HALF YEARLY EXAMINATION (2017-2018)

SUB: MATHEMATICS

Class : XI

SET - B

Time: 3 hrs

M.M: 100

Directions: All the questions are compulsory. Q 1 to Q 4 carries one mark each Q 5 to Q 12 carries 2 marks each Q 13 to Q 23 carries 4 marks each and Q 24 to Q 29 carries 6 marks each.

Q.1 Write the multiplicative inverse of $-2 - \sqrt{3}\, i$.

Q.2 State "True" or "False" for each of the following:

  (i) $\{1\} \in \{1,2,3\}$

  (ii) $\{b, c\} \subset \{a,\{b, c\}\}$.

Q.3 Give the definition of Modulus function.

Q.4 There are 12 buses running between Jammu and Delhi. In how many ways can a man go from Jammu to Delhi and return by a different bus?

Q.5 Evaluate: (i) $\sin 930^\circ$ (ii) $\cos(-870^\circ)$

Q.6 If the 21st and 22nd terms in $(1 + x)^{44}$ are equal, then find the value of x.

Q.7 How many three-digit numbers more than 600 can be formed by using the digits 2, 3, 4, 6, 7.

Q.8 Solve: $|x - 1| \le 5$

Q.9 Prove that: $i^{107} + i^{112} + i^{117} + i^{122} = 0$

Q.10 Solve: $9x^2 - 12x + 20 = 0$

Q. 11 If $f : R \to R$ is defined by $f(x) = \frac{x}{x^2 + 1}$, find $f(f(2))$.

Q.12 Write the following sets in the interval form:

  (i) $\{x \in R : 3 < 2x < 7\}$ (ii) $\{x \in R : -1 < 4x - 1 \le 9\}$.

Q.13 Prove that $\sum_{r=0}^{n} 3^r \ {}^nC_r = 4^n$

Q.14 Prove by using PMI for all $n \varepsilon N$

$$1 + 2 + 3 + \cdots + n < \frac{1}{8}(2n+1)^2$$

Q.15 Find the square root of $-7 - 24i$.

Q.16 Convert the complex number $\frac{-16}{1+i\sqrt{3}}$ into polar form.

OR

Find the conjugate and argument of $(3 - 2i)(3 + 2i)(1 + i)$.

Q.17 Solve the following system of inequalities graphically:

$3x + 2y \leq 150, \ x + 4y \leq 80, \ x \leq 15, \ x, y \geq 0.$

Q.18 Find n, if ${}^{2n}C_1, {}^{2n}C_2$ and ${}^{2n}C_3$ are in A.P.

Q.19 Find the term independent of x in the expansion of $(\frac{3}{2}x^2 - \frac{1}{3x})^6$.

OR

Prove that the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$.

Q. 20 Solve: $2cos^2 x + 3sinx = 0$

OR

Find the value of $\tan \frac{\pi}{8}$.

Q. 21 How many words with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?

Q.22 A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports?

Q.23 Let $f(x) = x^2 - x$ and $g(x) = x$ be two functions defined in the domain R^+ $\cup \{0\}$. Find:

(i) $(f + g)(0)$ (ii) $(f - g)(-1)$ (iii) $(fg)(\frac{1}{2})$ (iv) $\left(\frac{f}{g}\right)(4)$.

Q.24 Using PMI prove that for all $n \in N$: $4^n - 3n - 1$ is a multiple of 9.

OR

Using PMI prove that for all $n \in N$: $41^n - 14^n$ is a multiple of 27.

Q.25(i)   Let R be the set of real numbers. Define the real function $f : R \rightarrow R$ by $f(x) = x + 10$ and sketch the graph of this function.

(ii)   If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$.

Q.26 A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has:
(i)   no girl?

(ii)   at least one boy and one girl?

(iii)   At least 3 girls?

Q.27 A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more that 15% but less than 18%?

Q.28 Prove that: $cos^2 x + cos^2\left(x + \frac{\pi}{3}\right) + cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$

Q.29 Find a, b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290 and 30375, respectively.