EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. Let A = {a, b, c} and the relation R be defined on A as follows:

$$R = \{(a, a), (b, c), (a, b)\}$$

Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.

Sol. Here, $R = \{(a, a), (b, c), (a, b)\}$

for reflexivity; (b, b), (c, c) and for transitivity; (a, c)

Hence, the required ordered pairs are (b, b), (c, c) and (a, c)

Q2. Let D be the domain of the real valued function f defined by $f(x) = \sqrt{25 - x^2}$. Then write D.

Sol. Here, $f(x) = \sqrt{25 - x^2}$

For real value of $f(x)$, $25 - x^2 \geq 0$

$\Rightarrow \ -x^2 \geq -25 \ \Rightarrow x^2 \leq 25 \Rightarrow -5 \leq x \leq 5$

Hence, $D \in -5 \leq x \leq 5$ or $[-5, 5]$

Q3. Let $f, g : R \to R$ be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2 \ \forall \ x \in R$, respectively. Then find gof.

Sol. Here, $f(x) = 2x + 1$ and $g(x) = x^2 - 2$

$\therefore \quad gof = g[f(x)]$

$= [2x + 1]^2 - 2 = 4x^2 + 4x + 1 - 2 = 4x^2 + 4x - 1$

Hence, $gof = 4x^2 + 4x - 1$

Q4. Let $f : R \to R$ be the function defined by $f(x) = 2x - 3 \ \forall \ x \in R$. Write f^{-1}.

Sol. Here, $f(x) = 2x - 3$

Let $f(x) = y = 2x - 3$

$\Rightarrow \quad y + 3 = 2x \Rightarrow x = \dfrac{y+3}{2}$

$\therefore \quad f^{-1}(y) = \dfrac{y+3}{2}$ or $f^{-1}(x) = \dfrac{x+3}{2}$

Q5. If A = {a, b, c, d} and the function $f = \{(a, b), (b, d), (c, a), (d, c)\}$, write f^{-1}.

Sol. Let $y = f(x) \quad \therefore x = f^{-1}(y)$

$\therefore$ If $f = \{(a, b), (b, d), (c, a), (d, c)\}$

then $f^{-1} = \{(b, a), (d, b), (a, c), (c, d)\}$

Q6. If $f : R \to R$ is defined by $f(x) = x^2 - 3x + 2$, write $f[f(x)]$.

Sol. Here, $f(x) = x^2 - 3x + 2$

$\therefore \quad f[f(x)] = [f(x)]^2 - 3f(x) + 2$
$= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$
$= x^4 + 9x^2 + 4 - 6x^3 + 4x^2 - 12x - 3x^2 + 9x - 6 + 2$
$= x^4 - 6x^3 + 10x^2 - 3x$

Hence, $f[f(x)] = x^4 - 6x^3 + 10x^2 - 3x$

Q7. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? If g is described by $g(x) = \alpha x + \beta$, then what value should be assigned to α and β?

Sol. Yes, $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is a function.

Here, $g(x) = \alpha x + \beta$

For (1, 1), $g(1) = \alpha.1 + \beta$

$1 = \alpha + \beta$ ...(1)

For (2, 3), $g(2) = \alpha.2 + \beta$

$3 = 2\alpha + \beta$ ...(2)

Solving eqs. (1) and (2) we get, $\alpha = 2, \beta = -1$

Q8. Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective.

(*i*) $\{(x, y) : x$ is a person, y is the mother of $x\}$

(*ii*) $\{(a, b) : a$ is a person, b is an ancestor of $a\}$

Sol. (*i*) It represents a function. The image of distinct elements of x under f are not distinct. So, it is not injective but it is surjective.

(*ii*) It does not represent a function as every domain under mapping does not have a unique image.

Q9. If the mapping f and g are given by

$f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$ write fog.

Sol. $fog = f[g(x)]$
$= f[g(2)] = f(3) = 5$
$= f[g(5)] = f(1) = 2$
$= f[g(1)] = f(3) = 5$

Hence, $fog = \{(2, 5), (5, 2), (1, 5)\}$

Q10. Let C be the set of complex numbers. Prove that the mapping $f : C \to R$ given by $f(z) = |z|, \forall z \in C$, is neither one-one nor onto.

Sol. Here, $f(z) = |z| \quad \forall z \in C$

$f(1) = |1| = 1$

$f(-1) = |-1| = 1$

$f(1) = f(-1)$

But $1 \neq -1$

Therefore, it is not one-one.

Now, let $f(z) = y = |z|$. Here, there is no pre-image of negative numbers. Hence, it is not onto.

Q11. Let the function $f : R \to R$ be defined by $f(x) = \cos x$, $\forall\ x \in R$. Show that f is neither one-one nor onto.

Sol. Here, $f(x) = \cos x\ \forall\ x \in R$

Let $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \in f(x)$

$$f\left(-\frac{\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$$

$$\cos\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$$

$$f\left(-\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right) = 0$$

But $-\frac{\pi}{2} \neq \frac{\pi}{2}$

Therefore, the given function is not one-one. Also it is not onto function as no pre-image of any real number belongs to the range of $\cos x$ i.e., $[-1, 1]$.

Q12. Let X = {1, 2, 3} and Y = {4, 5}. Find whether the following subsets of X × Y are functions from X to Y or not.

(*i*) $f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$

(*ii*) $g = \{(1, 4), (2, 4), (3, 4)\}$

(*iii*) $h = \{(1, 4), (2, 5), (3, 5)\}$

(*iv*) $k = \{(1, 4), (2, 5)\}$

Sol. Here, given that X = {1, 2, 3}, Y = {4, 5}

$\therefore$ X × Y = {(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)}

(*i*) $f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$

f is not a function because there is no unique image of each element of domain under f.

(*ii*) $g = \{(1, 4), (2, 4), (3, 4)\}$

Yes, g is a function because each element of its domain has a unique image.

(*iii*) $h = \{(1, 4), (2, 5), (3, 5)\}$

Yes, it is a function because each element of its domain has a unique image.

(*iv*) $k = \{(1, 4), (2, 5)\}$

Clearly k is also a function.

