13.3 EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. For a loaded die, the probabilities of outcomes are given as under:

$P(1) = P(2) = 0.2, P(3) = P(5) = P(6) = 0.1$ and $P(4) = 0.3$

The die is thrown, two times. Let A and B be the events, 'same number each time'; and 'a total score is 10 or more'; respectively. Determine whether or not A and B are independent.

Sol. A loaded die is thrown such that

$P(1) = P(2) = 0.2, P(3) = P(5) = P(6) = 0.1$ and $P(4) = 0.3$ and die is thrown two times. Also given that:

A = Same number each time and

B = Total score is 10 or more.

$$\begin{aligned}
\text{So, } P(A) &= [P(1, 1) + P(2, 2) + P(3, 3) + P(4, 4) + P(5, 5) + P(6, 6)] \\
&= P(1).P(1) + P(2).P(2) + P(3).P(3) + P(4).P(4) \\
&\qquad + P(5).P(5) + P(6).P(6) \\
&= 0.2 \times 0.2 + 0.2 \times 0.2 + 0.1 \times 0.1 + 0.3 \times 0.3 + 0.1 \times 0.1 \\
&\qquad + 0.1 \times 0.1 \\
&= 0.04 + 0.04 + 0.01 + 0.09 + 0.01 + 0.01 = 0.20
\end{aligned}$$

$$\begin{aligned}
\text{Now } \quad B &= [(4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)] \\
P(B) &= [P(4).P(6) + P(6).P(4) + P(5).P(5) + P(5).P(6) \\
&\qquad + P(6).P(5) + P(6).P(6) \\
&= 0.3 \times 0.1 + 0.1 \times 0.3 + 0.1 \times 0.1 + 0.1 \times 0.1 + 0.1 \times 0.1 \\
&\qquad + 0.1 \times 0.1 \\
&= 0.03 + 0.03 + 0.01 + 0.01 + 0.01 + 0.01 = 0.10
\end{aligned}$$

A and B both events will be independent if

$$P(A \cap B) = P(A).P(B) \qquad ...(i)$$

Here, $\quad (A \cap B) = \{(5, 5), (6, 6)\}$

$$\begin{aligned}
\therefore \quad P(A \cap B) &= P(5, 5) + P(6, 6) = P(5).P(5) + P(6).P(6) \\
&= 0.1 \times 0.1 + 0.1 \times 0.1 = 0.02
\end{aligned}$$

From eq. (*i*) we get

$$0.02 = 0.20 \times 0.10$$
$$0.02 = 0.02$$

Hence, A and B are independent events.

Q2. Refer to Exercise 1 above. If the die were fair, determine whether or not, the events A and B are independent.

Sol. According to the solution of Q. 1, we have

A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

$\therefore$ n(A) = 6 and n(S) = 6 × 6 = 36

So, $$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

and B = {(4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)}

n(B) = 6 and n(S) = 36

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$A \cap B$ = {(5, 5), (6, 6)}

$$\therefore P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

Therefore, if A and B are independent, then

$P(A \cap B) = P(A) \,.\, P(B)$

$$\Rightarrow \quad \frac{1}{18} \neq \frac{1}{6} \times \frac{1}{6} \quad \Rightarrow \quad \frac{1}{18} \neq \frac{1}{36}$$

Hence, A and B are not independent events.

Q3. The probability that atleast one of the two events A and B occurs is 0.6. If A and B occurs simultaneously with probability 0.3, evaluate $P(\bar{A}) + P(\bar{B})$.

Sol. We know that:

$A \cup B$ denotes that atleast one of the events occurs

and $A \cap B$ denotes that the two events occur simultaneously.

$$\begin{aligned} \text{So,} \quad P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ \Rightarrow \quad 0.6 &= P(A) + P(B) - 0.3 \\ \Rightarrow \quad 0.9 &= P(A) + P(B) \\ \Rightarrow \quad 0.9 &= 1 - P(\bar{A}) + 1 - P(\bar{B}) \\ \Rightarrow \quad P(\bar{A}) + P(\bar{B}) &= 2 - 0.9 = 1.1 \end{aligned}$$

Hence, the required answer is 1.1.

Q4. A bag contains 5 red marbles and 3 black marbles. Three marbles are drawn one by one without replacement. What is the probability that atleast one of the three marbles drawn be black, if the first marble is red?

Sol. Let red marbles be represented with R and black marble with B. The following three conditions are possible, if atleast one of the three marbles drawn be black and the first marble is red.

(*i*) E_1 : II ball is black and III is red

(*ii*) E_2 : II ball is black and III is also black

(*iii*) E_3 : II ball is red and III is black

$$\therefore \quad P(E_1) = P(R_1).P(B_1/R_1).P(R_2/R_1B_1) = \frac{5}{8}.\frac{3}{7}.\frac{4}{6} = \frac{60}{336} = \frac{5}{28}$$

$$P(E_2) = P(R_1).P(B_1/R_1).P(B_2/R_1B_1) = \frac{5}{8}.\frac{3}{7}.\frac{2}{6} = \frac{30}{336} = \frac{5}{56}$$

$$\text{and } P(E_3) = P(R_1).P(R_2/R_1).B(B_1/R_1R_2) = \frac{5}{8}.\frac{4}{7}.\frac{3}{6} = \frac{60}{336} = \frac{5}{28}$$

$$\therefore \quad P(E) = P(E_1) + P(E_2) + P(E_3) = \frac{5}{28} + \frac{5}{56} + \frac{5}{28} = \frac{25}{56}$$

Hence the required probability is $\frac{25}{56}$.

Q5. Two dice are thrown together and the total score is noted. The events E, F and G are 'a total of 4' 'a total of 9 or more', and 'a total divisible by 5', respectively. Calculate P(E), P(F) and P(G) and decide which pairs of events, if any, are independent.

Sol. Two dice are thrown together

$\therefore n(S) = 36$

E = A total of 4 = {(2, 2), (1, 3), (3, 1)} $\therefore n(E) = 3$

F = A total of 9 or more

= {(3, 6), (6, 3), (5, 4), (4, 5), (5, 5), (4, 6), (6, 4), (5, 6), (6, 5), (6, 6)}

$\therefore n(F) = 10$; G = A total divisible by 5

= {(1, 4), (4, 1), (2, 3), (3, 2), (4, 6), (6, 4), (5, 5)} $\therefore n(G) = 7$

Here, we see that $(E \cap F) = \phi$ and $(E \cap G) = \phi$

and $(F \cap G) = \{(4, 6), (6, 4), (5, 5)\}$

$\therefore$ $n(F \cap G) = 3$ and $(E \cap F \cap G) = \phi$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{10}{36} = \frac{5}{18}; \; P(G) = \frac{n(G)}{n(S)} = \frac{7}{36}$$

$$P(F \cap G) = \frac{3}{36} = \frac{1}{12} \text{ and } P(F) \,.\, P(G) = \frac{5}{18} \cdot \frac{7}{36} = \frac{35}{648}$$

Since, $P(F \cap G) \neq P(F) \,.\, P(G)$

Hence, there is no pair of independent events.

Q6. Explain why the experiment of tossing a coin three times is said to have binomial distribution.

Sol. We know that random variable X takes values 0, 1, 2, 3, ..., n is said to be binomial distribution having parameters n and p, if the probability is given by

$P(X = r) = {}^nC_r \, p^r \, q^{n-r}$, where $q = 1 - p$ and $r = 0, 1, 2, 3, \ldots$

Similarly in case of tossing a coin 3 times,

$n = 3$ and X has the values 0, 1, 2, 3 with $p = \frac{1}{2}$, $q = \frac{1}{2}$.

Hence, it is said to have a binomial distribution.

Q7. A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$. Find:

(*i*) P(A/B) (*ii*) P(B/A) (*iii*) P(A′/B) (*iv*) P(A′/B′)

Sol. We have $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$

$$P(A') = 1 - \frac{1}{2} = \frac{1}{2},\ P(B') = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - \left[\frac{1}{2} + \frac{1}{3} - \frac{1}{4}\right] = 1 - \left[\frac{6 + 4 - 3}{12}\right] = 1 - \frac{7}{12} = \frac{5}{12}$$

(*i*) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$

(*ii*) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{1/2} = \frac{1}{2}$

(*iii*) $P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

$$= 1 - \frac{P(A \cap B)}{P(B)} = 1 - \frac{1/4}{1/3} = 1 - \frac{3}{4} = \frac{1}{4}$$

(*iv*) $P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{5/12}{2/3} = \frac{5}{12} \times \frac{3}{2} = \frac{5}{8}$

Q8. Three events A, B and C have probabilities $\frac{2}{5}$, $\frac{1}{3}$ and $\frac{1}{2}$ respectively. Given that $P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$, find the values of $P(C/B)$ and $P(A' \cap C')$.

Sol. We have $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{2}$

$P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$

$$\therefore \quad P(C/B) = \frac{P(B \cap C)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$$

$$P(A' \cap C') = 1 - P(A \cup C)$$

$$= 1 - [P(A) + P(C) - P(A \cap C)]$$

$$= 1 - \left[\frac{2}{5} + \frac{1}{2} - \frac{1}{5}\right] = 1 - \frac{7}{10} = \frac{3}{10}$$

Hence, the required probabilities are $\frac{3}{4}$ and $\frac{3}{10}$.

Q9. Let E_1 and E_2 be two independent events such that $P(E_1) = p_1$ and $P(E_2) = p_2$. Describe in words of the events whose probabilities are:

(*i*) p_1p_2 (*ii*) $(1 - p_1)p_2$

(*iii*) $1 - (1 - p_1)(1 - p_2)$ (*iv*) $p_1 + p_2 - 2p_1p_2$

Sol. Here, $P(E_1) = p_1$ and $P(E_2) = p_2$

(*i*) $p_1p_2 = P(E_1) \,.\, P(E_2) = P(E_1 \cap E_2)$

So, E_1 and E_2 occur.

(*ii*) $(1 - p_1).p_2 = P(E_1)'.P(E_2) = P(E_1' \cap E_2)$

So, E_1 does not occur but E_2 occurs.

(*iii*) $1 - (1 - p_1)\,(1 - p_2) = 1 - P(E_1)'P(E_2)' = 1 - P(E_1' \cap E_2')$

$= 1 - [1 - P(E_1 \cup E_2)] = P(E_1 \cup E_2)$

So, either E_1 or E_2 or both E_1 and E_2 occur.

(*iv*) $p_1 + p_2 - 2p_1p_2 = P(E_1) + P(E_2) - 2P(E_1).\,P(E_2)$

$= P(E_1) + P(E_2) - 2P(E_1 \cap E_2)$

$= P(E_1 \cup E_2) - 2P(E_1 \cap E_2)$

So, either E_1 or E_2 occurs but not both.

Q10. A discrete random variable X has the probability distribution given as below:

X	0.5	1	1.5	2
P(X)	k	k^2	$2k^2$	k

(*i*) Find the value of *k*.

