BHATNAGAR INT.

Code 041 (E) B

Time: 3 hrs

M. M. 100

GENERAL INSTRUCTIONS

(i) All questions are compulsory.

(ii) The question paper consists of 26 questions, divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However an internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic table, if required.

SECTION A

Question number 1 to 6 carry 1 mark each.

1. Let $A=\{1,2,3,4,5,6\}$. Define a relation R from A to A by $A=\{(x,y):y=x+1\}$

2. Insert 4 A.M.S between 4 and 19.

3. If $A=\{a_1,a_2\}$ and $B=\{b_1,b_2,b_3\}$ then write B X A .

4. Find $\cos 22\frac{1}{2}^{\circ}$

5. If $\frac{n+1}{\cos ecx}=n\cos x$ and $(2x+1)\sec y=\cos ecy$ then find $\tan(x+y)$.

6. Evaluate : $\sin(40^{\circ}+\theta).\cos(10^{\circ}+\theta)-\cos(40^{\circ}+\theta).\sin(10^{\circ}+\theta)$

SECTION B

Question number 7 to 19 carry 4 marks each.

7. If α and β are different complex numbers with $|\beta|=1$, find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$

8. Find all angles between 0° and 180° which satisfy the equation

$$\tan x+\tan 2x+\tan x.\tan 2x=1$$

9. Find the modulus, argument and polar form of complex number, -1-i

OR

Solve the equation $9x^2-12x+20=0$ by factorization method only.

10. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

11. Check the validity of the statement given below by contradiction method. "P: The sum of an irrational number and a rational number is irrational."

OR

If $\tan(\cot x) = \cot(\tan x)$ then prove that $\sin 2x = \frac{4}{(2n+1)\pi}$

12. Prove that by the principle of mathematical induction

$$\frac{1}{3.7} + \frac{1}{7.11} + \frac{1}{11.15} + \text{-------} + \frac{1}{(4n-1)(4n+3)} = \frac{n}{3(4n+3)}$$

OR

Prove by mathematical induction $(1+x)^n \geq 1+nx$ whenever x is positive and n is a positive integer.

13. (i) Prove that the sum to n terms of the series 11 + 103 + 1005 + ----- is $\frac{10}{9}(10^n - 1) + n^2$

(ii) A manufacturer reckons that the value of a machine, which costs him Rs. 15625, will depreciate each year of 20%. Finds the estimated value at the end of 5 years.

14. Let $x_1, x_2, \text{-------} x_n$ values of a variable x and let 'a' be a non-zero real number. Then, prove that the variance of the observations $ax_1, ax_2, \text{------}, ax_n$ is a^2 var (x). Also, find their standard deviation.

15. If $\cos\theta = \cos\alpha\cos\beta$ then prove that $\tan\left(\frac{\theta+\alpha}{2}\right).\tan\left(\frac{\theta-\alpha}{2}\right) = \tan^2\frac{\beta}{2}$ or Draw the graph of $y = \sin x$

16. Prove that $\frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} = \frac{1}{2}[\tan 9\theta - \tan\theta]$

17. Prove that : $2.7^n + 3.5^n - 5$ is divisible by 24, for all $n \in N$.

18. Let $f = \left(x, \frac{x^2}{1+x^2}\right) : x \in R$ be a function from R into R. Determine the range of f.

19. In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students that had (2)

(i) only chemistry 5
(ii) Mathematics and Physics but not Chemistry 12
(iii) only one of the subjects 3
(iv) at least one of the three subjects 11
(v) none of the subjects 2

OR

(2)

If A, B and C are any three sets, then prove that

(i) $A-(B\cap C)=(A-B)\cup(A-C)$

(ii) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

SECTION C

20. If $2\cos 2\theta + b\sin 2\theta = c$ has α and β its roots, them prove that

(i) $\tan\alpha+\tan\beta=\dfrac{2b}{a+c}$

(ii) $\tan\alpha.\tan\beta=\dfrac{c-a}{c+a}$

(i) $\tan(\alpha+\beta)=b/a$

OR

If $\tan x = -\frac{4}{3}$ and x is in quadrant II

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, $\tan\frac{x}{2}$

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21. Find the sum of the following series upto n terms.

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+------$$

OR

The ratio of the A.M. and G.M. of two positive numbers **a** and **b** is **m:n** then show that

$$a:b=\left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right)$$

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22. Solve the following systems of inequalities graphically:

$x+2y\le 10,\ x+2y\ge 1,\ x-y\le 0,\ x\ge 0,\ y\ge 0$

23. If a, b, c, d, and p are different real numbers such that:

$(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)\le 0$, then show that a, b, c, and d are in G.P.

Or Find the sum of $7+77+777+\ldots$ n terms

24. (i) Prove that $(a-b)\cos\dfrac{c}{2}=c\sin\left(\dfrac{A-B}{2}\right)$

(ii) The elevation of a tower at a station **A** due North of it is 'α' and a station **B** due west of **A** is 'β'. Prove that the height of the tower is

$$\frac{AB\sin\alpha\sin\beta}{\sqrt{\sin^2\alpha-\sin^2\beta}}$$

OR

If $\sin\alpha+\sin\beta=a$ and $\cos\alpha+\cos\beta=b$ then prove that

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(i) $\cos(\alpha-\beta)=\dfrac{a^2+b^2-2}{2}$

(ii) $\tan\left(\dfrac{\alpha-\beta}{2}\right)=\pm\sqrt{\dfrac{4-a^2-b^2}{a^2+b^2}}$

25. Find the variance and standard deviation for the following distribution.

X_i	4.5	14.5	24.5	34.5	44.5	54.5	64.5
F_i	1	5	12	22	17	9	4

26. Find real θ such that $\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

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