15-9-17

# ST. PAUL'S SCHOOL
# HALF YEARLY EXAMINATION 2017-18
# CLASS XI
# MATHEMATICS

**Time: 3 Hours** **M.M:100**

***General Instructions:***

(i) *All questions are compulsory.*

(ii) *The question paper consists of 29 questions divided into three sections, A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.*

(iii) *There is no overall choice.*

(iv) *Use of calculators is not permitted.*

## SECTION A

1. Write the number of arrangements of the word BANANA in which two N's come together

2. Solve the in equation: $\frac{1}{x-2} \geq 0$

3. Find in degrees the angle subtended at the centre of a circle of diameter 50cm by an arc of length 11cm

4. Evaluate $i^{-999}$

## SECTION B

5. If ${}^nP_r = 720$ and ${}^nC_r = 120$, find $r$

6. Solve the quadratic equation $\sqrt{5}x^2 + x + \sqrt{5} = 0$

7. Write the value of $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 2\theta}}}$ in the simplest form

8. If $R = \{(x, y): x, y \in Z, x^2 + y^2 \leq 4\}$ is a relation on $Z$, then find the domain of $R$

9. If A and B are two sets such that $n(A) = 115, n(B) = 326, n(A-B) = 47$, then write $n(A \cup B)$

10. Find the value of $\sin 1845°$

11. How many numbers are there between 100 and 1000 in which all the digits are distinct

12. If $x + iy = \frac{3+5i}{7-6i}$, then what is the value of $y$

## SECTION C

13. If $a+ib=\dfrac{c+i}{c-i}$, where $c$ is real, then prove that $a^2+b^2=1$ and $\dfrac{b}{a}=\dfrac{2c}{c^2-1}$

14. In how many ways can the letters of the word PERMUTATIONS be arranged, so that
   a. The words start with P and end in S
   b. There are always 4 letters between P and S

P _ _ _ S _ _ _

15. Using Principle of Mathematical Induction prove that
$$1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots+n}=\frac{2n}{n+1},\text{ for all } n\in N$$

16. Solve the following equation for $x$:
$\tan x+\tan 2x+\tan x\tan 2x=1$

17. Find the domain and range of:
   a. $f(x)=\dfrac{1}{\sqrt{x-5}}$
   b. $f(x)=\dfrac{x}{1+x^2}$

18. For any two sets A and B, prove using properties of sets that:
   a. $(A\cup B)-(A\cap B)=(A-B)\cup(B-A)$
   b. $A\cap(B-C)=(A\cap B)-(A\cap C)$

19. How many numbers greater than 1000000 can be formed by using the digits 1,2,0,2,4,2,4?

20. Convert the complex number $z=\dfrac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ in the polar form

21. Using Principle of Mathematical Induction prove that
$$1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3},\text{ for all } n\in N$$

22. Prove that $(\cos x+\cos y)^2+(\sin x+\sin y)^2=4\cos^2\left(\dfrac{x-y}{2}\right)$

23. A college awarded 38 medals in football, 15 in basketball and 20 to cricket. If these medals went to total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports.

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$$\begin{array}{r} 38 \\ 15 \\ \hline 53 \end{array}$$

$$\begin{array}{r} 38 \\ +\ 15 \\ 20 \\ 3 \\ \hline \end{array}$$

SECTION D

24. Solve the following system of inequations graphically:

$x + 2y \le 40, 3x + y \ge 30, 4x + 3y \ge 60, x \ge 0, y \ge 0$

25. If $(x+iy)^{1} = a+ib, x, y, a, b \in R$, show that:

a. $\dfrac{x}{a} + \dfrac{y}{b} = 4(a^2 - b^2)$

b. $\dfrac{x}{a} - \dfrac{y}{b} = -2(a^2 + b^2)$

26. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team includes

a. No girl $\quad {}^7C_5$

b. At least one boy and one girl

c. At least three girls $\quad {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$

27. (i) In a college, there are 1600 students. Out of these 450 play hockey, 500 play cricket and 325 play football. It is also known that 55 play both hockey and football, 40 play both cricket and hockey and 51 play both football and cricket. If 54 students play all three games, find the number of students who do not play at all.

(ii) Prove using properties $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

28. Prove that $\cos 36° \cos 72° \cos 108° \cos 144° = \dfrac{1}{16}$

29. Prove using Mathematical Induction:

$$1 \times 3 + 3 \times 5 + 5 \times 7 + \ldots + (2n-1)(2n+1) = \frac{n(4n^2 + 6n - 1)}{3}$$