(*ii*) Determine the mean of the distribution.

Sol. For a probability distribution, we know that if $P_i \geq 0$

(*i*) $\sum_{i=1}^{n} P_i = 1 \Rightarrow k + k^2 + 2k^2 + k = 1$

$\Rightarrow 3k^2 + 2k - 1 = 0 \Rightarrow 3k^2 + 3k - k - 1 = 0$

$\Rightarrow 3k(k + 1) - 1(k + 1) = 0 \Rightarrow (3k - 1)(k + 1) = 0$

$\therefore k = \frac{1}{3}$ and $k = -1$

But $k \geq 0 \quad \therefore k = \frac{1}{3}$

(*ii*) Mean of the distribution

$$E(X) = \sum_{i=1}^{n} X_iP_i = 0.5k + 1.k^2 + 1.5(2k^2) + 2k$$

$$= \frac{k}{2} + k^2 + 3k^2 + 2k = 4k^2 + \frac{5}{2}k$$

$$= 4\left(\frac{1}{3}\right)^2 + \frac{5}{2}\left(\frac{1}{3}\right) = \frac{4}{9} + \frac{5}{6} = \frac{23}{18}$$

Q11. Prove that

(*i*) $P(A) = P(A \cap B) + P(A \cap \bar{B})$

(*ii*) $P(A \cup B) = P(A \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap B)$

Sol. (*i*) To prove: $P(A) = P(A \cap B) + P(A \cap \bar{B})$

R.H.S. $= P(A \cap B) + P(A \cap \bar{B})$

$= P(A).P(B) + P(A).P(\bar{B}) = P(A)\,[P(B) + P(\bar{B})]$

$= P(A)\,.\,1$

$= P(A) =$ L.H.S. Hence proved.

(*ii*) To prove: $P(A \cup B) = P(A \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap B)$

R.H.S. $= P(A).P(B) + P(A).P(\bar{B}) + P(\bar{A}).P(B)$

$= P(A).P(B) + P(A)[1 - P(B)] + [1 - P(A)].P(B)$

$= P(A).P(B) + P(A) - P(A).P(B) + P(B) - P(A).P(B)$

$= P(A) + P(B) - P(A \cap B)$

$= P(A \cup B) =$ L.H.S. Hence proved.

Q12. If X is the number of tails in three tosses of a coin, determine the standard deviation of X.

Sol. Given that: X = 0, 1, 2, 3

$\therefore P(X = r) = {}^nC_r\, p^r\, q^{n-r}$

where $n = 3$, $p = \frac{1}{2}$, $q = \frac{1}{2}$ and $r = 0, 1, 2, 3$

$$P(X = 0) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8};\; P(X = 1) = 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$$

$$P(X = 2) = 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8};\; P(X = 3) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Probability distribution table is:

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$E(X) = 0 + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} = \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = \frac{3}{2}$$

$$E(X^2) = 0 + 1 \times \frac{3}{8} + 4 \times \frac{3}{8} + 9 \times \frac{1}{8} = \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = \frac{24}{8} = 3$$

We know that $\text{Var}(X) = E(X^2) - [E(X)]^2 = 3 - \left(\frac{3}{2}\right)^2 = 3 - \frac{9}{4} = \frac{3}{4}$

$\therefore$ Standard deviation $= \sqrt{\text{Var}(X)} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$.

Q13. In a dice game, a player pays a stake of Re 1 for each throw of a die. She receives ₹ 5 if the die shows a 3, ₹ 2, if the die shows a 1 or 6, and nothing otherwise. What is the player's expected profit per throw over a long series of throws?

Sol. Let X be the random variable of profit per throw.

X	-1	1	4
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Since, she loses ₹ 1 for getting any of 2, 4, 5.

So, $P(X = -1) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

$P(X = 1) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$ ($\because$ die showing 1 or 6)

$P(X = 4) = \frac{1}{6}$ ($\because$ die shows only a 3)

Player's expected profit $= \sum p_1 x_i$

$$= -1 \times \frac{1}{2} + 1 \times \frac{1}{3} + 4 \times \frac{1}{6} = -\frac{1}{2} + \frac{1}{3} + \frac{2}{3} = \frac{1}{2} = ₹\ 0.50$$

Q14. Three dice are thrown at the same time. Find the probability of getting three two's, if it is known that the sum of the numbers on the dice was six.

Sol. The dice is thrown three times

$\therefore$ Sample space $n(S) = (6)^3 = 216$

Let E_1 be the event when the sum of numbers on the dice was six and E_2 be the event when three two's occur.

$\Rightarrow E_1 = \{(1, 1, 4), (1, 2, 3), (1, 3, 2), (1, 4, 1), (2, 1, 3), (2, 2, 2),$
$(2, 3, 1), (3, 1, 2), (3, 2, 1), (4, 1, 1)\}$

$\Rightarrow n(E_1) = 10$ and $n(E_2) = 1$ [$\because E_2 = \{2, 2, 2\}$]

$\therefore P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{1/216}{10/216} = \frac{1}{10}$.

Q15. Suppose 10,000 tickets are sold in a lottery each for Re. 1. First prize is of ₹ 3,000 and the second prize is of ₹ 2,000. There are three third prizes of ₹ 500 each. If you buy one ticket, what is your expectation?

Sol. Let X be the random variable where X = 0, 500, 2000 and 3000

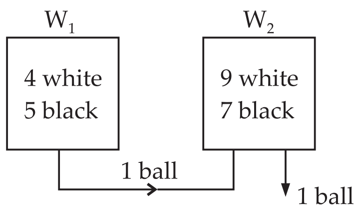
X	0	500	2000	3000
P(X)	$\frac{9995}{10000}$	$\frac{3}{10000}$	$\frac{1}{10000}$	$\frac{1}{10000}$

$$E(X) = 0\times\frac{9995}{10000}+500\times\frac{3}{10000}+2000\times\frac{1}{10000}+3000\times\frac{1}{10000}$$

$$= 0+\frac{1500}{10000}+\frac{2000}{10000}+\frac{3000}{10000} = \frac{6500}{10000}=\frac{13}{20} = ₹\,0.65$$

Hence, expectation is ₹ 0.65.

Q16. A bag contains 4 white and 5 black balls. Another bag contains 9 white and 7 black balls. A ball is transferred from the first bag to the second and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white.



Sol. Let W_1 and W_2 be two bags containing (4 W, 5 B) and (9 W, 7 B) balls respectively.

Let E_1 be the event that the transferred ball from the bag W_1 to W_2 is white and E_2 the event that the transferred ball is black.

And E be the event that the ball drawn from the second bag is white.

$$\therefore\ P(E/E_1) = \frac{10}{17},\quad P(E/E_2) = \frac{9}{17}$$

$$P(E_1) = \frac{4}{9}\quad \text{and}\quad P(E_2) = \frac{5}{9}$$

$$\therefore\quad P(E) = P(E_1).P(E/E_1)+P(E_2).P(E/E_2)$$

$$= \frac{4}{9}\times\frac{10}{17}+\frac{5}{9}\times\frac{9}{17} = \frac{40}{153}+\frac{45}{153} = \frac{85}{153} = \frac{5}{9}$$

Hence, the required probability is $\frac{5}{9}$.

Q17. Bat I contains 3 black and 2 white balls, bag II contains 2 black and 4 white balls. A bag and a ball is selected at random. Determine the probability of selecting a black ball.

Sol. Given that bag I = {3 B, 2 W}

and bag II = {2 B, 4 W}

Let E_1 = The event that bag I is selected

E_2 = The event that bag II is selected

and E = The event that a black ball is selected

$$\therefore \quad P(E_1) = \frac{1}{2},\ P(E_2) = \frac{1}{2},\ P(E/E_1) = \frac{3}{5} \text{ and } P(E/E_2) = \frac{1}{3}$$

$$P(E) = P(E_1).P(E/E_1) + P(E_2).P(E/E_2)$$

$$= \frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{1}{3} = \frac{3}{10}+\frac{1}{6} = \frac{9+5}{30} = \frac{14}{30} = \frac{7}{15}$$

Hence, the required probability is $\frac{7}{15}$.

Q18. A box has 5 blue and 4 red balls. One ball is drawn at random and not replaced. Its colour is also not noted. Then another ball is drawn at random. What is the probability of second ball being blue?

Sol. Given that the box has 5 blue and 4 red balls.

Let E_1 be the event that first ball drawn is blue

E_2 be the event that first ball drawn is red

and E is the event that second ball drawn is blue.

$$\therefore \quad P(E) = P(E_1).P(E/E_1) + P(E_2).P(E/E_2)$$

$$= \frac{5}{9}\times\frac{4}{8}+\frac{4}{9}\times\frac{5}{8} = \frac{20}{72}+\frac{20}{72} = \frac{40}{72} = \frac{5}{9}$$

Hence, the required probability is $\frac{5}{9}$.

Q19. Four cards are successively drawn without replacement from a deck of 52 playing cards. What is the probability that all the four cards are Kings?

Sol. Let E_1, E_2, E_3 and E_4 be the events that first, second, third and fourth card is King respectively.

$\therefore P(E_1 \cap E_2 \cap E_3 \cap E_4)$

$$= P(E_1).P(E_2/E_1).P\left[\frac{E_3}{(E_1 \cap E_2)}\right].P\left[\frac{E_4}{(E_1 \cap E_2 \cap E_3 \cap E_4)}\right]$$

$$= \frac{4}{52}\times\frac{3}{51}\times\frac{2}{50}\times\frac{1}{49} = \frac{24}{52\cdot 51\cdot 50\cdot 49} = \frac{1}{13\cdot 17\cdot 25\cdot 49} = \frac{1}{27075}$$

Hence, the required probability is $\frac{1}{27075}$.

Q20. A die is thrown 5 times. Find the probability that an odd number will come up exactly three times.

Sol. Here, $p = \frac{1}{6}+\frac{1}{6}+\frac{1}{6} = \frac{1}{2} \Rightarrow q = 1-\frac{1}{2} = \frac{1}{2}$ and $n = 5$

$\therefore \quad P(x = r) = {}^nC_r\, p^r\, q^{n-r}$

$$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} = \frac{5!}{3!2!}.\left(\frac{1}{2}\right)^3.\left(\frac{1}{2}\right)^2 = 10.\frac{1}{8}.\frac{1}{4} = \frac{5}{16}$$

Hence, the required probability is $\frac{5}{16}$.

Q21. Ten coins are tossed. What is the probability of getting atleast 8 heads?