Q13. If function $f : A \to B$ and $g : B \to A$ satisfy $gof = I_A$, then show that f is one-one and g is onto.

Sol. Let $x_1, x_2 \in gof$

$$gof\{f(x_1)\} = gof\{f(x_2)\}$$

$$\Rightarrow \quad g(x_1) = g(x_2) \qquad [\because gof = I_A]$$

$$\therefore \quad x_1 = x_2$$

Hence, f is one-one. But g is not onto as there is no pre-image of A in B under g.

Q14. Let $f : R \to R$ be the function defined by $f(x) = \dfrac{1}{2 - \cos x}$, $\forall\, x \in R$. Then, find the range of f.

Sol. Given function is $f(x) = \dfrac{1}{2 - \cos x}$, $\forall\, x \in R$.

Range of $\cos x$ is $[-1, 1]$

Let $\quad f(x) = y = \dfrac{1}{2 - \cos x}$

$$\Rightarrow \quad 2y - y\cos x = 1 \quad \Rightarrow \quad y \cos x = 2y - 1$$

$$\Rightarrow \quad \cos x = \frac{2y-1}{y} = 2 - \frac{1}{y}$$

Now $\quad -1 \le \cos x \le 1$

$$\Rightarrow \quad -1 \le 2 - \frac{1}{y} \le 1 \quad \Rightarrow \quad -1 - 2 \le -\frac{1}{y} \le 1 - 2$$

$$\Rightarrow \quad -3 \le -\frac{1}{y} \le -1 \quad \Rightarrow 3 \ge \frac{1}{y} \ge 1 \quad \Rightarrow \quad \frac{1}{3} \le y \le 1$$

Hence, the range of $f = \left[\mathbf{\frac{1}{3}, 1}\right]$.

Q15. Let n be a fixed positive integer. Define a relation R in Z as follows $\forall\, a, b \in Z$, a R b if and only if $a - b$ is divisible by n. Show that R is an equivalence relation.

Sol. Here, $\forall\, a, b \in Z$ and a R b if and only if $a - b$ is divisible by n. The given relation is an equivalence relation if it is reflexive, symmetric and transitive.

(*i*) Reflexive:

a R $a \quad \Rightarrow \quad (a - a) = 0$ divisible by n

So, R is reflexive.

(*ii*) Symmetric:

a R b = b R $a \quad \forall\, a, b \in Z$

$a - b$ is divisible by n (Given)

$\Rightarrow -(b - a)$ is divisible by n

$\Rightarrow b - a$ is divisible by n

$\Rightarrow b \text{ R } a$

Hence, R is symmetric.

(*iii*) Transitive:

$a \text{ R } b$ and $b \text{ R } c \quad \Leftrightarrow \quad a \text{ R } c \quad \forall\ a, b, c \in \text{Z}$

$a - b$ is divisible by n

$b - c$ is also divisible by n

$\Rightarrow (a - b) + (b - c)$ is divisible by n

$\Rightarrow (a - c)$ is divisible by n

Hence, R is transitive.

So, R is an equivalence relation.

LONG ANSWER TYPE QUESTIONS

Q16. If A = {1, 2, 3, 4}, define relations on A which have properties of being.

(*a*) reflexive, transitive but not symmetric.

(*b*) symmetric but neither reflexive nor transitive

(*c*) reflexive, symmetric and transitive.

Sol. Given that A = {1, 2, 3, 4}

$\therefore \quad A R A = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), (2, 1), (3, 1), (4, 1), (3, 2), (4, 2), (4, 3)\}$

(*a*) Let $R_1 = \{(1, 1), (2, 2), (1, 2), (2, 3), (1, 3)\}$

So, R_1 is reflexive and transitive but not symmetric.

(*b*) Let $R_2 = \{(2, 3), (3, 2)\}$

So, R_2 is only symmetric.

(*c*) Let $R_3 = \{(1, 1), (1, 2), (2, 1), (2, 4), (1, 4)\}$

So, R_3 is reflexive, symmetric and transitive.

Q17. Let R be relation defined on the set of natural number N as follows:

$R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

Sol. Given that $x \in N$, $y \in N$ and $2x + y = 41$

$\therefore \quad$ Domain of R = {1, 2, 3, 4, 5, ..., 20}

and $\quad$ Range = {39, 37, 35, 33, 31, ..., 1}

Here, $\quad (3, 3) \notin R$

as $\quad 2 \times 3 + 3 \neq 41$

So, R is not reflexive.

R is not symmetric as $(2, 37) \in R$ but $(37, 2) \notin R$

R is not transitive as $(11, 19) \in R$ and $(19, 3) \in R$ but $(11, 3) \notin R$.

Hence, R is neither reflexive, nor symmetric and nor transitive.

Q18. Given A = {2, 3, 4}, B = {2, 5, 6, 7}, construct an example of each of the following:

(*i*) an injective mapping from A to B.

(*ii*) a mapping from A to B which is not injective

(*iii*) a mapping from B to A.

Sol. Here, A = {2, 3, 4} and B = {2, 5, 6, 7}

(*i*) Let $f : A \to B$ be the mapping from A to B

$f = \{(x, y) : y = x + 3\}$

$\therefore f = \{(2, 5), (3, 6), (4, 7)\}$ which is an injective mapping.

(*ii*) Let $g : A \to B$ be the mapping from $A \to B$ such that

$g = \{(2, 5), (3, 5), (4, 2)\}$ which is not an injective mapping.

(*iii*) Let $h : B \to A$ be the mapping from B to A

$h = \{(y, x) : x = y - 2\}$

$h = \{(5, 3), (6, 4), (7, 3)\}$ which is the mapping from B to A.

Q19. Give an example of a map

(*i*) which is one-one but not onto.

(*ii*) which is not one-one but onto.

(*iii*) which is neither one-one nor onto.

Sol. (*i*) Let $f : N \to N$ given by $f(x) = x^2$

Let $x_1, x_2 \in N$ then $f(x_1) = x_1^2$ and $f(x_2) = x_2^2$

Now, $f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 + x_2)(x_1 - x_2) = 0$

Since $x_1, x_2 \in N$, so $x_1 + x_2 = 0$ is not possible.

