

Tanishka
XI-D 18

THE MOTHER'S INTERNATIONAL SCHOOL
HALF YEARLY EXAMINATION (2017-2018)
CLASS - XI
SUBJECT: MATHEMATICS

TIME: 3 HOURS

M.M:100

Note:

1. Q1 – Q4 are of 1 mark each.
2. Q5 – Q12 are of 2 marks each.
3. Q13 – Q23 are of 4 marks each.
- Q24 – Q29 are of 6 marks each.
5. All questions have to be attempted in order.

Section A

- Q1. ✓ If $C(n, 2) = C(n, 3)$, find $P(8, n)$
- Q2. ✓ Prove: $\cos^2 2x - \cos^2 6x = \sin 4x \cdot \sin 8x$
- Q3. If $\tan x = 2 \tan y$, show that $\sin(x + y) = 3 \sin(x - y)$
- Q4. ✓ Given two complex numbers z_1 and z_2 such that $|z_1| = |z_2| = 1$, prove that:
 $|z_1 + z_2| = \left| \frac{1}{z_1} + \frac{1}{z_2} \right|$

Section B

- Q5. ✓ Prove: $\frac{1 + \cos 2x}{\cot \frac{x}{2} - \tan \frac{x}{2}} = \frac{1}{2} \sin 2x$

Q6. Modinager is 64 kms from Delhi. Find, to the nearest second, the angle subtended at the centre of Earth by the arc joining the two cities; Earth being regarded as a sphere with 6400 kms radius.

Q7. Evaluate :
$$\frac{\tan (270^\circ + \theta) \cdot \operatorname{cosec} (450^\circ + \theta)}{\sin (\theta - 180^\circ) \cdot \cot (90^\circ + \theta)}$$

Q8. Solve the equation : $|z| + z = 2 + i$, where z is a complex number.

Q9. A square is drawn by joining the mid - points of the sides of a given square. A third square is drawn inside the second square in the same way and this process is continued indefinitely. If the side of the first square is 4cm; determine the sum of the areas of all the squares.

Q10. Find n , given that the 4th term in the expansion of $\left(\frac{x}{2} + \frac{1}{x}\right)^n$ is $\frac{5}{2}$.

Q11. Express $z = -\sqrt{3} - i$ in the polar form.

Q12. Rita invites 8 persons to dinner and places 5 of them on one round table and the remaining on another. In how many ways can she seat the guests?

Section C

Q13. A complex number $z = \frac{(1-i)(2-i)(3-i)}{1+i}$.

Write z in standard form and hence find its conjugate and multiplicative inverse.

Q14. Solve the given system of linear inequations simultaneously

$$|x - 1| + |x - 2| \geq 3 \quad \text{and} \quad \frac{13 + x}{2x + 7} \geq 1$$

Q15. In an increasing GP, the sum of the first and last term is 99; the product of the second and the second last term is 288 and the sum of all the terms is 189. How many terms are there in the GP?

Q16. Prove:
$$\frac{\sin A \sin 2A + \sin 3A \sin 6A + \sin 4A \sin 13A}{\sin A \cos 2A + \sin 3A \cos 6A + \sin 4A \cos 13A} = \tan 9A$$

Q17. Prove: $\cos^3 A \cdot \cos 3A + \sin^3 A \cdot \sin 3A = \cos^3 2A$

Q18. Find n, if the coefficients of 5^{th} , 6^{th} and 7^{th} terms in the expansion of $(1+x)^n$ are in AP.

Q19. Find the coefficient of x^3 in the expansion of $(1+2x)^4 \cdot (3-x)^5$

Q20. Prove, by The Principle of Mathematical Induction, that $3^{4n+1} + 2^{2n+2}$ is divisible by 7, for all $n \in \mathbb{N}$.

Q21. The different permutations of the word PARALLEL are listed as in a dictionary.

a) How many words are there before the first word starting with L?

b) How many words can be made using all the letters of the given word in which all the vowels are not together?

Q22. In a group of 15 boys, there are 6 hockey players. In how many ways can 12 boys be selected so as to include at least 4 hockey players?

Q23. Solve the following system of linear inequations graphically:

$$x - y + 2 \geq 0, \quad 5x + 4y \leq 20, \quad 4x - 3y < 12, \quad x \geq 0$$

Section D

Q24. (a) Given that the sum of n terms of two APs are in the ratio $(7n - 5) : (5n + 17)$, find the ratio of their 6th terms.

(b) Prove, by The Principle of Mathematical Induction, that, for all $n \in \mathbb{N}$:

$$1.2.3 + 2.3.4 + 3.4.5 + \dots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

Q25. (a) If $\sin \theta = \frac{1}{2} \left(x + \frac{1}{x} \right)$, then show that $\sin 3\theta = -\frac{1}{2} \left(x^3 + \frac{1}{x^3} \right)$

(b) Solve for x : $\sin x \tan x - 1 = \tan x - \sin x$.

Q26. (a) If $2 \tan A + \cot A = \tan B$, prove that $\cot A = 2 \tan (B - A)$

(b) How many 6 - digit numbers can be formed using the digits of the number 764899 which are

(i) less than the given number?

(ii) odd?

Q27. Solve for x : $x^2 - x + 1 + i = 0$

Q28. Find (i) $\sin(A + 2B)$ (ii) $\tan \frac{A}{2}$, given that

$$\sin A = \frac{3}{5}, \frac{\pi}{2} < A < \pi \quad \text{and} \quad \cos B = \frac{-12}{13}, \pi < B < \frac{3\pi}{2}$$

29. Find the sum: $1 \cdot 4^2 + 2 \cdot 5^2 + 3 \cdot 6^2 + \dots$ upto n terms.

*****The End*****