Sol. Here, $n = 10$, $p = \frac{1}{2}$, $q = \quad -\frac{1}{2} = \frac{1}{2}$

$$P(X \geq 8) = P(x = 8) + P(x = 9) + P(x = 10)$$

$$= {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{10-8} + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{10-9} + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$$

$$= \frac{10!}{8!2!}.\left(\frac{1}{2}\right)^8.\left(\frac{1}{2}\right)^2 + \frac{10!}{9!1!}\left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^{10}$$

$$= 45.\left(\frac{1}{2}\right)^{10} + 10.\left(\frac{1}{2}\right)^{10} + \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{10}.(45 + 10 + 1)$$

$$= 56\left(\frac{1}{2}\right)^{10} = 56 \times \frac{1}{1024} = \frac{7}{128}$$

Hence, the required probability is $\frac{7}{128}$.

Q22. The probability of a man hitting a target is 0.25. He shoots 7 times. What is the probability of his hitting at least twice?

Sol. Here $n = 7$, $p = 0.25 = \frac{25}{100} = \frac{1}{4}$ and $q = 1 - \frac{1}{4} = \frac{3}{4}$

$$P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[{}^7C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^7 + {}^7C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^6\right]$$

$$= 1 - \left[\left(\frac{3}{4}\right)^7 + \frac{7}{4}\left(\frac{3}{4}\right)^6\right] = 1 - \left(\frac{3}{4}\right)^6 \left(\frac{3}{4} + \frac{7}{4}\right)$$

$$= 1 - \left(\frac{3}{4}\right)^6 \left(\frac{10}{4}\right) = 1 - \frac{729}{4096} \times \frac{10}{4} = 1 - \frac{7290}{16384}$$

$$= \frac{16384 - 7290}{16384} = \frac{9094}{16384} = \frac{4547}{8192}$$

Hence, the required probability is $\frac{4547}{8192}$.

Q23. A lot of 100 watches is known to have 10 defective watches. If 8 watches are selected (one by one with replacement) at random, what is the probability that there will be atleast one defective watch?

Sol. Probability of defective watch out of 100 watches = $\frac{10}{100} = \frac{1}{10}$.

Here, $n = 8$, $p = \frac{1}{10}$ and $q = 1 - \frac{1}{10} = \frac{9}{10}$ and $r \geq 1$

$$P(X \geq 1) = 1 - P(x = 0) = 1 - {}^8C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{8-0} = 1 - \left(\frac{9}{10}\right)^8$$

Hence, the required probability is $1 - \left(\frac{9}{10}\right)^8$.

Q24. Consider the probability distribution of a random variable X:

X	0	1	2	3	4
P(X)	0.1	0.25	0.3	0.2	0.15

Calculate: (*i*) $V\left(\frac{X}{2}\right)$ (*ii*) Variance of X.

Sol. Here, we have

X	0	1	2	3	4
P(X)	0.1	0.25	0.3	0.2	0.15

We know that: $\text{Var}(X) = E(X^2) - [E(X)]^2$

where $E(X) = \sum_{i=1}^{n} x_i p_i$ and $E(X^2) = \sum_{i=1}^{n} p_i x_i^2$

$$\therefore \quad E(X) = 0 \times 0.1 + 1 \times 0.25 + 2 \times 0.3 + 3 \times 0.2 + 4 \times 0.15$$
$$= 0 + 0.25 + 0.6 + 0.6 + 0.6 = 2.05$$
$$E(X^2) = 0 \times 0.1 + 1 \times 0.25 + 4 \times 0.3 + 9 \times 0.2 + 16 \times 0.15$$
$$= 0 + 0.25 + 1.2 + 1.8 + 2.40 = 5.65$$

(*i*) $$V\left(\frac{X}{2}\right) = \frac{1}{4}V(X) = \frac{1}{4}[5.65 - (2.05)^2] = \frac{1}{4}[5.65 - 4.2025]$$
$$= \frac{1}{4} \times 1.4475 = 0.361875$$

$$\left[\because V\left(\frac{X}{2}\right) = \frac{1}{4}V(X)\right]$$

(*ii*) $\text{Var}(X) = 1.4475$

Q25. The probability distribution of a random variable X is given below:

X	0	1	2	3
P(X)	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

(*i*) Determine the value of k.
(*ii*) Determine $P(X \le 2)$ and $P(X > 2)$
(*iii*) Find $P(X \le 2) + P(X > 2)$.

Sol. (*i*) We know that $P(0) + P(1) + P(2) + P(3) = 1$

$$\Rightarrow \quad k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} = 1$$

$$\Rightarrow \quad \frac{8k + 4k + 2k + k}{8} = 1 \quad \Rightarrow 15k = 8$$

$$\therefore \quad k = \frac{\;}{15}$$

(*ii*)
$$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$
$$= k + \frac{k}{2} + \frac{k}{4} = \frac{7k}{4} = \frac{7}{4} \times \frac{8}{15} = \frac{14}{15}$$

and
$$P(X > 2) = P(X = 3) = \frac{k}{8} = \frac{1}{8} \times \frac{8}{15} = \frac{1}{15}$$

(*iii*) $$P(X \le 2) + P(X > 2) = \frac{14}{15} + \frac{1}{15} = \frac{14+1}{15} = \frac{15}{15} = 1.$$

Q26. For the following probability distribution, determine standard deviation of the random variable X.

X	2	3	4
P(X)	0.2	0.5	0.3

Sol. We know that: Standard deviation (S.D.) = $\sqrt{\text{Variance}}$

$\therefore \quad Var(X) = E(X^2) - [E(X)]^2$

$$E(X) = 2 \times 0.2 + 3 \times 0.5 + 4 \times 0.3 = 0.4 + 1.5 + 1.2 = 3.1$$
$$E(X^2) = 4 \times 0.2 + 9 \times 0.5 + 16 \times 0.3 = 0.8 + 4.5 + 4.8 = 10.1$$

So $\quad V(X) = 10.1 - (3.1)^2 = 10.1 - 9.61 = 0.49$

$\therefore \quad S.D. = \sqrt{Var(X)} = \sqrt{0.49} = 0.7$

Q27. A biased die is such that $P(4) = \frac{1}{10}$ and other scores being equally likely. The die is tossed twice. If X is the 'number of fours seen', find the variance of the random variable X.

Sol. Here, random variable X = 0, 1, 2

$$P(4) = \frac{1}{10}, \quad P(\bar{4}) = 1 - \frac{1}{10} = \frac{9}{10}$$

$$P(X = 0) = P(\bar{4}).P(\bar{4}) = \frac{9}{10} \times \frac{9}{10} = \frac{81}{100}$$

$$P(X = 1) = P(\bar{4}).P(4) + P(4).P(\bar{4}) = \frac{9}{10} \times \frac{1}{10} + \frac{1}{10} \times \frac{9}{10} = \frac{18}{100}$$

$$P(X = 2) = P(4).P(4) = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$$

X	0	1	2
P(X)	$\frac{81}{100}$	$\frac{18}{100}$	$\frac{1}{100}$

We know that $V(X) = E(X^2) - [E(X)]^2$

$$E(X) = 0 \times \frac{81}{100} + 1 \times \frac{18}{100} + \frac{2}{100} = \frac{20}{100} = \frac{1}{5}$$

$$E(X^2) = 0 \times \frac{81}{100} + 1 \times \frac{18}{100} + 4 \times \frac{1}{100} = \frac{22}{100} = \frac{11}{50}$$

$$\therefore \quad Var(X) = \frac{11}{50} - \left(\frac{1}{5}\right)^2 = \frac{11}{50} - \frac{1}{25} = \frac{9}{50} = 0.18$$

Hence, the required variance = 0.18.

Q28. A die is thrown three times. Let X be the 'number of twos seen', find the expectation of X.

Sol. Here, we have X = 0, 1, 2, 3 [$\because$ die is thrown 3 times]

and $p = \frac{1}{6}$, $q = \frac{5}{6}$

$$\therefore \quad P(X = 0) = P(\text{not } 2) . P(\text{not } 2) . P(\text{not } 2) = \frac{5}{6} . \frac{5}{6} . \frac{5}{6} = \frac{125}{216}$$

$$P(X = 1) = P(2) . P(\text{not } 2) . P(\text{not } 2) + P(\text{not } 2) . P(2) . P(\text{not } 2) + P(\text{not } 2) . P(\text{not } 2) . P(2)$$

$$= \frac{1}{6} . \frac{5}{6} . \frac{5}{6} + \frac{5}{6} . \frac{1}{6} . \frac{5}{6} + \frac{5}{6} . \frac{5}{6} . \frac{1}{6} = \frac{25}{216} + \frac{25}{216} + \frac{25}{216} = \frac{75}{216}$$

$$P(X = 2) = P(2) . P(2) . P(\text{not } 2) + P(2) . P(\text{not } 2) . P(2) + P(\text{not } 2) . P(2) . P(2)$$

$$= \frac{1}{6} . \frac{1}{6} . \frac{5}{6} + \frac{1}{6} . \frac{5}{6} . \frac{1}{6} + \frac{5}{6} . \frac{1}{6} . \frac{1}{6} = \frac{5}{216} + \frac{5}{216} + \frac{5}{216} = \frac{15}{216}$$

$$P(X = 3) = P(2) . P(2) . P(2) = \frac{1}{6} . \frac{1}{6} . \frac{1}{6} = \frac{1}{216}$$

Now $$E(X) = \sum_{i=1}^{n} p_i x_i$$

$$= 0 \times \frac{125}{216} + 1 \times \frac{75}{216} + 2 \times \frac{15}{216} + 3 \times \frac{1}{216}$$

$$= 0 + \frac{75}{216} + \frac{30}{216} + \frac{3}{216} = \frac{75 + 30 + 3}{216} = \frac{108}{216} = \frac{1}{2}$$

Hence, the required expectation is $\frac{1}{2}$.

Q29. Two biased dice are thrown together. For the first die $P(6) = \frac{1}{2}$, the other scores being equally likely while for the second die, $P(1) = \frac{2}{5}$ and the other scores are equally likely. Find the probability distribution of 'the number of ones seen'.

Sol. Given that: for the first die, $P(6) = \frac{1}{2}$ and $P(\bar{6}) = 1 - \frac{1}{2} = \frac{1}{2}$

$\Rightarrow P(1) + P(2) + P(3) + P(4) + P(5) = \frac{1}{2}$

But $P(1) = P(2) = P(3) = P(4) = P(5)$

$\therefore\ 5.P(1) = \frac{1}{2} \Rightarrow P(1) = \frac{1}{10}$ and $P(\bar{1}) = 1 - \frac{1}{10} = \frac{9}{10}$

For the second die, $P(1) = \frac{2}{5}$ and $P(\bar{1}) = 1 - \frac{2}{5} = \frac{3}{5}$

Let X be the number of one's seen

$\therefore\ X = 0, 1, 2$

$$\Rightarrow \quad P(X=0) = \underset{\text{I}}{P(\bar{1})}.\underset{\text{II}}{P(\bar{1})} = \frac{9}{10}.\frac{3}{5} = \frac{27}{50} = 0.54$$

$$P(X=1) = \underset{\text{I}}{P(\bar{1})}\cdot\underset{\text{II}}{P(1)} + \underset{\text{I}}{P(1)}\cdot\underset{\text{II}}{P(\bar{1})}$$

$$= \frac{9}{10}.\frac{2}{5} + \frac{1}{10}.\frac{3}{5} = \frac{18+3}{50} = \frac{21}{50} = 0.42$$

$$P(X=2) = \underset{\text{I}}{P(1)}\,.\,\underset{\text{II}}{P(1)} = \frac{1}{10}.\frac{2}{5} = \frac{2}{50} = 0.04$$

Hence, the required probability distribution is

X	0	1	2
P(X)	0.54	0.42	0.04

Q30. Two probability distributions of the discrete random variable X and Y are given below.