$\therefore \quad x_1 - x_2 = 0 \quad \Rightarrow x_1 = x_2$

$\therefore \quad f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

So, $f(x)$ is one to one function.

Now, Let $f(x) = 5 \in N$

then $x^2 = 5 \quad \Rightarrow \quad x = \pm\sqrt{5} \notin N$

So, f is not onto.

Hence, $f(x) = x^2$ is one-one but not onto.

(*ii*) Let $f : N \times N$, defined by $f(n) = \begin{cases} \dfrac{n+1}{2} & \text{if } n \text{ is odd} \\ \dfrac{n}{2} & \text{if } n \text{ is even} \end{cases}$

Since $f(1) = f(2)$ but $1 \neq 2$,

So, f is not one-one.

Now, let $y \in N$ be any element.

Then $f(n) = y$

$$\Rightarrow \left.\begin{matrix} \dfrac{n+1}{2} & \text{if } n \text{ is odd} \\ \dfrac{n}{2} & \text{if } n \text{ is even} \end{matrix}\right\} = y$$

$\Rightarrow \quad n = 2y - 1 \quad$ if y is even
$\quad n = 2y \quad$ if y is odd or even

$$\Rightarrow \quad n = \begin{cases} 2y - 1 & \text{if } y \text{ is even} \\ 2y & \text{if } y \text{ is odd or even} \end{cases} \in \mathrm{N}\ \forall\ y \in \mathrm{N}$$

$\therefore$ Every $y \in$ N has pre-image

$$n = \begin{cases} 2y - 1 & \text{if } y \text{ is even} \\ 2y & \text{if } y \text{ is odd or even} \end{cases} \in \mathrm{N}$$

$\therefore$ f is onto.

Hence, f is not one-one but onto.

(*iii*) Let $f : \mathrm{R} \to \mathrm{R}$ be defined as $f(x) = x^2$

Let $x_1 = 2$ and $x_2 = -2$

$$f(x_1) = x_1^2 = (2)^2 = 4$$
$$f(x_2) = x_2^2 = (-2)^2 = 4$$
$$f(2) = f(-2) \quad \text{but } 2 \neq -2$$

So, it is not one-one function.

Let $f(x) = -2 \quad \Rightarrow \quad x^2 = -2 \quad \therefore \quad x = \pm\sqrt{-2} \notin \mathrm{R}$

Which is not possible, so f is not onto.

Hence, f is neither one-one nor onto.

Q20. Let A = R – {3}, B = R – {1}. Let $f : \mathrm{A} \to \mathrm{B}$ be defined by $f(x) = \dfrac{x-2}{x-3}$, $\forall\ x \in$ A. Then, show that f is bijective.

Sol. Here, A $\in$ R – {3}, B = R – {1}

Given that $f : \mathrm{A} \to \mathrm{B}$ defined by $f(x) = \dfrac{x-2}{x-3}$ $\forall\ x \in$ A.

Let $x_1, x_2 \in f(x)$

$$\therefore \quad f(x_1) = f(x_2)$$
$$\Rightarrow \quad \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$
$$\Rightarrow \quad (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$
$$\Rightarrow \quad \cancel{x_1 x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_1 x_2} - 3x_2 - 2x_1 + \cancel{6}$$
$$\Rightarrow \quad -x_1 = -x_2 \quad \Rightarrow \quad x_1 = x_2$$

So, it is injective function.

Now, Let $\quad y = \dfrac{x-2}{x-3}$

$$\Rightarrow \quad xy - 3y = x - 2 \quad \Rightarrow \quad xy - x = 3y - 2$$
$$\Rightarrow \quad x(y-1) = 3y - 2 \quad \Rightarrow x = \frac{3y-2}{y-1}$$

$$f(x) = \frac{x-2}{x-3} = \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} \Rightarrow \frac{3y-2-2y+2}{3y-2-3y+3} \Rightarrow y$$

$\Rightarrow$ $f(x) = y \in$ B.

So, $f(x)$ is surjective function.

Hence, $f(x)$ is a bijective function.

Q21. Let A = [– 1, 1], then discuss whether the following functions defined on A are one-one, onto or bijective.

(*i*) $f(x) = \frac{x}{2}$ (*ii*) $g(x) = |x|$ (*iii*) $h(x) = x\,|x|$ (*iv*) $k(x) = x^2$

Sol. (*i*) Given that $-1 \le x \le 1$

Let $x_1, x_2 \in f(x)$

$$f(x_1) = \frac{1}{x_1} \quad \text{and} \quad f(x_2) = \frac{1}{x_2}$$

$$f(x_1) = f(x_2) \Rightarrow \frac{1}{x_1} = \frac{1}{x_2} \Rightarrow x_1 = x_2$$

So, $f(x)$ is one-one function.

Let $f(x) = y = \frac{x}{2} \Rightarrow x = 2y$

For $y = 1$, $x = 2 \notin [-1, 1]$

So, $f(x)$ is not onto. Hence, $f(x)$ is not bijective function.

(*ii*) Here, $g(x) = |x|$

$$g(x_1) = g(x_2) \Rightarrow |x_1| = |x_2| \Rightarrow x_1 = \pm x_2$$

So, $g(x)$ is not one-one function.

Let $g(x) = y = |x| \Rightarrow x = \pm y \notin A \ \forall\ y \in A$

So, $g(x)$ is not onto function.

Hence, $g(x)$ is not bijective function.

(*iii*) Here, $h(x) = x|x|$

$$h(x_1) = h\,f(x_2)$$

$\Rightarrow$ $x_1|x_1| = x_2|x_2| \Rightarrow x_1 = x_2$

So, $h(x)$ is one-one function.

Now, let $h(x) = y = x|x| = x^2$ or $-x^2$

$\Rightarrow$ $x = \pm\sqrt{-y} \notin A\ \forall\ y \in A$

$\therefore$ $h(x)$ is not onto function.

Hence, $h(x)$ is not bijective function.

(*iv*) Here, $k(x) = x^2$

$$k(x_1) = k(x_2)$$

$\Rightarrow$ $x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

So, $k(x)$ is not one-one function.

