Roll. No. .... Name ....

NEW GREEN FIELDS SCHOOL
HALF YEARLY EXAMINATION, 2015-16
MATHEMATICS

Time : 3 hrs.   Class – XI   M.M. :

General Instructions—

(1) All questions are compulsory.

(2) The question paper consists of 26 questions divided into three sections A, B & C. Section A comprises 6 questions of one mark each section B comprises 13 questions of four marks each & section C comprises 7 questions of 6 marks each.

(3) There is no over all choice. However, internal choice have been provided in 4 questions of four marks each & 2 questions of 6 marks each.

(4) Use of calculator is not permitted.

SECTION—A

1   Write the following sets in roaster form :

     $A = \{x : x^4 - 5x^2 + 6 = 0\}$

2   Given $\sin (A + B) = \dfrac{\sqrt{3}}{2}$ & $\tan (A - B) = \dfrac{1}{\sqrt{3}}$, find A & B.

3   Simplify :

     $\sqrt{-4} \times (1 - \sqrt{-64})$

4   Find the multiplicative inverse of $Z = 4 - 3i$.

5   Find the slope of the line passing through the points $(3, -2)$ & $(-1, 4)$.

6   Find the equation of the line which makes intercepts (-3) & 2 on the x & y axis respectively.

SECTION—B

Question numbers 7 to 19 carry 4 marks each.

7 Two finite sets have m and n elements. The total number of subset of the first set is 56 more than the total no. of subsets of the second set. Find the values of m & n.

8 Let f = {(1, 1), (2, 3), (3, 5), (4, 7)} be a function from Z into Z defined by f(x) = ax + b for a, b ∈ Z. Determine a & b.

9 If U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 4, 6, 8} & B = {2, 3, 5, 7}, verify that—

(a) $(A \cup B)' = A' \cap B'$ (b) $(A \cap B)' = A' \cup B'$

10 **Prove that :**

$\tan 50° = 2 \tan 10° + \tan 40°$

11 If $(x + iy)^3 = a + ib$, x, y, a & b ∈ R. Show that :

$$\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$$

12 Prove that :

$$\cos 2\theta . \cos \frac{\theta}{2} - \cos 3\theta . \cos \frac{9\theta}{2} = \sin 5\theta . \sin \frac{5\theta}{2}$$

13 Solve the trigonometrical equation :

$\cos \theta + \cos 3\theta - 2 \cos 2\theta = 0$

OR

Solve :

$\sqrt{3} \cos \theta + \sin \theta = \sqrt{2}$

14 If $\frac{(a+i)^2}{2a-i} = p + iq$, show that :

$$p^2 + q^2 = \frac{(a^2+1)^2}{4a^2+1}$$

OR

If $x + iy = \sqrt{\dfrac{a+ib}{c+id}}$, then prove that :

$$(x^2 + y^2)^2 = \dfrac{a^2 + b^2}{c^2 + d^2}$$

15 Write $Z = \dfrac{i-1}{\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}}$ into the polar form.

16 If $|z| = 1$, prove that $\dfrac{z-1}{z+1}(z \neq -1)$ is purely imaginary number. What can you conclude

is $z = 1$.

OR

For what values of x & y are the numbers $3 + ix^2y$ & $x^2 + y + 4i$ conjugate complexes ? x, y are reals.

17 Find the coordinates of the foot of the perpendicular from the point (–1, 3) to the line $3x - 4y - 16 = 0$.

OR

Find the distance of the line $4x - y = 0$ from the point (4, 1) measured along the line making an angle of 135° with positive x-axis.

18 Find the equation of the line passing through the point (2, 2) & cutting off intercepts on the axis whose sum is 9.

19 Find the equation of the line passing through the intersection of the lines $4x + 7y - 3 = 0$ & $2x - 3y + 1 = 0$ that has equal intercept on the axis.

SECTION—C

Question numbers 20 to 26 carry 6 marks each.

20 Solve :

$$x^2 - (\sqrt{3} - 2\sqrt{2}i)x - 2\sqrt{6}i = 0$$

21 By using the principle of mathematical induction prove that $2.7^n + 3.5^n - 5$ is divisible by 24 for all $n \in N$.

OR

Prove that :

$$\sum_{i=1}^{n-1} t(t+1) = \frac{n(n-1)(n+1)}{3} \ \forall \ n \ge 2, \ n \in N$$

22   Find the value of—

   (a) $\tan \dfrac{\pi}{8}$ (b) $\tan \dfrac{5\pi}{12}$

23   Prove that :

$$\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{16}$$

24   In a town of 10,000 families, it was found that 40% families buy newspaper A, 20% newspaper B & 10% newspaper C, 5% families buy A & B, 3% buy B & C & 4% buy A & C. If 2% families buy all the three newspapers, find the number of families which buy—

   (a) A only   (b) B only

   (c) None of A, B & C

25   (a) Find the domain and the range of the function $f(x) = \sqrt{16 - x^2}$.

   (b) Draw the graph of the function :

$$f(x) = \begin{cases} 1 - x & , & x < 0 \\ 1 & , & x = 0 \\ x + 1 & , & x > 0 \end{cases}$$

26   Find the image of the point (3, 8) with respect to the line x + 3y = 7 assuming that the line to be a plane mirror.

OR

A line is such that its segment between the lines 5x − y + 4 = 0 & 3x + 4y − 4 = 0 is bisected at the point (1, 5). Obtain its equation.