X	0	1	2	3
P(X)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

(Y)	0	1	2	3
P(Y)	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

Prove that: $E(Y^2) = 2E(X)$.

Sol. First probability distribution is given by

X	0	1	2	3
P(X)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

We know that, $E(X) = \sum_{i=1}^{n} P_i X_i$

$$\Rightarrow \quad E(X) = 0 \cdot \frac{1}{5} + 1 \cdot \frac{2}{5} + 2 \cdot \frac{1}{5} + 3 \cdot \frac{1}{5} = 0 + \frac{2}{5} + \frac{2}{5} + \frac{3}{5} = \frac{7}{5}$$

For the second probability distribution,

Y	0	1	2	3
P(Y)	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

$$E(Y^2) = 0 \cdot \frac{1}{5} + 1 \cdot \frac{3}{10} + 4 \cdot \frac{2}{5} + 9 \cdot \frac{1}{10}$$

$$= 0 + \frac{3}{10} + \frac{8}{5} + \frac{9}{10} = \frac{28}{10} = \frac{14}{5}$$

Now $E(Y^2) = \frac{14}{5}$ and $2\,E(X) = 2.\frac{7}{5} = \frac{14}{5}$

Hence, $E(Y^2) = 2E(X)$.

Q31. A factory produces bulbs. The probability that any one bulb is defective is $\frac{1}{50}$ and they are packed in boxes of 10. From the single box, find the probability that (*i*) none of the bulbs is defective (*ii*) exactly two bulbs are defective (*iii*) more than 8 bulbs work properly.

Sol. Let X be the random variable denoting a bulb to be defective.

Here, $n = 10$, $p = \frac{1}{50}$, $q = 1 - \frac{1}{50} = \frac{49}{50}$

We know that $P(X = r) = {}^nC_r p^r q^{n-r}$

(*i*) None of the bulbs is defective, i.e., $r = 0$

$$P(x = 0) = {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10-0} = \left(\frac{49}{50}\right)^{10}$$

(*ii*) Exactly two bulbs are defective

$$\therefore \quad P(x = 2) = {}^{10}C_2 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^{10-2}$$

$$= 45.\frac{(49)^8}{(50)^{10}} = 45 \times \left(\frac{1}{50}\right)^{10} \times (49)^8$$

(*iii*) More than 8 bulbs work properly

We can say that less than 2 bulbs are defective

$$P(x < 2) = P(x = 0) + P(x = 1)$$

$$= {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10} + {}^{10}C_1 \left(\frac{1}{50}\right)^1 \left(\frac{49}{50}\right)^9 = \left(\frac{49}{50}\right)^{10} + \frac{1}{5}\left(\frac{49}{50}\right)^9$$

$$= \left(\frac{49}{50}\right)^9 \left(\frac{49}{50} + \frac{1}{5}\right) = \left(\frac{49}{50}\right)^9 \left(\frac{59}{50}\right) = \frac{59\,(49)^9}{(50)^{10}}.$$

Q32. Suppose you have two coins which appear identical in your pocket. You know that one is fair and one is 2-headed coin. If you take one out, toss it and get a head, what is the probability that it was a fair coin?

Sol. Let E_1 = Event that the coin is fair

E_2 = Event that the coin is 2-headed

and H = Event that the tossed coin gets head.

$$P(E_1) = \frac{1}{2}, \quad P(E_2) = \frac{1}{2}, \quad P(H/E_1) = \frac{1}{2}, \quad P(H/E_2) = 1$$

$\therefore$ Using Bayes' Theorem, we get

$$P(E_1/H) = \frac{P(E_1).P(H/E_1)}{P(E_1).P(H/E_1) + P(E_2).P(H/E_2)}$$

$$= \frac{\frac{1}{2}.\frac{1}{2}}{\frac{1}{2}.\frac{1}{2} + \frac{1}{2}.1} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Hence the required probability is $\frac{1}{3}$.

Q33. Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed, 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O?

Sol. Let E_1 = The event that a person selected is of blood group O

E_2 = The event that the person selected is of other group

and H = The event that selected person is left handed

$\therefore \quad P(E_1) = 0.30$ and $P(E_2) = 0.70$

$P(H/E_1) = 0.06 \qquad P(H/E_2) = 0.10$

So, from Bayes' Theorem

$$P\left(\frac{E_1}{H}\right) = \frac{P(E_1).P(H/E_1)}{P(E_1).P(H/E_1) + P(E_2).P(H/E_2)}$$

$$= \frac{0.30 \times 0.06}{0.30 \times 0.06 + 0.70 \times 0.10} = \frac{0.018}{0.018 + 0.070} = \frac{0.018}{0.088} = \frac{9}{44}$$

Hence, the required probability is $\frac{9}{44}$.

Q34. Two natural numbers r and s are drawn one at a time, without replacement from the set S = {1, 2, 3, ..., n}. Find $P(r \leq p/s \leq p)$, where $p \in$ S.

Sol. Given that: S = {1, 2, 3, ..., n}

$$\therefore \quad P(r \leq p/s \leq p) = \frac{P(P \cap S)}{P(S)} = \frac{p-1}{n} \times \frac{n}{n-1} = \frac{p-1}{n-1}$$

Hence, the required probability is $\frac{p-1}{n-1}$.

Q35. Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice. Determine also the mean of the distribution.

Sol. Let X be the random variable scores when a die is thrown twice.

$$X = 1, 2, 3, 4, 5, 6$$

and S = {(1, 1), (1, 2), (2, 1), (2, 2), (1, 3), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), ..., (6, 6)}

So, $P(X = 1) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$$P(X = 2) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{3}{36}$$

$$P(X = 3) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{5}{36}$$

Similarly

$$P(X = 4) = \frac{7}{36};\ P(X = 5) = \frac{9}{36} \text{ and } P(X = 6) = \frac{11}{36}$$

So, the required distribution is

X	1	2	3	4	5	6
P(X)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

Now, the mean $E(X) = \sum_{i=1}^{n} x_i p_i$

$$= 1 \times \frac{1}{36} + 2 \times \frac{3}{36} + 3 \times \frac{5}{36} + 4 \times \frac{7}{36} + 5 \times \frac{9}{36} + 6 \times \frac{11}{36}$$

$$= \frac{1}{36} + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{45}{36} + \frac{66}{36} = \frac{161}{36}$$

Hence, the required mean = $\frac{161}{36}$.

Q36. The random variable X can take only the values 0, 1, 2. If $P(X=0) = P(X=1) = p$ and $E(X^2) = E(X)$, then find the value of p.

Sol. Given that: $X = 0, 1, 2$

and P(X) at X = 0 and 1 is p. Let P(X) at X = 2 is x

$\Rightarrow \quad p + p + x = 1 \quad \Rightarrow \quad x = 1 - 2p$

Now we have the following distributions.

X	0	1	2
P(X)	p	p	$1-2p$

$\therefore \quad E(X) = 0.p + 1.p + 2(1-2p) = p + 2 - 4p = 2 - 3p$

and $\quad E(X^2) = 0.p + 1.p + 4(1-2p) = p + 4 - 8p = 4 - 7p$

Given that: $\quad E(X^2) = E(X)$

$\therefore \quad 4 - 7p = 2 - 3p \quad \Rightarrow \quad 4p = 2 \quad \Rightarrow \quad p = \frac{1}{2}$

Hence, the required value of p is $\frac{1}{2}$.

Q37. Find the variance of the distribution:

X	0	1	2	3	4	5
P(X)	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

Sol. We know that:

$$\text{Variance } (X) = E(X^2) - [E(X)]^2$$

$$E(X) = \sum_{i=1}^{n} p_i x_i$$

$$= 0 \times \frac{1}{6} + 1 \times \frac{5}{18} + 2 \times \frac{2}{9} + 3 \times \frac{1}{6} + 4 \times \frac{1}{9} + 5 \times \frac{1}{18}$$

$$= 0 + \frac{5}{18} + \frac{4}{9} + \frac{3}{6} + \frac{4}{9} + \frac{5}{18} = \frac{5+8+9+8+5}{18} = \frac{35}{18}$$

$$E(X^2) = 0 \times \frac{1}{6} + 1 \times \frac{5}{18} + 4 \times \frac{2}{9} + 9 \times \frac{1}{6} + 16 \times \frac{1}{9} + 25 \times \frac{1}{18}$$

$$= \frac{5}{18} + \frac{8}{9} + \frac{9}{6} + \frac{16}{9} + \frac{25}{18} = \frac{5+16+27+32+25}{18} = \frac{105}{18}$$

$$\therefore \text{Var } (X) = \frac{105}{18} - \frac{35}{18} \times \frac{35}{18} = \frac{1890 - 1225}{324} = \frac{665}{324}$$

Hence, the required variance is $\frac{665}{324}$.

Q38. A and B throw a pair of dice alternately. A wins the game if he gets a total of 6 and B wins if she gets a total of 7. If A starts the game, find the probability of winning the game by A in third throw of the pair of dice.

Sol. Let A_1 be the event of getting a total of 6

$= \{(2, 4), (4, 2), (1, 5), (5, 1), (3, 3)\}$

and B_1 be the event of getting a total of 7

$= \{(2, 5), (5, 2), (1, 6), (6, 1), (3, 4), (4, 3)\}$

Let $P(A_1)$ is the probability, if A wins in a throw $= \frac{5}{36}$

and $P(B_1)$ is the probability, if B wins in a throw $= \frac{1}{6}$

$\therefore$ The required probability of winning A in his third throw

$$= P(\bar{A}_1) . P(\bar{B}_1) . P(A_1) = \frac{31}{36} . \frac{5}{6} . \frac{5}{36} = \frac{775}{7776}.$$

Q39. Two dice are tossed. Find whether the following two events A and B are independent. $A = \{(x, y) : x + y = 11\}$ and $B = \{(x, y) : x \neq 5\}$ where (x, y) denotes a typical sample point.