Now, let $k(x) = y = x^2 \Rightarrow x = \pm\sqrt{y}$

If $y = -1 \Rightarrow x = \pm\sqrt{-1} \notin A \;\forall\; y \in A$

$\therefore$ $k(x)$ is not onto function.

Hence, $k(x)$ is not a bijective function.

Q22. Each of the following defines a relation of N

(*i*) x is greater than y, $x, y \in$ N

(*ii*) $x + y = 10$, $x, y \in$ N

(*iii*) xy is square of an integer $x, y \in$ N

(*iv*) $x + 4y = 10$, $x, y \in$ N.

Determine which of the above relations are reflexive, symmetric and transitive.

Sol. (*i*) x is greater than y, $x, y \in$ N

For reflexivity $x > x \;\forall\; x \in$ N which is not true

So, it is not reflexive relation.

Now, $x > y$ but $y \not> x \;\; \forall\; x, y \in$ N

$\Rightarrow x \text{ R } y$ but $y \not\text{R}\, x$

So, it is not symmetric relation.

For transitivity, $x \text{ R } y, y \text{ R } z \Rightarrow x \text{ R } z \;\forall\; x, y, z \in$ N

$\Rightarrow x > y, y > z \Rightarrow x > z$

So, it is transitive relation.

(*ii*) Here, $\text{R} = \{(x, y) : x + y = 10 \;\forall\; x, y \in \text{N}\}$

$\text{R} = \{(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)\}$

For reflexive: $5 + 5 = 10$, $5 \text{ R } 5 \Rightarrow (x, x) \in \text{R}$

So, R is reflexive.

For symmetric: $(1, 9) \in \text{R}$ and $(9, 1) \in \text{R}$

So, R is symmetric.

For transitive: $(3, 7) \in \text{R}$, $(7, 3) \in \text{R}$ but $(3, 3) \notin \text{R}$

So, R is not transitive.

(*iii*) Here, $\text{R} = \{(x, y) : xy$ is a square of an integer, $x, y \in \text{N}\}$

For reflexive: $x \text{ R } x = x \,.\, x = x^2$ is an integer

[$\because$ Square of an integer is also an integer]

So, R is reflexive.

For symmetric: $x \text{ R } y = y \text{ R } x \;\forall\; x, y \in$ N

$\therefore$ $xy = yx$ (integer)

So, it is symmetric.

For transitive: $x \text{ R } y$ and $y \text{ R } z \Rightarrow x \text{ R } z$

Let $xy = k^2$ and $yz = m^2$

$$x = \frac{k^2}{y} \quad \text{and} \quad z = \frac{m^2}{y}$$

$\therefore$ $xz = \dfrac{k^2 m^2}{y^2}$ which is again a square of an integer.

So, R is transitive.

(*iv*) Here, $R = \{(x, y) : x + 4y = 10, x, y \in N\}$

$R = \{(2, 2), (6, 1)\}$

For reflexivity: $(2, 2) \in R$

So, R is reflexive.

For symmetric: $(x, y) \in R$ but $(y, x) \notin R$

$(6, 1) \in R$ but $(1, 6) \notin R$

So, R is not symmetric.

For transitive: $(x, y) \in R$ but $(y, z) \notin R$ and $(x, z) \in R$

So, R is not transitive.

Q23. Let $A = \{1, 2, 3, ..., 9\}$ and R be the relation in $A \times A$ defined by $(a, b)\, R\, (c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation and also obtain equivalent class $[(2, 5)]$.

Sol. Here, $A = \{1, 2, 3, ..., 9\}$

and $R \to A \times A$ defined by $(a, b)\, R\, (c, d) \Rightarrow a + d = b + c$

$\forall\, (a, b), (c, d) \in A \times A$

For reflexive: $(a, b)\, R\, (a, b) = a + b = b + a \quad \forall\, a, b \in A$ which is true. So, R is reflexive.

For symmetric: $(a, b)\, R\, (c, d) = (c, d)\, R\, (a, b)$

L.H.S. $a + d = b + c$

R.H.S. $c + b = d + a$

L.H.S. = R.H.S. So, R is symmetric.

For transitive: $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f) \Leftrightarrow (a, b)\, R\, (e, f)$

$\Rightarrow \quad a + d = b + c$ and $c + f = d + e$

$\Rightarrow \quad a + d = b + c$ and $d + e = c + f$

$\Rightarrow \quad (a + d) - (d + e) = (b + c) - (c + f)$

$\Rightarrow \quad a - e = b - f$

$\Rightarrow \quad a + f = b + e$

$\Rightarrow \quad (a, b)\, R\, (e, f)$

So, R is transitive.

Hence, R is an equivalence relation.

Equivalent class of $\{(2, 5)\}$ is $\{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$

Q24. Using the definition, prove that the function $f : A \to B$ is invertible if and only if f is both one-one and onto.

Sol. A function $f : X \to Y$ is said to be invertible if there exists a function $g : Y \to X$ such that $gof = I_X$ and $fog = I_Y$ and then the inverse of f is denoted by f^{-1}.

A function $f : X \to Y$ is said to be invertible iff f is a bijective function.

Q25. Function $f, g : R \to R$ are defined, respectively, by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$, find

(*i*) *fog* (*ii*) *gof* (*iii*) *fof* (*iv*) *gog*

Sol. (*i*) $fog \Rightarrow f[g(x)] = [g(x)]^2 + 3[g(x)] + 1$

$= (2x - 3)^2 + 3(2x - 3) + 1$

$= 4x^2 + 9 - 12x + 6x - 9 + 1 = 4x^2 - 6x + 1$

(*ii*) $gof \Rightarrow g[f(x)] = 2[x^2 + 3x + 1] - 3$

$= 2x^2 + 6x + 2 - 3 = 2x^2 + 6x - 1$

(*iii*) $fof \Rightarrow f[f(x)] = [f(x)]^2 + 3[f(x)] + 1$

$= (x^2 + 3x + 1)^2 + 3(x^2 + 3x + 1) + 1$

$= x^4 + 9x^2 + 1 + 6x^3 + 6x + 2x^2 + 3x^2 + 9x + 3 + 1$

$= x^4 + 6x^3 + 14x^2 + 15x + 5$

(*iv*) $gog \Rightarrow g[g(x)] = 2[g(x)] - 3 = 2(2x - 3) - 3 = 4x - 6 - 3 = 4x - 9$

Q26. Let $*$ be the binary operation defined on Q. Find which of the following binary operations are commutative.

