length of the latus rectum of the ellipse $3x^2 + y^2 = 12$ is

(*a*) 4 (*b*) 3

(*c*) 8 (*d*) $4/\sqrt{3}$

Sol. Equation of the ellipse is

$$3x^2 + y^2 = 12$$

$$\Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{12} = 1$$

Here $a^2 = 4 \Rightarrow a = 2$

$b^2 = 12 \Rightarrow b = 2\sqrt{3}$

$$\text{Length of the latus rectum} = \frac{2a^2}{b} = \frac{2 \times 4}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$$

Hence, the correct option is (*d*).

Q56. If e is the eccentricity of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (where $a < b$) then

(*a*) $b^2 = a^2(1 - e^2)$ (*b*) $a^2 = b^2(1 - e^2)$

(*c*) $a^2 = b^2(e^2 - 1)$ (*d*) $b^2 = a^2(e^2 - 1)$

Sol. Given equation is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \quad (a < b)$

$$\therefore \quad \text{Eccentricity } e = \sqrt{1 - \frac{a^2}{b^2}} \Rightarrow e^2 = 1 - \frac{a^2}{b^2}$$

$$\Rightarrow \quad \frac{a^2}{b^2} = (1 - e^2) \Rightarrow a^2 = b^2(1 - e^2)$$

Hence, the correct option is (*b*).

Q57. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half of the distance between the foci is

(*a*) $\frac{4}{3}$ (*b*) $\frac{4}{\sqrt{3}}$

(*c*) $\frac{2}{\sqrt{3}}$ (*d*) None of these

Sol. Length of the latus rectum of the hyperbola

$$= \frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a \quad ...(i)$$

Distance between the foci = $2ae$

Transverse axis = $2a$

and Conjugate axis = $2b$

$$\therefore \quad \frac{1}{2}(2ae) = 2b \Rightarrow ae = 2b \Rightarrow b = \frac{ae}{2} \quad ...(ii)$$

$$\Rightarrow \quad b^2 = \frac{a^2e^2}{4}$$

$$\Rightarrow \quad 4a = \frac{a^2e^2}{4} \quad \text{[from eq. } (i)]$$

$$\Rightarrow \quad 16 = ae^2 \quad \therefore \quad a = \frac{16}{e^2}$$

Now $b^2 = a^2(e^2 - 1)$

$$\Rightarrow \quad 4a = a^2(e^2 - 1)$$

$$\Rightarrow \quad \frac{4}{a} = e^2 - 1 \Rightarrow \frac{4}{16/e^2} = e^2 - 1$$

$$\Rightarrow \quad \frac{e^2}{4} = e^2 - 1 \Rightarrow e^2 - \frac{e^2}{4} = 1$$

$$\Rightarrow \quad \frac{3e^2}{4} = 1 \Rightarrow e^2 = \frac{4}{3}$$

$$\therefore \quad e = \frac{2}{\sqrt{3}}$$

Hence, the correct option is (*c*).

Q58. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is

(*a*) $x^2 - y^2 = 32$ (*b*) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(*c*) $2x^2 - 3y^2 = 7$ (*d*) None of these

Sol. We know that the distance between the foci = $2ae$

$\therefore \quad 2ae = 16 \Rightarrow ae = 8$

Given that $\quad e = \sqrt{2}$

$\therefore \quad \sqrt{2}a = 8 \Rightarrow a = 4\sqrt{2}$

Now $\quad b^2 = a^2(e^2 - 1)$

$\Rightarrow \quad b^2 = 32(2 - 1) \Rightarrow b^2 = 32$

So, the equation of the hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{32} - \frac{y^2}{32} = 1$$

$$\Rightarrow x^2 - y^2 = 32$$

Hence, the correct option is (*a*).

Q59. Equation of the hyperbola with eccentricity $\frac{3}{2}$ and foci at $(\pm 2, 0)$ is

(*a*) $\frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$ (*b*) $\frac{x^2}{9} - \frac{y^2}{9} = \frac{4}{9}$

(*c*) $\frac{x^2}{4} - \frac{y^2}{9} = 1$ (*d*) None of these

Sol. Given that $\quad e = \frac{3}{2}$

and $\quad \text{foci} = (\pm ae, 0) = (\pm 2, 0)$

$\therefore \quad ae = 2$

$$a \times \frac{3}{2} = 2 \Rightarrow a = \frac{4}{3}$$

Now we know that $\quad b^2 = a^2(e^2 - 1)$

$$b^2 = \frac{16}{9}\left(\frac{9}{4} - 1\right) \Rightarrow b^2 = \frac{16}{9} \times \frac{5}{4}$$

$\Rightarrow \quad b^2 = \frac{20}{9}$

So, the equation of the hyperbola is

$$\frac{x^2}{(4/3)^2} - \frac{y^2}{20/9} = 1$$

$\Rightarrow \quad \frac{9x^2}{16} - \frac{9y^2}{20} = 1 \Rightarrow \frac{x^2}{16} - \frac{y^2}{20} = \frac{1}{9}$

$\Rightarrow \quad \frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$

Hence, the correct option is (*a*).