Sol. Given that:

$A = \{(x, y) : x + y = 11\}$ and $B = \{(x, y) : x \neq 5\}$

$\therefore \quad A = \{(5, 6), (6, 5)\},$

$B = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

$\Rightarrow \quad n(A) = 2, n(B) = 30$ and $n(A \cap B) = 1$

$\therefore \quad P(A) = \frac{2}{36} = \frac{1}{18}$ and $P(B) = \frac{30}{36} = \frac{5}{6}$

$\Rightarrow \quad P(A). P(B) = \frac{1}{18} . \frac{5}{6} = \frac{5}{108}$ and $P(A \cap B) = \frac{1}{36}$

Since $P(A). P(B) \neq P(A \cap B)$

Hence, A and B are not independent.

Q40. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. Show that the probability of drawing a white ball now does not depend on k.

Sol. Let A be the event having m white and n black balls

$E_1 = \{$first ball drawn of white colour$\}$

$E_2 = \{$first ball drawn of black colour$\}$

E_3 = {second ball drawn of white colour}

$\therefore \quad P(E_1) = \dfrac{m}{m+n}$ and $P(E_2) = \dfrac{n}{m+n}$

$$P(E_3/E_1) = \frac{m+k}{m+n+k} \text{ and } P(E_3/E_2) = \frac{m}{m+n+k}$$

Now $P(E_3) = P(E_1) \,.\, P(E_3/E_1) + P(E_2) \,.\, P(E_3/E_2)$

$$= \frac{m}{m+n} \times \frac{m+k}{m+n+k} + \frac{n}{m+n} \times \frac{m}{m+n+k}$$

$$= \frac{m}{m+n+k}\left[\frac{m+k}{m+n} + \frac{n}{m+n}\right]$$

$$= \frac{m}{m+n+k}\left[\frac{m+n+k}{m+n}\right] = \frac{m}{m+n}$$

Hence, the probability of drawing a white ball does not depend upon k.

LONG ANSWER TYPE QUESTIONS

Q41. Three bags contain a number of red and white balls as follows, Bag I: 3 red balls, Bag II: 2 red balls and 1 white ball and Bag III: 3 white balls. The probability that bag i will be chosen and a ball is selected from it is $\dfrac{i}{6}$, where i = 1, 2, 3. What is the probability that (*i*) a red ball will be selected (*ii*) a white ball is selected?

Sol. Given that:

Bag I: 3 red balls and no white ball

Bag II: 2 red balls and 1 white ball

Bag III: no red ball and 3 white balls

Let E_1, E_2 and E_3 be the events of choosing Bag I, Bag II and Bag III respectively and a ball is drawn from it.

$\therefore P(E_1) = \dfrac{1}{6}$, $P(E_2) = \dfrac{2}{6}$ and $P(E_3) = \dfrac{3}{6}$

(*i*) Let E be the event that red ball is selected

$$\therefore P(E) = P(E_1).P(E/E_1) + P(E_2).P(E/E_2) + P(E_3).P(E/E_3)$$

$$= \frac{1}{6}.\frac{3}{3} + \frac{2}{6}.\frac{2}{3} + \frac{3}{6}.0 = \frac{3}{18} + \frac{4}{18} = \frac{7}{18}$$

(*ii*) Let F be the event that a white ball is selected

$\therefore P(F) = 1 - P(E)$ [P(E) + P(F) = 1]

$$= 1 - \frac{7}{18} = \frac{11}{18}$$

Hence, the required probabilities are $\frac{7}{18}$ and $\frac{11}{18}$.

Q42. Refer to Exercise Q.41 above. If a white ball is selected, what is the probability that it came from

(*i*) Bag II (*ii*) Bag III?

Sol. Referring to Exercise Q.41, we will use here Bayes' Theorem

$$(i)\ P(E_2/F) = \frac{P(E_2)\,.\,P(F/E_2)}{P(E_1)\,.\,P(F/E_1) + P(E_2)\,.\,P(F/E_2) + P(E_3)\,.\,P(F/E_3)}$$

$$= \frac{\frac{2}{6}\cdot\frac{1}{3}}{\frac{1}{6}\,.\,0 + \frac{2}{6}\cdot\frac{1}{3} + \frac{3}{6}\,.\,1} = \frac{\frac{2}{18}}{\frac{2}{18} + \frac{3}{6}} = \frac{2}{11}$$

$$(ii)\ P(E_3/F) = \frac{P(E_3)\,.\,P(F/E_3)}{P(E_1)\,.\,P(F/E_1) + P(E_2)\,.\,P(F/E_2) + P(E_3)\,.\,P(F/E_3)}$$

$$= \frac{\frac{3}{6}\,.\,1}{\frac{1}{6}\,.\,0 + \frac{2}{6}\cdot\frac{1}{3} + \frac{3}{6}\,.\,1} = \frac{\frac{3}{6}}{\frac{2}{18} + \frac{3}{6}} = \frac{3}{6} \times \frac{18}{11} = \frac{9}{11}$$

Hence, the required probabilities are $\frac{2}{11}$ and $\frac{9}{11}$.

Q43. A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture, where the proportions are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35%. Calculate the probability

(*i*) of a randomly chosen seed to germinate

(*ii*) that it will not germinate given that the seed is of type A_3

(*iii*) that it is of the type A_2 given that a randomly chosen seed does not germinate.

Sol. Given that $A_1 : A_2 : A_3 = 4 : 4 : 2$

$\therefore$ $P(A_1) = \frac{4}{10}$, $P(A_2) = \frac{4}{10}$ and $P(A_3) = \frac{2}{10}$

where A_1, A_2 and A_3 are the three types of seeds.

Let E be the event that a seed germinates and $\bar{E}$ be the event that a seed does not germinate

$$\therefore\ P\left(\frac{E}{A_1}\right) = \frac{45}{100},\ P\left(\frac{E}{A_2}\right) = \frac{60}{100} \text{ and } P\left(\frac{E}{A_3}\right) = \frac{35}{100}$$

$$\text{and } P\left(\frac{\bar{E}}{A_1}\right) = \frac{55}{100},\ P\left(\frac{\bar{E}}{A_2}\right) = \frac{40}{100} \text{ and } P\left(\frac{\bar{E}}{A_3}\right) = \frac{65}{100}$$

(*i*) $$P(E) = P(A_1).P\left(\frac{E}{A_1}\right) + P(A_2).P\left(\frac{E}{A_2}\right) + P(A_3).P\left(\frac{E}{A_3}\right)$$
$$= \frac{4}{10}\cdot\frac{45}{100} + \frac{4}{10}\cdot\frac{60}{100} + \frac{2}{10}\cdot\frac{35}{100}$$
$$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{1000} = \frac{490}{1000} = 0.49$$

(*ii*) $$P\left(\bar{E}/A_3\right) = 1 - P\left(E/A_3\right) = 1 - \frac{35}{100} = \frac{65}{100} = 0.65$$

(*iii*) Using Bayes' Theorem, we get

$$P\left(A_2/\bar{E}\right) = \frac{P(A_2)\,.\,P\left(\bar{E}/A_2\right)}{P(A_1).P\left(\bar{E}/A_1\right)+P(A_2).P\left(\bar{E}/A_2\right)+P(A_3).P\left(\bar{E}/A_3\right)}$$
$$= \frac{\frac{4}{10}\cdot\frac{40}{100}}{\frac{4}{10}\cdot\frac{55}{100} + \frac{4}{10}\cdot\frac{40}{100} + \frac{2}{10}\cdot\frac{65}{100}}$$
$$= \frac{\frac{160}{1000}}{\frac{220}{1000} + \frac{160}{1000} + \frac{130}{1000}} = \frac{160}{220 + 160 + 130} = \frac{160}{510} = \frac{16}{51} = 0.314$$

Hence, the required probability is $\frac{16}{51}$ or 0.314.

Q44. A letter is known to have come either from TATA NAGAR or from CALCUTTA. On the envelope, just two consecutive letters TA are visible. What is the probability that the letter came from TATA NAGAR?

Sol. Let E_1 : The event that the letter comes from TATA NAGAR

and E_2 : The event that the letter comes from CALCUTTA

Also E_3 : The event that on the letter, two consecutive letters TA are visible

$\therefore$ $P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{2}$ and $P\left(\frac{E_3}{E_1}\right) = \frac{2}{8}$ and $P\left(\frac{E_3}{E_2}\right) = \frac{1}{7}$

[$\because$ For TATA NAGAR, the two consecutive letters visible are TA, AT, TA, AN, NA, AG, GA, AR] $\therefore$ $P\left(E_3/E_1\right) = \frac{2}{8}$

and [For CALCUTTA, the two consecutive letters visible are CA, AL, LC, CU, UT, TT and TA] So, $P\left(E_3/E_2\right) = \frac{1}{7}$

Now using Bayes' Theorem, we have

$$P\left(E_1/E_3\right) = \frac{P(E_1)\,.\,P\left(E_3/E_1\right)}{P(E_1)\,.\,P\left(E_3/E_1\right) + P(E_2)\,.\,P\left(E_3/E_2\right)}$$

$$= \frac{\frac{1}{2}\cdot\frac{2}{8}}{\frac{1}{2}\cdot\frac{2}{8}+\frac{1}{2}\cdot\frac{1}{7}} = \frac{\frac{1}{8}}{\frac{1}{8}+\frac{1}{14}} = \frac{\frac{1}{8}}{\frac{7+4}{56}} = \frac{7}{11}$$

Hence, the required probability is $\frac{7}{11}$.

Q45. There are two bags, one of which contains 3 black and 4 white balls while the other contains 4 black and 3 white balls. A die is thrown. If it shows up 1 or 3, a ball is taken from the first bag but if it shows up any other number, a ball is chosen from the second bag. Find the probability of choosing a black ball.

Sol. Let E_1 be the event of selecting Bag I
and E_2 be the event of selecting Bag II
Let E_3 be the event that black ball is selected

$$\therefore \quad P(E_1) = \frac{2}{6} = \frac{1}{3} \text{ and } P(E_2) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(E_3/E_1) = \frac{3}{7} \text{ and } P(E_3/E_2) = \frac{4}{7}$$

$$\therefore \quad P(E_3) = P(E_1) . P(E_3/E_1) + P(E_2) . P(E_3/E_2)$$

$$= \frac{1}{3}\cdot\frac{3}{7}+\frac{2}{3}\cdot\frac{4}{7} = \frac{3+8}{21} = \frac{11}{21}$$

Hence, the required probability is $\frac{11}{21}$.

Q46. There are three urns containing 2 white and 3 black balls, 3 white and 2 black balls and 4 white and 1 black balls, respectively. There is an equal probability of each urn being chosen. A ball is drawn at random from the chosen urn and it is found to be white. Find the probability that the ball drawn was from the second urn.

Sol. We have 3 urns:

2 W 3 B	3 W 2 B	4 W 1 B
U_1	U_2	U_3

$\therefore$ Probabilities of choosing either of the urns are

$$P(U_1) = P(U_2) = P(U_3) = \frac{1}{3}$$

Let H be the event of drawing white ball from the chosen urn.