(*i*) $a * b = a - b \ \forall \ a, b \in Q$ (*ii*) $a * b = a^2 + b^2 \ \forall \ a, b \in Q$

(*iii*) $a * b = a + ab \ \forall \ a, b \in Q$ (*iv*) $a * b = (a - b)^2 \ \forall \ a, b \in Q$

Sol. (*i*) $a * b = a - b \in Q \quad \forall \ a, b \in Q.$

So, $*$ is binary operation.

$a * b = a - b$ and $b * a = b - a \quad \forall \ a, b \in Q$

$a - b \neq b - a$

So, $*$ is not commutative.

(*ii*) $a * b = a^2 + b^2 \in Q$, so $*$ is a binary operation.

$a * b = b * a$

$\Rightarrow \quad a^2 + b^2 = b^2 + a^2 \quad \forall \ a, b \in Q$

Which is true. So, $*$ is commutative.

(*iii*) $a * b = a + ab \in Q$, so $*$ is a binary operation.

$a * b = a + ab$ and $b * a = b + ba$

$a + ab \neq b + ba \Rightarrow a * b \neq b * a \quad \forall \ a, b \in Q.$

So, $*$ is not commutative.

(*iv*) $a * b = (a - b)^2 \in Q$, so $*$ is binary operation.

$a * b = (a - b)^2$ and $b * a = (b - a)^2$

$a * b = b * a \Rightarrow (a - b)^2 = (b - a)^2 \quad \forall \ a, b \in Q.$

So, $*$ is commutative.

Q27. If $*$ be binary operation defined on R by $a * b = 1 + ab \ \forall \ a, b \in R$. Then, the operation $*$ is

(*i*) commutative but not associative

(*ii*) associative but not commutative

(*iii*) neither commutative nor associative

(*iv*) both commutative and associative

Sol. (*i*): Given that

$a * b = 1 + ab \quad \forall \ a, b \in R$

and $b * a = 1 + ba \quad \forall \ a, b \in R$

$a * b = b * a = 1 + ab$

So, $*$ is commutative.

Now $a * (b * c) = (a * b) * c \qquad \forall \ a, b, c \in R$

L.H.S. $a * (b * c) = a * (1 + bc) = 1 + a(1 + bc) = 1 + a + abc$
R.H.S. $(a * b) * c = (1 + ab) * c = 1 + (1 + ab) . c = 1 + c + abc$
L.H.S. ≠ R.H.S.
So, $*$ is not associative.
Hence, $*$ is commutative but not associative.

OBJECTIVE TYPE QUESTIONS

Choose the correct answer out of the given four options in each of the Exercises from 28 to 47 (M.C.Q.)

Q28. Let T be the set of all triangles in the Euclidean plane and let a relation R on T be defined as a R b, if a is congruent to b, $\forall\ a, b \in$ T. Then R is
(*a*) Reflexive but not transitive
(*b*) Transitive but not symmetric
(*c*) Equivalence
(*d*) None of these

Sol. If $a \cong b\ \forall\ a, b \in$ T
then a R $a \Rightarrow a \cong a$ which is true for all $a \in$ T
So, R is reflexive.
Now, a R b and b R a.
i.e., $a \cong b$ and $b \cong a$ which is true for all $a, b \in$ T
So, R is symmetric.
Let a R b and b R c.
$\Rightarrow a \cong b$ and $b \cong a \Rightarrow a \cong c\ \forall\ a, b, c \in$ T
So, R is transitive.
Hence, R is equivalence relation.
So, the correct answer is (*c*).

Q29. Consider the non-empty set consisting of children in a family and a relation R defined as a R b, if a is brother of b. Then R is
(*a*) symmetric but not transitive
(*b*) transitive but not symmetric
(*c*) neither symmetric nor transitive
(*d*) both symmetric and transitive

Sol. Here, a R $b \Rightarrow a$ is a brother of b.
a R $a \Rightarrow a$ is a brother of a which is not true.
So, R is not reflexive.
a R $b \Rightarrow a$ is a brother of b.
b R $a \Rightarrow$ which is not true because b may be sister of a.
$\Rightarrow a$ R $b \neq b$ R a
So, R is not symmetric.
Now, a R b, b R $c \Rightarrow a$ R c
$\Rightarrow a$ is the brother of b and b is the brother of c.
$\therefore a$ is also the brother of c.

So, R is transitive.
Hence, correct answer is (*b*).

Q30. The maximum number of equivalence relations on the set A = {1, 2, 3} are

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 5

Sol. Here, A = {1, 2, 3}
The number of equivalence relations are as follows:
R_1 = {(1, 1), (1, 2), (2, 1), (2, 3), (1, 3)}
R_2 = {(2, 2), (1, 3), (3, 1), (3, 2), (1, 2)}
R_3 = {(3, 3), (1, 2), (2, 3), (1, 3), (3, 2)}
Hence, correct answer is (*d*)

Q31. If a relation R on the set {1, 2, 3} be defined by R = {(1, 2)}, then R is

(*a*) reflexive (*b*) transitive
(*c*) symmetric (*d*) None of these

Sol. Given that: R = {(1, 2)}
$a \not R\, a$, so it is not reflexive.
a R b but $b \not R\, a$, so it is not symmetric.
a R b and b R $c \Rightarrow a$ R c which is true.
So, R is transitive.
Hence, correct answer is (*b*).

Q32. Let us define a relation R in R as a R b if $a \geq b$. Then R is

(*a*) an equivalence relation
(*b*) reflexive, transitive but not symmetric
(*c*) symmetric, transitive but not reflexive
(*d*) neither transitive nor reflexive but symmetric.

