Mid Term Examination
MATHEMATICS
MT-2017-11(B)

Time : 3 hrs. **M. Marks : 100**

General Instructions :

(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Questions 1-4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Questions 5-12 in Section B are short-answer type questions carrying 2 marks each.
(v) Questions 13-23 in Section C are long-answer type questions carrying 4 marks each.
(vi) Questions 24-29 in Section D are long answer type questions carrying 6 marks each.
(vii) Use of calculator is not allowed.

SECTION - A

1. Find the coefficient of $\left(\frac{1}{x^2}\right)$ in the expansion of $\left(\frac{1}{x^3}+x\right)^{17}$, if any.

2. Prove that : $\frac{1+\sin A+\cos A}{1+\sin A-\cos A}=\cot A/2$.

3. In how many ways can 5 boys and 3 girls be seated in a row so that no two girls are together.

4. Evaluate : $\sum_{K=1}^{\infty}\left[\left(\frac{1}{2}\right)^{K+2}\right]$.

SECTION - B

5. If $\cos x=-\frac{1}{4}$, x lies in III quadrant. Find the value of $\sin\frac{x}{2},\cos\frac{x}{2}$.

6. Prove that : $\tan 50^0=\tan 40^0+2\tan 10^0$

7. If $Z_1=2-i, Z_2=1+i$, find $\left|\frac{Z_1-Z_2+1}{Z_1+Z_2+1}\right|$.

8. A man wants to cut three lengths from a single piece of board of length 91cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice

as long as the shortest. What are the possible lengths of the shortest board, if the third piece is to be at least 5 cm longer than the second?

9. How many four digit numbers divisible by 4 can be formed with 3, 4, 5, 6, 7, 8, 9 if repetition of digits is not allowed.

10. The english alphabet has 5 vowels and 21 consonants. How many words with or without meaning, with one vowel and three different consonants can be formed from these alphabets.

11. How many terms of the $AP: -6, \frac{-11}{2}, -5$ ...... are added to give the sum (–25).

12. If the sum of n terms of an A.P. is $(pn + qn^2)$, where p and q are constants, find the common difference.

SECTION - C

13. The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in order, we obtain an A.P. Find the numbers.

14. If α and β are roots of $x^2 - 3x + K = 0$ and γ and δ are roots of $x^2 - 12x + m = 0$ where $\alpha, \beta \gamma, \delta$ are in G.P. Prove that $\frac{m+k}{m-k} = \frac{17}{15}$.

15. Show that : $\frac{1.3.5.7..........(2K-1)}{K!}.2^K.y^K$ is the middle term in the expansion of $(y+1)^{2K}$, where K is a positive integer.

16. Show that the coefficient of the middle terms in the expansion of $(1+x)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1+x)^{2n-1}$.

17. A tea party is arranged for 18 people along two sides of a long table with 9 chairs on each side. Four men wish to sit on one particular side and two on the other side. In how many ways can they be seated.

18. From a class of 23 students, 10 are to be chosen for an excursion party. There are three students who decide that either all of them will join or none of them will join. In how many ways can the excursion party be arranged.

19. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of 8% solution, how many litres of the 2% solution will have to be added.

OR

Solve the following system of equations graphically :

$x + 2y \le 8,\ 2x + y \le 8,\ x - y \ge 0,\ x \ge 0, y \ge 0$

20. If Z_1 and Z_2 are different complex numbers with $|Z_2|=1$, then find the value of $\left|\dfrac{Z_2-Z_1}{1-\bar{Z}_1 Z_2}\right|$.

21. Convert the complex number $\dfrac{16}{1+i\sqrt{3}}$ into polar form.

22. Prove by using the principle of mathematical induction for $n \in N$.

$1.2 + 2.2^2 + 3.2^3 + \dots\dots\dots + n.2^n = (n-1)2^{n+1} + 2$

OR

Prove by Principle of mathematical induction for $n \in N$

$\left(x^{2n} - y^{2n}\right)$ is divisible by $(x+y)$

23. Prove that :

$$4\cos\frac{x+y}{2}.\cos\frac{y+z}{2}.\cos\frac{z+x}{2} = \cos x + \cos y + \cos z + \cos(x+y+z)$$

OR

Solve : $\sqrt{2}\,\text{cosec}\, x + \cot x - \sqrt{3} = 0$

SECTION - D

24. Prove that : $\sin 80^0 . \sin 60^0 . \sin 40^0 . \sin 20^0 = \dfrac{3}{16}$.

25. If $z\cos\left(\theta + \dfrac{4\pi}{3}\right) = y\cos\left(\theta + \dfrac{2\pi}{3}\right) = x\cos\theta$, then find the value of $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}$.

OR

Prove that : $\cos^2 x + \cos^2\left(x + \dfrac{\pi}{3}\right) + \cos^2\left(x - \dfrac{\pi}{3}\right) = \dfrac{3}{2}$

26. Solve : $2x^2 - (3+7i)x - (3-9i) = 0$, over the set of complex number.

27. How many four letter words can be formed using the letters of the word TRIANGLE so that

(i) T is included in each word.

(ii) T is not included in any word

(iii) there are 2 vowels and 2 consonants in each word.

OR

Find the number of ways in which an arrangement of 4 letters can be made from the letters of the word MATHEMATICS.

28. Find a, b and n in the expansion of $(a + b)^n$ if first three terms of the expansion are 729, 7290, 30375 respectively.

29. Find the sum of first n terms of the series :

$5 + 11 + 19 + 29 + 41 + \dots\dots\dots$

OR

Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2 . R^n = S^n$.