$\therefore\ P(H/U_1) = \frac{2}{5},\ P(H/U_2) = \frac{3}{5}$ and $P(H/U_3) = \frac{4}{5}$

$$\therefore P(U_2/H) = \frac{P(U_2).P(H/U_2)}{P(U_1).P(H/U_1)+P(U_2).P(H/U_2)+P(U_3).P(H/U_3)}$$

$$= \frac{\frac{1}{3}.\frac{3}{5}}{\frac{1}{3}.\frac{2}{5}+\frac{1}{3}.\frac{3}{5}+\frac{1}{3}.\frac{4}{5}} = \frac{\frac{3}{5}}{\frac{2}{5}+\frac{3}{5}+\frac{4}{5}} = \frac{3}{9} = \frac{1}{3}$$

Hence, the required probability is $\frac{1}{3}$.

Q47. By examining the chest X-ray, the probability that TB is detected when a person is actually suffering is 0.99. The probability of an healthy person diagnosed to have TB is 0.001. In a certain city, 1 in 1000 people suffer from TB. A person is selected at random and is diagnosed to have TB. What is the probability that he actually has TB?

Sol. Let E_1 : Event that a person has TB

E_2 : Event that a person does not have TB

and H : Event that the person is diagnosed to have TB.

$$\therefore \quad P(E_1) = \frac{1}{1000} = 0.001,\ P(E_2) = 1 - \frac{1}{1000} = \frac{999}{1000} = 0.999$$

$$P(H/E_1) = 0.99,\ P(H/E_2) = 0.001$$

$$\therefore P(E_1/H) = \frac{P(E_1)\,.\,P(H/E_1)}{P(E_1)\,.\,P(H/E_1) + P(E_2)\,.\,P(H/E_2)}$$

$$= \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.001} = \frac{0.99}{0.99 + 0.999}$$

$$= \frac{0.990}{0.990 + 0.999} = \frac{990}{1989} = \frac{110}{221}$$

Hence, the required probability is $\frac{110}{221}$.

Q48. An item is manufactured by three machines A, B and C. Out of the total number of items manufactured during a specified period, 50% are manufactured on machine A, 30% on B and 20% on C. 2% of items produced on A and 2% of items produced on B are defective and 3% of these produced on machine C are defective. All the items are stored at one godown. One item is drawn at random and is found to be defective. What is the probability that it was manufactured on machine A?

Sol. Let E_1 : The event that the item is manufactured on machine A

E_2 : The event that the item is manufactured on machine B

E_3 : The event that the item is manufactured on machine C

Let H be the event that the selected item is defective.

$\therefore$ Using Bayes' Theorem,

$$P(E_1) = \frac{50}{100},\ P(E_2) = \frac{30}{100},\ P(E_3) = \frac{20}{100}$$

$$P(H/E_1) = \frac{2}{100},\ P(H/E_2) = \frac{2}{100} \text{ and } P(H/E_3) = \frac{3}{100}$$

$$\therefore P(E_1/H) = \frac{P(E_1).P(H/E_1)}{P(E_1).P(H/E_1)+P(E_2).P(H/E_2)+P(E_3).P(H/E_3)}$$

$$= \frac{\frac{50}{100}\times\frac{2}{100}}{\frac{50}{100}\times\frac{2}{100}+\frac{30}{100}\times\frac{2}{100}+\frac{20}{100}\times\frac{3}{100}}$$

$$= \frac{100}{100+60+60} = \frac{100}{220} = \frac{10}{22} = \frac{5}{11}$$

Hence, the required probability is $\frac{5}{11}$.

Q49. Let X be a discrete random variable whose probability distribution is defined as follows:

$$P(X = x) = \begin{cases} k(x+1) & \text{for } x = 1, 2, 3, 4 \\ 2kx & \text{for } x = 5, 6, 7 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant—Calculate:

(*i*) the value of k (*ii*) E(X) (*iii*) Standard deviation of X.

Sol. (*i*) Here, $P(X = x) = k(x + 1)$ for $x = 1, 2, 3, 4$

So, $P(X = 1) = k(1 + 1) = 2k;\ P(X = 2) = k(2 + 1) = 3k$

$P(X = 3) = k(3 + 1) = 4k;\ P(X = 4) = k(4 + 1) = 5k$

Also, $P(X = x) = 2kx$ for $x = 5, 6, 7$

$P(X = 5) = 2(5)k = 10k;\ P(X = 6) = 2(6)k = 12k$

$P(X = 7) = 2(7)k = 14k$

and for otherwise it is 0.

$\therefore$ The probability distribution is given by

X	1	2	3	4	5	6	7	otherwise
P(X)	$2k$	$3k$	$4k$	$5k$	$10k$	$12k$	$14k$	0

We know that $\sum_{i=1}^{n} P(X_i) = 1$

So, $2k + 3k + 4k + 5k + 10k + 12k + 14k = 1$

$\Rightarrow \quad 50k = 1 \Rightarrow k = \frac{1}{50}$

Hence, the value of k is $\frac{1}{50}$

(*ii*) Now the probability distribution is

X	1	2	3	4	5	6	7
P(X)	$\frac{2}{50}$	$\frac{3}{50}$	$\frac{4}{50}$	$\frac{5}{50}$	$\frac{10}{50}$	$\frac{12}{50}$	$\frac{14}{50}$

$$E(X) = 1\times\frac{2}{50}+2\times\frac{3}{50}+3\times\frac{4}{50}+4\times\frac{5}{50}+5\times\frac{10}{50}+6\times\frac{12}{50}+7\times\frac{14}{50}$$

$$= \frac{2}{50}+\frac{6}{50}+\frac{12}{50}+\frac{20}{50}+\frac{50}{50}+\frac{72}{50}+\frac{98}{50} = \frac{260}{50} = \frac{26}{5} = 5.2$$

(*iii*) We know that Standard deviation (SD) = $\sqrt{\text{Variance}}$

Variance= $E(X^2) - [E(X)]^2$

$$E(X^2) = 1\times\frac{2}{50}+4\times\frac{3}{50}+9\times\frac{4}{50}+16\times\frac{5}{50}+25\times\frac{10}{50}$$
$$36\times\frac{12}{50}+49\times\frac{14}{50}$$

$$= \frac{2}{50}+\frac{12}{50}+\frac{36}{50}+\frac{80}{50}+\frac{250}{50}+\frac{432}{50}+\frac{686}{50} = \frac{1498}{50}$$

$$\therefore \text{ Variance (X)} = \frac{1498}{50} - \left(\frac{26}{5}\right)^2$$

$$= \frac{1498}{50} - \frac{676}{25} = \frac{1498-1352}{50} = \frac{146}{50} = 2.92$$

$\therefore \quad$ S.D = $\sqrt{2.92}$ = 1.7 (approx.)

Q50. The probability distribution of a discrete random variable X is given as under:

X	1	2	4	2A	3A	5A
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Calculate: (*i*) The value of A if E(X) = 2.94; (*ii*) Variance of X

Sol. (*i*) We know that: $E(X) = \sum_{i=1}^{n} P_i X_i$

$$\therefore \quad E(X) = 1\times\frac{1}{2}+2\times\frac{1}{5}+4\times\frac{3}{25}+2A\times\frac{1}{10}+3A\times\frac{1}{25}+5A\times\frac{1}{25}$$

$$2.94 = \frac{1}{2}+\frac{2}{5}+\frac{12}{25}+\frac{A}{5}+\frac{3A}{25}+\frac{A}{5}$$

$$\Rightarrow \quad 2.94 = 0.5 + 0.4 + 0.48 + \frac{13A}{25} = 1.38 + \frac{13A}{25}$$

$$\Rightarrow \quad 2.94 - 1.38 = \frac{13A}{25} \Rightarrow 1.56 = \frac{13A}{25}$$

$$\Rightarrow \quad A = \frac{1.56 \times 25}{13} = 0.12 \times 25$$

$$\therefore \quad A = 3$$

(*ii*) Now the distribution becomes

X	1	2	4	6	9	15
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

$$E(X^2) = 1\times\frac{1}{2}+4\times\frac{1}{5}+16\times\frac{3}{25}+36\times\frac{1}{10}+81\times\frac{1}{25}+225\times\frac{1}{25}$$

$$= \frac{1}{2}+\frac{4}{5}+\frac{48}{25}+\frac{36}{10}+\frac{81}{25}+\frac{225}{25}$$

$$= 0.5 + 0.8 + 1.92 + 3.6 + 3.24 + 9.00 = 19.06$$

$$\text{Variance }(X) = E(X^2) - [E(X)]^2$$

$$= 19.06 - (2.94)^2 = 19.06 - 8.64 = 10.42$$

Q51. The probability distribution of a random variable X is given as under:

$$P(X = x) = \begin{cases} kx^2 & \text{for } x = 1, 2, 3 \\ 2kx & \text{for } x = 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. Calculate:

(*i*) E(X) (*ii*) $E(3X^2)$ (*iii*) $P(X \geq 4)$

Sol. Given that: $P(X = x) = \begin{cases} kx^2 & \text{for } x = 1, 2, 3 \\ 2kx & \text{for } x = 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$

$\therefore$ Probability distribution of random variable X is

X	1	2	3	4	5	6	otherwise
P(X)	k	$4k$	$9k$	$8k$	$10k$	$12k$	0

We know that $\sum_{i=1}^{n} P(X_i) = 1$

$\therefore \quad k + 4k + 9k + 8k + 10k + 12k = 1 \Rightarrow 44k = 1 \quad \Rightarrow \quad k = \frac{1}{44}$

(*i*) $E(X) = \sum_{i=1}^{n} P_i X_i = 1 \times k + 2 \times 4k + 3 \times 9k + 4 \times 8k + 5 \times 10k + 6 \times 12k$

$$= k + 8k + 27k + 32k + 50k + 72k = 190k$$

$$= 190 \times \frac{1}{44} = \frac{95}{22} = 4.32 \text{ (approx.)}$$

(*ii*) $E(3X^2) = 3[k + 4 \times 4k + 9 \times 9k + 16 \times 8k + 25 \times 10k + 36 \times 12k]$

$$= 3[k + 16k + 81k + 128k + 250k + 432k] = 3[908k]$$

$$= 3 \times 908 \times \frac{1}{44} = \frac{2724}{44} = 61.9 \text{ (approx.)}$$

(*iii*) $P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$

$$= 8k + 10k + 12k = 30k$$

$$= 30 \times \frac{1}{44} = \frac{15}{22}.$$

Q52. A bag contains $(2n + 1)$ coins. It is known that n of these coins have a head on both sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n.

Sol. Given that n coins are two headed coins and the remaining $(n + 1)$ coins are fair.