Sol. Here, a R b if $a \geq b$
$\Rightarrow a$ R $a \Rightarrow a \geq a$ which is true, so it is reflexive.
Let a R $b \Rightarrow a \geq b$, but $b \not\geq a$, so $b \not R\, a$
R is not symmetric.
Now, $a \geq b$, $b \geq c \Rightarrow a \geq c$ which is true.
So, R is transitive.
Hence, correct answer is (*b*).

Q33. Let A = {1, 2, 3} and consider the relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}, then R is

(*a*) reflexive but not symmetric
(*b*) reflexive but not transitive
(*c*) symmetric and transitive
(*d*) neither symmetric nor transitive.

Sol. Given that: R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

Here, 1 R 1, 2 R 2 and 3 R 3, so R is reflexive.
1 R 2 but $2 \not R 1$ or 2 R 3 but $3 \not R 2$, so, R is not symmetric.
1 R 1 and 1 R 2 ⇒ 1 R 3, so, R is transitive.
Hence, the correct answer is (*a*).

Q34. The identity element for the binary operation ∗ defined on $Q \sim \{0\}$ as $a * b = \frac{ab}{2}\ \forall\, a, b \in Q \sim \{0\}$ is

(*a*) 1 (*b*) 0 (*c*) 2 (*d*) None of these

Sol. Given that: $a * b = \frac{ab}{2}\ \forall\, a, b \in Q - \{0\}$

Let e be the identity element

$$\therefore \quad a * e = \frac{ae}{2} = a \Rightarrow e = 2$$

Hence, the correct answer is (*c*).

Q35. If the set A contains 5 elements and set B contains 6 elements, then the number of one-one and onto mapping from A to B is

(*a*) 720 (*b*) 120 (*c*) 0 (*d*) None of these

Sol. If A and B sets have m and n elements respectively, then the number of one-one and onto mapping from A to B is

$n!$ if $m = n$

and 0 if $m \neq n$

Here, $m = 5$ and $n = 6$

$5 \neq 6$

So, number of mapping = 0
Hence, the correct answer is (*c*).

Q36. Let A = {1, 2, 3, ..., n} and B = {a, b}. Then the number of surjections from A to B is

(*a*) ${}^nP_2$ (*b*) $2^n - 2$ (*c*) $2^n - 1$ (*d*) None of these

Sol. Here, A = {1, 2, 3, ..., n} and B = {a, b}
Let m be the number of elements of set A
and n be the number of elements of set B
∴ Number of surjections from A to B is
${}^nC_m \times m!$ as $n \geq m$
Here, $m = 2$ (given)
∴ Number of surjections from A to B = ${}^nC_2 \times 2!$

$$= \frac{n!}{2!(n-2)!} \times 2! = \frac{n(n-1)(n-2)!}{2!(n-2)!} \times 2 = n(n-1) = n^2 - n$$

Hence, the correct answer is (*d*).

Q37. Let $f : R \to R$ be defined by $f(x) = \frac{1}{x},\ \forall\, x \in R$ then f is

(*a*) one-one (*b*) onto

(*c*) bijective (*d*) *f* is not defined

Sol. Given that $f(x) = \frac{1}{x}$

Put $x = 0$ $\therefore$ $f(x) = \frac{1}{0} = \infty$

So, $f(x)$ is not defined.

Hence, the correct answer is (*d*).

Q38. Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2+1}$, then gof is

(*a*) $\frac{3x^2-5}{9x^4-30x^2+26}$ (*b*) $\frac{3x^2-5}{9x^4-6x^2+26}$

(*c*) $\frac{3x^2}{x^4+2x^2-4}$ (*d*) $\frac{3x^2}{9x^4+30x^2-2}$

Sol. Here, $f(x) = 3x^2 - 5$ and $g(x) = \frac{x}{x^2+1}$

$\therefore$ $gof = gof(x) = g[3x^2 - 5]$

$$= \frac{3x^2-5}{(3x^2-5)^2+1} = \frac{3x^2-5}{9x^4+25-30x^2+1}$$

$\therefore$ $gof = \frac{3x^2-5}{9x^4-30x^2+26}$

Hence, the correct answer is (*a*).

Q39. Which of the following functions from Z to Z are bijections?

(*a*) $f(x) = x^3$ (*b*) $f(x) = x + 2$

(*c*) $f(x) = 2x + 1$ (*d*) $f(x) = x^2 + 1$

Sol. Given that $f : Z \to Z$

Let $x_1, x_2 \in f(x) \Rightarrow f(x_1) = x_1 + 2, f(x_2) = x_2 + 2$

$f(x_1) = f(x_2) \Rightarrow x_1 + 2 = x_2 + 2 \Rightarrow x_1 = x_2$

So, $f(x)$ is one-one function.

Now, let $y = x + 2$ $\therefore$ $x = y - 2 \in Z \ \forall \ y \in Z$

So, $f(x)$ is onto function.

$\therefore$ $f(x)$ is bijective function.

Hence, the correct answer is (*b*).

Q40. Let $f : R \to R$ be the functions defined by $f(x) = x^3 + 5$. Then $f^{-1}(x)$ is

(*a*) $(x + 5)^{1/3}$ (*b*) $(x - 5)^{1/3}$ (*c*) $(5 - x)^{1/3}$ (*d*) $5 - x$

Sol. Given that $f(x) = x^3 + 5$

Let $y = x^3 + 5 \Rightarrow x^3 = y - 5$

$\therefore$ $x = (y - 5)^{1/3} \Rightarrow f^{-1}(x) = (x - 5)^{1/3}$

Hence, the correct answer is (*b*).

Q41. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be the bijective functions. Then $(gof)^{-1}$ is

(*a*) $f^{-1}og^{-1}$ (*b*) $f\,og$ (*c*) $g^{-1}of^{-1}$ (*d*) gof

Sol. Here, $f : A \rightarrow B$ and $g : B \rightarrow C$

$\therefore \quad (gof)^{-1} = f^{-1}og^{-1}$

Hence, the correct answer is (*a*).

