Amity International - Saket

First Terminal Examination 2014 - 2015

Class – XI

Subject – Mathematics Chetan

Time : 3 Hours **Max. Marks : 100**

General Instructions :

1. All questions are compulsory.
2. The question paper consists of 26 questions.
3. This question paper is divided into three sections A, B and C.
4. Section A comprises of 6 questions of one mark each
5. Section B comprises of 13 questions of four marks each
6. Section C comprises of 7 questions of 6 marks each
7. There is no overall choice. However, internal choice has been provided in 4 questions of 4 marks each and 2 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

SECTION – 'A' (1×6=6)

1. Let A = {1, 2} and B = {3, 4}. Find the number of relations from A to B.

2. Write down the power set of A, where $A = \{\phi, \{\phi\}\}$.

3. Find x and y if $(2x + y, x - y) = (8, 3)$.

4. Simplify :

$$\frac{\cos x}{\sin\left(\frac{\pi}{2} + x\right)} + \frac{\sin(-x)}{\sin(\pi + x)} - \frac{\tan\left(\frac{\pi}{2} + x\right)}{\cot x}$$

5. Solve :

$$\frac{5 - 2x}{3} \leq \frac{x}{6} - 5 \text{ for } x \in R.$$

6. Express :

$$\left\{i^{17} - \left(\frac{1}{i}\right)^{34}\right\}^2 \text{ in the form of } a + ib$$

+ve −ve +ve

0 1

SECTION – 'B' (4×13=52)

7. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that $A = B$.

OR

A and B are two sets such that $n(A - B) = 14 + x$, $n(B - A) = 3x$ and $n(A \cap B) = x$. Draw a Venn diagram to illustrate information and if $n(A) = n(B)$, find the value of x.

8. Using the principle of mathematical induction, prove that $(41^n - 14^n)$ is a multiple of 27 for all $n \in N$.

9. Let $A = \{x \in N : x^2 - 5x + 6 = 0\}$, $B = \{x \in W : 0 \leq x < 2\}$ and $C = \{x \in N : x < 3\}$, then find :

(a) $(A \times B) \cup (A \times C)$ (b) $A \times (B \cap C)$

(c) $(A \cap C) \times (B \cup C)$

$(x^2+y^2)^2$

10. Find the domain and range of the real function $f(x) = \dfrac{1}{1 - x^2}$.

$3n^2+5n+6n+10$

11. Find the square root of $4 - 4\sqrt{3}\,i$.

$3n(n+2)+5(n+2)$

OR

If $(x + iy)^{\frac{1}{3}} = a + ib$, where $x, y, a, b \in R$, show that $\dfrac{x}{a} - \dfrac{y}{b} = -2(a^2 + b^2)$.

12. Convert the complex number $Z = \dfrac{1 + 7i}{(2 - i)^2}$ into polar form.

13. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n + 1)^{th}$ to $(2n)^{th}$ term is $\dfrac{1}{r^n}$.

$\dfrac{a(r^3-1)}{(r-1)}$

OR

$\dfrac{a(r^n-1)}{(r-1)} = 13$

Find three numbers in G.P. whose sum is 13 and the sum of whose squares is 91.

14. Define the Signum function, draw the graph of it, and also find its domain and range.

15. A man wants to cut three lengths from a single tree of length 43 feet. The second length is to be 3 feet longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest piece if the third piece is to be at least 5 feet longer than the second? By cutting tree what value the man is violating?

16. Find the general solution of the trigonometric equation :

$$\sin x \tan x - 1 = \tan x - \sin x$$

17. In any triangle ABC prove that :

$$\frac{(b^2 - c^2)}{a^2} \sin 2A + \frac{(c^2 - a^2)}{b^2} \sin 2B + \frac{(a^2 - b^2)}{c^2} \sin 2C = 0.$$

OR

In any triangle ABC prove that :

$$\frac{\sin(B - C)}{\sin(B + C)} = \frac{b^2 - c^2}{a^2}$$

$(a-b)(a^2+ab+b^2)$

$a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$

$a^3 - b^3$

18. Prove that $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$.

19. 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on. It took 8 more days to finish the work. Find the number of days in which the work was completed.

SECTION – 'C' (6×7=42)

20. Show that :

$$\frac{1 \times 2^2 + 2 \times 3^2 + \ldots\ldots\ldots\ldots + n \times (n + 1)^2}{1^2 \times 2 + 2^2 \times 3 + \ldots\ldots\ldots\ldots + n^2 \times (n + 1)} = \frac{3n + 5}{3n + 1}$$

13. 21. Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the geometric mean between a and b.

OR

If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the arithmetic mean between a and b, then find the value of n.

22. In a town of 10,000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, find the number of families which buy :

(a) newspaper A only.

(b) none of the newspapers A, B and C.

(c) newspaper B only.

Write the importance of reading newspaper.

23. Using the principle of mathematical induction, prove that :

$$1.3 + 2.3^2 + 3.3^3 + \ldots\ldots\ldots\ldots + n.3^n = \frac{(2n-1)3^{n+1}+3}{4}, \text{ for all } n \in N.$$

24. Show that $2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) = \cos 2\alpha$.

25. Prove that :

$$\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}.$$

OR

Prove that :

$$\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma) = 4\cos\frac{\alpha+\beta}{2}\cos\frac{\beta+\gamma}{2}\cos\frac{\gamma+\alpha}{2}$$

26. Solve the following system of inequalities graphically :

$x + 2y \leq 10, \quad x + y \geq 1, \quad x \leq y, \quad x \geq 0, \quad y \geq 0$

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