Let E_1 : the event that unfair coin is selected

E_2 : the event that the fair coin is selected

E : the event that the toss results in a head

$$\therefore \quad P(E_1) = \frac{n}{2n+1} \text{ and } P(E_2) = \frac{n+1}{2n+1}$$

$$P(E/E_1) = 1 \text{ (sure event) and } P(E/E_2) = \frac{1}{2}$$

$$\therefore \quad P(E) = P(E_1).P(E/E_1) + P(E_2).P(E/E_2)$$

$$= \frac{n}{2n+1}.1 + \frac{n+1}{2n+1}.\frac{1}{2} = \frac{1}{2n+1}\left(n + \frac{n+1}{2}\right)$$

$$= \frac{1}{2n+1}\left(\frac{2n+n+1}{2}\right) = \frac{3n+1}{2(2n+1)}$$

But $\quad P(E) = \frac{31}{42}$ (given)

$$\therefore \frac{3n+1}{2(2n+1)} = \frac{31}{42} \Rightarrow \frac{3n+1}{2n+1} = \frac{31}{21}$$

$$\Rightarrow 63n + 21 = 62n + 31$$

$$\Rightarrow n = 10$$

Hence, the required value of n is 10.

Q53. Two cards are drawn successively without replacement from a well shuffled deck of cards. Find the mean and standard deviation of the random variable X where X is the number of aces.

Sol. Let X be the random variable such that X = 0, 1, 2

and E = the event of drawing an ace

and F = the event of drawing non-ace.

$$\therefore P(E) = \frac{4}{52} \text{ and } P(\bar{E}) = \frac{48}{52}$$

Now $P(X=0) = P(\bar{E}).P(\bar{E}) = \frac{48}{52}.\frac{47}{51} = \frac{188}{221}$

$$P(X=1) = P(E).P(\bar{E}) + P(\bar{E}).P(E) = \frac{4}{52} \times \frac{48}{51} + \frac{48}{52} \times \frac{4}{51} = \frac{32}{221}$$

$$P(X=2) = P(E).P(E) = \frac{4}{52}.\frac{3}{51} = \frac{1}{221}$$

We have Distribution Table:

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Now, Mean $E(X) = 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2 \times \frac{1}{221} = \frac{32}{221} + \frac{2}{221} = \frac{34}{221} = \frac{2}{13}$

$$E(X^2) = 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 4 \times \frac{1}{221} = \frac{32}{221} + \frac{4}{221} = \frac{36}{221}$$

$$\therefore \text{Variance} = E(X^2) - [E(X)]^2$$

$$= \frac{36}{221} - \left(\frac{2}{13}\right)^2 = \frac{36}{221} \quad \frac{4}{169} = \frac{468-68}{13 \times 221} = \frac{400}{2873}$$

Standard deviation $= \sqrt{\frac{400}{2873}} = 0.377$ (approx.)

Q54. A die is tossed twice. A 'success' is getting an even number on a toss. Find the variance of the number of successes.

Sol. Let E be the event of getting even number on tossing a die.

$\therefore$ $P(E) = \frac{3}{6} = \frac{1}{2}$ and $P(\bar{E}) = 1 - \frac{1}{2} = \frac{1}{2}$

Here X = 0, 1, 2

$$P(X = 0) = P(\bar{E}).P(\bar{E}) = \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$$

$$P(X = 1) = P(E).P(\bar{E}) + P(\bar{E}).P(E) = \frac{1}{2}.\frac{1}{2} + \frac{1}{2}.\frac{1}{2} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}$$

$$P(X = 2) = P(E).P(E) = \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$$

$\therefore$ Probability distribution table is

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

$$E(X) = 0 \times \frac{1}{4} + 1 \times \frac{2}{4} + 2 \times \frac{1}{4} = \frac{2}{4} + \frac{2}{4} = 1$$

$$E(X^2) = 0 \times \frac{1}{4} + 1 \times \frac{2}{4} + 4 \times \frac{1}{4} = \frac{3}{2}$$

$\therefore$ Variance $(X) = E(X^2) - [E(X)]^2 = \frac{3}{2} - 1 = \frac{1}{2} = 0.5$

Q55. There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denotes the sum of the numbers on two cards drawn. Find the mean and variance of X.

Sol. Here, sample space S = {(1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2), (1, 4), (4, 1), (1, 5), (5, 1), (2, 4), (4, 2), (2, 5), (5, 2), (3, 4), (4, 3), (3, 5), (5, 3), (5, 4), (4, 5)}

$\therefore$ $n(S) = 20$

Let X be the random variable denoting the sum of the numbers on two cards drawn.

$\therefore$ $X = 3, 4, 5, 6, 7, 8, 9$

So, $P(X = 3) = \frac{2}{20}$ $\quad$ $P(X = 4) = \frac{2}{20}$

$P(X = 5) = \frac{4}{20}$ $\quad$ $P(X = 6) = \frac{4}{20}$

$P(X = 7) = \frac{4}{20}$ $\quad$ $P(X = 8) = \frac{2}{20}$

$$P(X = 9) = \frac{2}{20}$$

$$\therefore \text{ Mean, } E(X) = 3\times\frac{2}{20}+4\times\frac{2}{20}+5\times\frac{4}{20}+6\times\frac{4}{20}+7\times\frac{4}{20}+8\times\frac{2}{20}+9\times\frac{2}{20}$$

$$= \frac{6}{20}+\frac{8}{20}+\frac{20}{20}+\frac{24}{20}+\frac{28}{20}+\frac{16}{20}+\frac{18}{20} = \frac{120}{20} = 6$$

$$E(X^2) = 9\times\frac{2}{20}+16\times\frac{2}{20}+25\times\frac{4}{20}+36\times\frac{4}{20}+49\times\frac{4}{20}+64\times\frac{2}{20}+81\times\frac{2}{20}$$

$$= \frac{18}{20}+\frac{32}{20}+\frac{100}{20}+\frac{144}{20}+\frac{196}{20}+\frac{128}{20}+\frac{162}{20} = \frac{780}{20} = 39$$

$$\therefore \text{Variance } (X) = E(X^2) - [E(X)]^2 = 39 - (6)^2 = 39 - 36 = 3$$

OBJECTIVE TYPE QUESTIONS

Choose the correct answer from the given four options in each of the Exercises from Q. 56 to 93.

Q56. If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ is equal to

(*a*) $\frac{1}{10}$ (*b*) $\frac{1}{8}$ (*c*) $\frac{7}{8}$ (*d*) $\frac{17}{20}$

Sol. Given that: $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$

$$\therefore \quad P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{7/10}{4/5} = \frac{7}{8}$$

Hence, the correct option is (*c*).

Q57. If $P(A \cap B) = \frac{7}{10}$ and $P(B) = \frac{17}{20}$, then $P(A/B)$ equals

(*a*) $\frac{14}{17}$ (*b*) $\frac{17}{20}$ (*c*) $\frac{7}{8}$ (*d*) $\frac{1}{8}$

Sol. Given that: $P(A \cap B) = \frac{7}{10}$ and $P(B) = \frac{17}{20}$

$$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{7/10}{17/20} = \frac{14}{17}$$

Hence, the correct option is (*a*).

Q58. If $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P(B/A) + P(A/B)$ equals to

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{5}{12}$ (*d*) $\frac{7}{12}$

Sol. Here, $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad \frac{3}{5} = \frac{3}{10} + \frac{2}{5} - P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = \frac{3}{10} + \frac{2}{5} - \frac{3}{5} = \frac{3+4-6}{10} = \frac{1}{10}$$

Now $P(A/B) + P(B/A) = \frac{P(A \cap B)}{P(B)} + \frac{P(A \cap B)}{P(A)}$

$$= \frac{1/10}{2/5} + \frac{1/10}{3/10} = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$$

Hence, the correct option is (*d*).

Q59. If $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P(A'/B').\ P(B'/A')$ is equal to

(*a*) $\frac{5}{6}$ (*b*) $\frac{5}{7}$ (*c*) $\frac{25}{42}$ (*d*) 1

Sol. Given that: $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$

$$P(A') = 1 - \frac{2}{5} = \frac{3}{5},\ P(B') = 1 - \frac{3}{10} = \frac{7}{10}$$

and $P(A' \cap B') = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$

$$= 1 - \left[\frac{2}{5} + \frac{3}{10} - \frac{1}{5}\right] = 1 - \left[\frac{1}{5} + \frac{3}{10}\right] = 1 - \frac{5}{10} = \frac{1}{2}$$

$$\therefore \quad P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1/2}{7/10} = \frac{5}{7}$$

and $P(B'/A') = \frac{P(A' \cap B')}{P(A')} = \frac{1/2}{3/5} = \frac{5}{6}$

$$\therefore P(A'/B').P(B'/A') = \frac{5}{7} \times \frac{5}{6} = \frac{25}{42}$$

Hence, the correct option is (*c*).

Q60. If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$, then $P(A' \cap B')$ equals

(*a*) $\frac{1}{12}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{3}{16}$

Sol. Given that: $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{1}{4} = \frac{P(A \cap B)}{1/3} \Rightarrow P(A \cap B) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

Now $P(A' \cap B') = 1 - P(A \cup B)$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - \left[\frac{1}{2} + \frac{1}{3} - \frac{1}{12}\right] = 1 - \left[\frac{5}{6} - \frac{1}{12}\right] = 1 - \frac{9}{12} = \frac{3}{12} = \frac{1}{4}$$

Hence, the correct option is (*c*).

Q61. If P(A) = 0.4, P(B) = 0.8 and $P(B/A) = 0.6$ then $P(A \cup B)$ is equal to

(*a*) 0.24 (*b*) 0.3 (*c*) 0.48 (*d*) 0.96

Sol. Given that: P(A) = 0.4, P(B) = 0.8 and $P(B/A) = 0.6$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \Rightarrow 0.6 = \frac{P(A \cap B)}{0.4}$$

$\therefore$ $P(A \cap B) = 0.6 \times 0.4 = 0.24$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.4 + 0.8 - 0.24 = 1.20 - 0.24 = 0.96$$

Hence, the correct option is (*d*).

Q62. If A and B are two events and $A \neq \phi$, $B \neq \phi$, then

(*a*) $P(A/B) = P(A).P(B)$ (*b*) $P(A/B) = \frac{P(A \cap B)}{P(B)}$

(*c*) $P(A/B).P(B/A) = 1$ (*d*) $P(A/B) = \frac{P(A)}{P(B)}$

Sol. Given that: $A = \phi$ and $B \neq \phi$,

then $$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Hence, the correct option is (*b*).

Q63. A and B are events such that P(A) = 0.4, P(B) = 0.3 and $P(A \cup B)$ = 0.5. Then $P(B' \cap A)$ equals

(*a*) $\frac{2}{3}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{3}{10}$ (*d*) $\frac{1}{5}$

Sol. Given that: P(A) = 0.4, P(B) = 0.3 and P(A ∪ B) = 0.5

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.5 = 0.4 + 0.3 - P(A \cap B)$$
$$P(A \cap B) = 0.4 + 0.3 - 0.5 = 0.2$$
$$\therefore \quad P(B' \cap A) = P(A) - P(A \cap B)$$
$$= 0.4 - 0.2 = 0.2 = \frac{1}{5}$$

Hence, the correct option is (*d*).