Q42. Let $f : R - \left\{\frac{3}{5}\right\} \rightarrow R$ be defined by $f(x) = \frac{3x+2}{5x-3}$, then

(*a*) $f^{-1}(x) = f(x)$ (*b*) $f^{-1}(x) = -f(x)$

(*c*) $(fof)x = -x$ (*d*) $f^{-1}(x) = \frac{1}{19}f(x)$

Sol. Given that $f(x) = \frac{3x+2}{5x-3} \quad \forall\, x \neq \frac{3}{5}$

Let $y = \frac{3x+2}{5x-3}$

$\Rightarrow \quad y(5x-3) = 3x+2$

$\Rightarrow \quad 5xy - 3y = 3x+2$

$\Rightarrow \quad 5xy - 3x = 3y+2$

$\Rightarrow \quad x(5y-3) = 3y+2$

$\Rightarrow \quad x = \frac{3y+2}{5y-3}$

$\Rightarrow \quad f^{-1}(x) = \frac{3x+2}{5x-3}$

$\Rightarrow \quad f^{-1}(x) = f(x)$

Hence, the correct answer is (*a*).

Q43. Let $f : [0, 1] \rightarrow [0, 1]$ be defined by $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational} \end{cases}$.

Then $(fof)x$ is

(*a*) constant (*b*) $1 + x$

(*c*) x (*d*) None of these

Sol. Given that $f : [0, 1] \rightarrow [0, 1]$

$\therefore \quad f = f^{-1}$

So, $(fof)x = x$ (identity element)

Hence, correct answer is (*c*).

Q44. Let $f : [2, \infty) \rightarrow R$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range of f is

(*a*) R (*b*) $[1, \infty)$ (*c*) $[4, \infty)$ (*d*) $[5, \infty)$

Sol. Given that $f(x) = x^2 - 4x + 5$

Let $y = x^2 - 4x + 5$

$\Rightarrow \quad x^2 - 4x + 5 - y = 0$

$$\Rightarrow \quad x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (5-y)}}{2 \times 1}$$

$$= \frac{4 \pm \sqrt{16 - 20 + 4y}}{2}$$

$$= \frac{4 \pm \sqrt{4y - 4}}{2} = \frac{4 \pm 2\sqrt{y-1}}{2} = 2 \pm \sqrt{y-1}$$

$\therefore$ For real value of x, $y - 1 \geq 0 \Rightarrow y \geq 1$.

So, the range is $[1, \infty)$.

Hence, the correct answer is (*b*).

Q45. Let $f : \text{N} \to \text{R}$ be the function defined by $f(x) = \frac{2x-1}{2}$ and $g : \text{Q} \to \text{R}$ be another function defined by $g(x) = x + 2$ then, $gof\left(\frac{3}{2}\right)$ is

(*a*) 1 (*b*) -1 (*c*) $\frac{7}{2}$ (*d*) None of these

Sol. Here, $f(x) = \frac{2x-1}{2}$ and $g(x) = x + 2$

$$\therefore \quad gof(x) = g[(f(x)]$$

$$= f(x) + 2$$

$$= \frac{2x-1}{2} + 2 = \frac{2x+3}{2}$$

$$gof\left(\frac{3}{2}\right) = \frac{2 \times \frac{3}{2} + 3}{2} = 3$$

Hence, the correct answer is (*d*).

Q46. Let $f : \text{R} \to \text{R}$ be defined by $f(x) = \begin{cases} 2x & : x > 3 \\ x^2 & : 1 < x \leq 3 \\ 3x & : x \leq 1 \end{cases}$

then $f(-1) + f(2) + f(4)$ is

(*a*) 9 (*b*) 14 (*c*) 5 (*d*) None of these

Sol. Given that:

$$f(x) = \begin{cases} 2x & : x > 3 \\ x^2 & : 1 < x \leq 3 \\ 3x & : x \leq 1 \end{cases}$$

$\therefore f(-1) + f(2) + f(4) = 3(-1) + (2)^2 + 2(4) = -3 + 4 + 8 = 9$

Hence, the correct answer is (*a*).

Q47. If $f : R \to R$ be given by $f(x) = \tan x$, then $f^{-1}(1)$ is

(*a*) $\frac{\pi}{4}$ (*b*) $\left\{n\pi + \frac{\pi}{4} : n \in Z\right\}$

(*c*) does not exist (*d*) None of these

Sol. Given that $f(x) = \tan x$

Let $f(x) = y = \tan x \Rightarrow x = \tan^{-1} y$

$\Rightarrow f^{-1}(x) = \tan^{-1}(x)$

$\Rightarrow f^{-1}(1) = \tan^{-1}(1)$

$\Rightarrow f^{-1}(1) = \tan^{-1}\left[\tan\left(\frac{\pi}{4}\right)\right] = \frac{\pi}{4}$

Hence, the correct answer is (*a*).

Fill in the Blanks in Each of the Exercises 48 to 52.

Q48. Let the relation R be defined in N by $a \, R \, b$ if $2a + 3b = 30$. Then R = ...

Sol. Given that $a \, R \, b : 2a + 3b = 30$

$\Rightarrow 3b = 30 - 2a$

$\Rightarrow b = \frac{30 - 2a}{3}$

for $a = 3, b = 8$

$a = 6, b = 6$

$a = 9, b = 4$

$a = 12, b = 2$

Hence, R = {(3, 8), (6, 6), (9, 4), (12, 2)}

Q49. Let the relation R be defined on the set A = {1, 2, 3, 4, 5} by $R = \{(a, b) : |a^2 - b^2| < 8\}$. Then R is given by ....

Sol. Given that A = {1, 2, 3, 4, 5} and $R = \{(a, b) : |a^2 - b^2| < 8\}$

So, clearly, R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (4, 3) (3, 4), (4, 4), (5, 5)}

Q50. Let f = {(1, 2), (3, 5), (4, 1)} and g = {(2, 3), (5, 1), (1, 3)}. Then gof = ... and fog = ...

