Khushi

TAFS

HALF YEARLY EXAMINATION

Session - 2017-18

Class - XI

Subject – Mathematics

Time: 3 Hrs. **M.M.: 100**

Instructions:

a. *All questions are compulsory.*

b. *The question paper contains 29 questions*

c. *Questions 1-4 in section A are very short –answer type questions carrying 1 mark each.*

d. *Questions 5-12 in Section B are short- answer type questions carrying 2 marks each.*

e. *Questions 13-23 in Section C are long-answer-I type questions carrying 4 marks each.*

f. *Questions 24-29 in Section D are long- answer –II type questions carrying 6 marks each.*

Section – A

1. Write the set A = $\{\frac{1}{5}, \frac{2}{7}, \frac{3}{9}, \frac{4}{11}\}$ in the set builder form.
2. Find the principal solution of cosec$\theta = \frac{-2}{\sqrt{3}}$.
3. Find the value of $i^{108} + i^{122}$
4. If the sum of n terms of the G.P. 3, 6, 12,... is 381, then find the value of n.

Section – B

5. Let f be the subset of **Z** x **Z** defined by $f(x) = \{(ab, a+b): a, b \in Z\}$. Is f a function from **Z** x **Z**? Justify your answer.
6. In an A.P. if m^{th} term is n and the n^{th} term is m, where $m \neq n$, find the p^{th} term.
7. Show that $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta,\ 0 < \theta < \frac{\pi}{8}$.
8. Find real θ, such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.
9. A Let $A = \{1,2,3\}$ and $B = \{5,6\}$. Find the number of relations from A to B and A x B.

$(\sqrt{3}+1)^2$
$3+1+2\sqrt{3}$
$4+2\sqrt{3}$

10. Find the value of $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$.

$1+\cos 2\theta = 2\sin^2\theta$

11. Prove that :

$6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times \ldots.\infty = 6$

12. The longest side of a triangle is three times the shortest side and the third side is 2cm shorter than the longest side. If the perimeter of the triangle is at least 61cm, find the minimum length of the shortest side.

$\frac{\sqrt{6}-\sqrt{2}}{4}$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

$\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}$

Section – C

13. Prove that : $\cos 5A = 16cos^5A - 20cos^3A + 5\cos A$

14. If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$ then find the value of $xy+yz+xz$.

OR

Prove that :

$$2sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) = \cos 2\alpha$$

15. If $(x+iy)^3 = u+iv$, then prove that $\frac{u}{x}+\frac{v}{y} = 4(x^2-y^2)$.

$\sin(A+A)$

$\sin A\cos A + \cos A\sin A$

OR

Let z_1 and z_2 be two complex numbers such that

$|1-\bar{z}_1z_2|^2 - |z_1-z_2|^2 = k(1-|z_1|^2)(1-|z_2|^2)$. Find the value of k.

16. Find polar form of the complex number : $\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$.

17. Using Principle of mathematical induction, prove that :

$$1+\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+\cdots+\frac{1}{1+2+3+\cdots+n}=\frac{2n}{n+1}$$

for all $n \in \mathbf{N}$.

18. A college awarded 58 medals in football, 20 in basketball and 25 in cricket. If these went to a total of 78 students and only 5 students got medals in all the three sports, how, many received medals in exactly two of the three sports? **What values dosports inculcate?**

19. Draw the graph of the function f defined as :

$$f(x)=\begin{cases} x, & 0\le x\le 1 \\ \frac{4-x}{3}, & 1\le x\le 4 \\ -x+4, & 4\le x\le 5 \end{cases}$$

Also find its domain and range.

$\cos^2A - \sin^2A$

$\sin^2A = 1-\cos^2A$

$\cos^2A - 1 + \cos^2A$

$2\cos^2A - 1$

$\cos 3A =$

$\cos(2A+A) = \cos 2A\cos A - \sin 2A\sin A$

$(2\cos^2A - 1)\cos A -$

20. Find the sum to n terms of the sequence 6, 66, 666, 6666,....

21. Solve : $\cot x + cosec\, x = \sqrt{3}$

22. If there are (2n+1) terms in an A.P. then prove that the ratio of the sum of odd terms and the sum of even terms is (n+1) : n.

OR

The sums of the n terms of two A.P.s are in the ratio (3n+8) : (7n+15). Find the ratio of their 12^{th} terms.

23. Solve for x:

$$\frac{|x-2|-1}{|x-2|-2} \leq 0$$

3

Section – D

24. Prove that:

$$cos^4\frac{\pi}{8} + cos^4\frac{3\pi}{8} + cos^4\frac{5\pi}{8} + cos^4\frac{7\pi}{8} = \frac{3}{2}$$

OR

Prove that:

$$(1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1+\cos\frac{5\pi}{8})(1+\cos\frac{7\pi}{8}) = \frac{1}{8}$$

25. Solve for x : $x^2 - (7-i)x + (18-i) = 0$

26. Solve the simultaneous linear inequations graphically:

$x + y \leq 5,\ 4x + y \geq 4,\ y > 2x,\ x \leq 4,\ y \leq 3.$

27. Using principle of mathematical induction, prove that $2.7^n + 3.5^n - 5$ is divisible by 24, for all n$\in$**N**.

28. Show that $\dfrac{1\times 2^2 + 2\times 3^2 + 3\times 4^2 + \ldots\ldots + n\times(n+1)^2}{1^2\times 2 + 2^2\times 3 + 3^2\times 4 + \ldots\ldots + n^2\times(n+1)} = \dfrac{3n+5}{3n+1}$

OR

If p, q, r are in G.P. and the equations, $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P.

$S_n = a$

4, 8, 16, 32.

$a = 4$

$r = 2$

28. i. Let $U = \{x : x \in \mathbf{N} \text{ and } x \le 8\}$, $A = \{x : \sqrt{5} < x^2 < \sqrt{50}\}$ and $B = \{x : x \text{ is prime}\}$. Draw a Venn diagram to show the relationship between the given sets U, A and B. Also verify that $(A \cup B)' = A' \cap B'$.

ii. Assuming that P(A) = P(B), prove that A = B.

4, 8, 16,

$a \quad \frac{4(3^{2-1}+n)}{2-1}$ $\quad 4 \times 9 \quad 36$

$\frac{2(2^{3-1}-1)}{2n-1}$

$\frac{4(8-1)}{1}$