Sol. Here, f = {(1, 2), (3, 5), (4, 1)} and g = {(2, 3), (5, 1), (1, 3)}

$gof(1) = g[f(1)] = g(2) = 3$

$gof(3) = g[f(3)] = g(5) = 1$

$gof(4) = g[f(4)] = g(1) = 3$

$\therefore gof = \{(1, 3), (3, 1), (4, 3)\}$

$fog(2) = f[g(2)] = f(3) = 5$

$$fog(5) = f[g(5)] = f(1) = 2$$
$$fog(1) = f[g(1)] = f(3) = 5$$
$$\therefore \quad fog = \{(2, 5), (5, 2), (1, 5)\}$$

Q51. Let f : R → R be defined by $f(x) = \dfrac{x}{\sqrt{1+x^2}}$, then

$(fofof)(x) =$ ....

Sol. Here, $f(x) = \dfrac{x}{\sqrt{1+x^2}} \quad \forall\, x \in R$

$$fofof(x) = fof[f(x)] = f[f\{f(x)\}]$$

$$= f\left[f\left(\frac{x}{\sqrt{1+x^2}}\right)\right] = f\left[\frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x^2}{1+x^2}}}\right]$$

$$= f\left[\frac{\frac{x}{\sqrt{1+x^2}}}{\frac{\sqrt{1+x^2+x^2}}{\sqrt{1+x^2}}}\right] = f\left[\frac{x}{\sqrt{1+2x^2}}\right]$$

$$= \left[\frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1+\frac{x^2}{1+2x^2}}}\right] = \left[\frac{\frac{x}{\sqrt{1+2x^2}}}{\frac{\sqrt{1+2x^2+x^2}}{\sqrt{1+2x^2}}}\right] = \frac{x}{\sqrt{1+3x^2}}$$

Hence, $fofof(x) = \dfrac{x}{\sqrt{3x^2+1}}$

Q52. If $f(x) = [4-(x-7)^3]$, then $f^{-1}(x) =$ ....

Sol. Given that, $\quad f(x) = [4-(x-7)^3]$

Let $\quad y = [4-(x-7)^3]$

$\Rightarrow \quad (x-7)^3 = 4-y$

$\Rightarrow \quad x-7 = (4-y)^{1/3} \Rightarrow x = 7+(4-y)^{1/3}$

Hence, $\quad f^{-1}(x) = 7+(4-x)^{1/3}$

State True or False for the Statements in each of the Exercises 53 to 62.

Q53. Let R = {(3, 1), (1, 3), (3, 3)} be a relation defined on the set A = {1, 2, 3}. Then R is symmetric, transitive but not reflexive.

Sol. Here, R = {(3, 1), (1, 3), (3, 3)}
(3, 3) $\in$ R, so R is reflexive.
(3, 1) $\in$ R and (1, 3) $\in$ R, so R is symmetric.
Now, (3, 1) $\in$ R and (1, 3) $\in$ R but (1, 1) $\notin$ R
So, R is not transitive.
Hence, the statement is 'False'.

Q54. Let f : R $\to$ R be the function defined by
$f(x) = \sin(3x + 2)\ \forall\ x \in$ R, then f is invertible.

Sol. Given that: $f(x) = \sin(3x + 2)\ \forall\ x \in$ R,
$f(x)$ is not one-one.
Hence, the statement is 'False'.

Q55. Every relation which is symmetric and transitive is also reflexive.

Sol. Let R be any relation defined on A = {1, 2, 3}
R = {(1, 2), (2, 1), (2, 3), (1, 3)}
Here, (1, 2) $\in$ R and (2, 1) $\in$ R, so R is symmetric.
(1, 2) $\in$ R, (2, 3) $\in$ R $\Rightarrow$ (1, 3) $\in$ R, so R is transitive.
But (1, 1) $\notin$ R, (2, 2) $\notin$ R and (3, 3) $\notin$ R.
Hence, the statement is 'False'.

Q56. An integer m is said to be related to another integer n if m is an integral multiple of n. This relation in Z is reflexive, symmetric and transitive.

Sol. Here, $m = kn$ (where k is an integer)
If $k = 1$ $m = n$, so z is reflexive.
Clearly z is not symmetric but z is transitive.
Hence, the statement is 'False'.

Q57. Let A = {0, 1} and N be the set of natural numbers then the mapping f : N $\to$ A defined by $f(2n - 1) = 0$, $f(2n) = 1$, $\forall\ n \in$ N is onto.

Sol. Given that A = [0, 1]
$f(2n - 1) = 0$ and $f(2n) = 1\ \forall\ n \in$ N
So, f : N $\to$ A is a onto function.
Hence, the statement is 'True'.

Q58. The relation R on the set A = {1, 2, 3} defined as
R = {(1, 1), (1, 2), (2, 1), (3, 3)} is reflexive, symmetric and transitive.

Sol. Here, R = {(1, 1), (1, 2), (2, 1), (3, 3)}
Here, (1, 1) $\in$ R, so R is Reflexive.
(1, 2) $\in$ R and (2, 1) $\in$ R, so R is Symmetric.

$(1, 2) \in R$ but $(2, 3) \notin R$
So, R is not transitive.
Hence, the statement is 'False'.

Q59. The composition of functions is commutative.

Sol. Let $f(x) = x^2$ and $g(x) = 2x + 3$

$$fog(x) = f[g(x)] = (2x + 3)^2 = 4x^2 + 9 + 12x$$
$$gof(x) = g[f(x)] = 2x^2 + 3$$

So, $fog(x) \neq gof(x)$

Hence, the statement is 'False'.

Q60. The composition of functions is associative.

Sol. Let $f(x) = 2x$, $g(x) = x - 1$ and $h(x) = 2x + 3$

$$fo\{goh(x)\} = fo\{g(2x + 3)\}$$
$$= f(2x + 3 - 1) = f(2x + 2) = 2(2x + 2) = 4x + 4.$$

and $$(fog)oh(x) = (fog)\{h(x)\}$$
$$= fog(2x + 3)$$
$$= f(2x + 3 - 1) = f(2x + 2) = 2(2x + 2) = 4x + 4$$

So, $fo\{goh(x)\} = \{(fog)oh(x)\} = 4x + 4$

Hence, the statement is 'True'.

Q61. Every function is invertible.

Sol. Only bijective functions are invertible.

Hence, the statement is 'False'.

Q62. A binary operation on a set has always the identity element.

Sol. '+' is a binary operation on the set N but it has no identity element.

Hence, the statement is 'False'.