Q64. If A and B are two events such that $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then P(A) equals

(*a*) $\frac{3}{10}$ (*b*) $\frac{1}{5}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{3}{5}$

Sol. Given that: $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$

We know that $P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow \frac{1}{2} = \frac{P(A \cap B)}{3/5}$

$$\therefore \quad P(A \cap B) = \frac{3}{10}$$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{4}{5} = P(A) + \frac{3}{5} - \frac{3}{10}$$

$$\Rightarrow \quad P(A) = \frac{4}{5} - \frac{3}{5} + \frac{3}{10} = \frac{1}{5} + \frac{3}{10} = \frac{5}{10} = \frac{1}{2}$$

Hence, the correct option is (*c*).

Q65. In Exercise 64 above, $P(B/A')$ is equal to

(*a*) $\frac{1}{5}$ (*b*) $\frac{3}{10}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{3}{5}$

Sol. According to Exercise 64, we have

$P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$, $P(A \cup B) = \frac{4}{5}$

$$P(B/A') = \frac{P(B \cap A')}{P(A')} = \frac{P(B) - P(A \cap B)}{1 - P(A)} = \frac{\frac{3}{5} - \frac{3}{10}}{1 - \frac{1}{2}} = \frac{\frac{3}{10}}{\frac{1}{2}} = \frac{3}{5}$$

Hence, the correct option is (*d*).

Q66. If $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then

$P(A \cup B)' + P(A' \cup B) =$

(*a*) $\frac{1}{5}$ (*b*) $\frac{4}{5}$ (*c*) $\frac{1}{2}$ (*d*) 1

Sol. Given that: $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{P(A \cap B)}{3/5} \Rightarrow P(A \cap B) = \frac{3}{10}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{4}{5} = P(A) + \frac{3}{5} - \frac{3}{10}$$

$$\therefore \quad P(A) = \frac{4}{5} - \frac{3}{5} + \frac{3}{10} = \frac{1}{5} + \frac{3}{10} = \frac{5}{10} = \frac{1}{2}$$

Now $P(A \cup B)' + P(A' \cup B)$

$$= 1 - P(A \cup B) + 1 - P(A \cap B')$$

$$= 2 - \frac{4}{5} - P(A).P(B')$$

$$= \frac{6}{5} - \frac{1}{2}.\left(1 - \frac{3}{5}\right) = \frac{6}{5} - \frac{1}{2} \times \frac{2}{5} = \frac{6}{5} - \frac{1}{5} = \frac{5}{5} = 1$$

Hence, the correct option is (*d*).

Q67. Let $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$. Then $P(A'/B)$ is equal to

(*a*) $\frac{6}{13}$ (*b*) $\frac{4}{13}$ (*c*) $\frac{4}{9}$ (*d*) $\frac{5}{9}$

Sol. Given that: $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$

$$P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{\frac{9}{13} - \frac{4}{13}}{\frac{9}{13}} = \frac{\frac{5}{13}}{\frac{9}{13}} = \frac{5}{9}$$

Hence, the correct option is (*d*).

Q68. If A and B are such events that $P(A) > 0$ and $P(B) \neq 1$ then $P(A'/B')$ equals

(*a*) $1 - P(A/B)$ (*b*) $1 - P(A'/B)$

(*c*) $\frac{1 - P(A \cup B)}{P(B')}$ (*d*) $\frac{P(A')}{P(B')}$

Sol. Given that: P(A) > 0 and P(B) ≠ 1

$\therefore \quad P(A'/B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{1 - P(A \cup B)}{P(B')}$

Hence, the correct option is (*c*).

Q69. If A and B are two independent events with $P(A) = \dfrac{3}{5}$ and $P(B) = \dfrac{4}{9}$, then $P(A' \cap B')$ equals

(*a*) $\dfrac{4}{15}$ (*b*) $\dfrac{8}{45}$ (*c*) $\dfrac{1}{3}$ (*d*) $\dfrac{2}{9}$

Sol. Given that: A and B are independent events

such that $P(A) = \dfrac{3}{5} \quad \therefore P(A') = 1 - \dfrac{3}{5} = \dfrac{2}{5}$

$P(B) = \dfrac{4}{9} \quad \therefore P(B') = 1 - \dfrac{4}{9} = \dfrac{5}{9}$

$\therefore \quad P(A' \cap B') = P(A') \,.\, P(B') = \dfrac{2}{5} \cdot \dfrac{5}{9} = \dfrac{2}{9}$

Hence, the correct option is (*d*).

Q70. If two events are independent, then

(*a*) they must be mutually exclusive
(*b*) the sum of their probabilities must be equal to 1
(*c*) (*a*) and (*b*) both are correct
(*d*) none of the above is correct

Sol. For independent events A and B, $P(A).P(B) = P(A \cap B)$

So, they will not be mutually exclusive.

If P(A) + P(B) = 1, they are exhaustive events and for independent events A and $P(A \cap B) \neq 0$.

Hence, the correct option is (*d*).

Q71. Let A and B be two events such that $P(A) = \dfrac{3}{8}$, $P(B) = \dfrac{5}{8}$ and $P(A \cup B) = \dfrac{3}{4}$. Then $P(A/B).P(A'/B)$ is equal to

(*a*) $\dfrac{2}{5}$ (*b*) $\dfrac{3}{8}$ (*c*) $\dfrac{3}{20}$ (*d*) $\dfrac{6}{25}$

Sol. Given that: $P(A) = \dfrac{3}{8}$, $P(B) = \dfrac{5}{8}$ and $P(A \cup B) = \dfrac{3}{4}$

$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\dfrac{3}{4} = \dfrac{3}{8} + \dfrac{5}{8} - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = \dfrac{3}{8} + \dfrac{5}{8} - \dfrac{3}{4} = \dfrac{1}{4}$

$$\text{Now } P(A/B).P(A'/B) = \frac{P(A \cap B)}{P(B)} \cdot \frac{P(A' \cap B)}{P(B)}$$

$$= \frac{P(A \cap B)}{P(B)} \cdot \frac{P(B) - P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{4}}{\frac{5}{8}} \cdot \frac{\left(\frac{5}{8} - \frac{1}{4}\right)}{\frac{5}{8}} = \frac{2}{5} \cdot \frac{3}{5} = \frac{6}{25}$$

Hence, the correct option is (*d*).

Q72. If the events A and B are independent, then $P(A \cap B)$ is equal to

(*a*) $P(A) + P(B)$ (*b*) $P(A) - P(B)$ (*c*) $P(A).P(B)$ (*d*) $\frac{P(A)}{P(B)}$

Sol. Since A and B are two independent events

$\therefore$ $P(A \cap B) = P(A).P(B)$

Hence, the correct option is (*c*).

Q73. Two events E and F are independent. If $P(E) = 0.3$ and $P(E \cup F) = 0.5$, then $P(E/F) - P(F/E)$ equals

(*a*) $\frac{2}{7}$ (*b*) $\frac{3}{35}$ (*c*) $\frac{1}{70}$ (*d*) $\frac{1}{7}$

Sol. Given that: E and F are independent events such that

$P(E) = 0.3$ and $P(E \cup F) = 0.5$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$0.5 = 0.3 + P(F) - P(E).P(F)$$

$$\Rightarrow \quad 0.5 - 0.3 = P(F)\,[1 - P(E)] \Rightarrow 0.2 = P(F)\,(1 - 0.3)$$

$$\Rightarrow \quad 0.2 = P(F).(0.7)$$

$$\therefore \quad P(F) = \frac{0.2}{0.7} = \frac{2}{7}$$

$$\text{Now } P(E/F) - P(F/E) = \frac{P(E \cap F)}{P(F)} - \frac{P(E \cap F)}{P(E)}$$

$$= \frac{P(E).P(F)}{P(F)} - \frac{P(E).P(F)}{P(E)} = P(E) - P(F) = \frac{3}{10} - \frac{2}{7} = \frac{1}{70}$$

Hence, the correct option is (*c*).

Q74. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then the probability of getting exactly one red ball is

(*a*) $\frac{45}{196}$ (*b*) $\frac{135}{392}$ (*c*) $\frac{15}{56}$ (*d*) $\frac{15}{29}$

Sol. Given that: Bag contains 5 red and 3 blue balls.

Probability of getting exactly one red ball if 3 balls are randomly drawn without replacement

= P(R) . P(B) . P(B) + P(B) . P(R) . P(B) + P(B) . P(B) . P(R)

$$= \frac{5}{8}\cdot\frac{3}{7}\cdot\frac{2}{6}+\frac{3}{8}\cdot\frac{5}{7}\cdot\frac{2}{6}+\frac{3}{8}\cdot\frac{2}{7}\cdot\frac{5}{6} = \frac{30}{336}+\frac{30}{336}+\frac{30}{336} = \frac{90}{336}=\frac{15}{56}$$

Hence, the correct option is (*c*).

Q75. Refer to Question 74 above. The probability that exactly two of the three balls were red, the first ball being red is

(*a*) $\frac{1}{3}$ (*b*) $\frac{4}{7}$ (*c*) $\frac{15}{28}$ (*d*) $\frac{5}{28}$

Sol. According to Question 74,

Let E_1 be the event that first ball is red.

E_2 be the event that exactly two of the three balls are red.

$$\therefore \quad P(E_1) = P(R).P(R).P(B)+P(R).P(R).P(R)+P(R).P(B).P(R) + P(R).P(B).P(B)$$

$$= \frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6}+\frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6}+\frac{5}{8}\cdot\frac{3}{7}\cdot\frac{4}{6}+\frac{5}{8}\cdot\frac{3}{7}\cdot\frac{2}{6}$$

$$= \frac{60}{336}+\frac{60}{336}+\frac{60}{336}+\frac{30}{336}=\frac{210}{336}$$

$$P(E_1 \cap E_2) = P(R).P(B).P(R)+P(R).P(R).P(B)$$

$$= \frac{5}{8}\cdot\frac{3}{7}\cdot\frac{4}{6}+\frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6}=\frac{60}{336}+\frac{60}{336}=\frac{120}{336}$$

$$\therefore P\left(E_2/E_1\right) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{120/336}{210/336}=\frac{4}{7}$$

Hence, the correct option is (*b*).

Q76. Three persons A, B and C fire at a target in turn, starting with A. Their probabilities of hitting the target are 0.4, 0.3 and 0.2 respectively. The probabilities of two hits is

(*a*) 0.024 (*b*) 0.188 (*c*) 0.336 (*d*) 0.452

Sol. Given that: P(A) = 0.4, P(B) = 0.3 and P(C) = 0.2

Also $P(\bar{A}) = 1 - 0.4 = 0.6$, $P(\bar{B}) = 1 - 0.3 = 0.7$

and $P(\bar{C}) = 1 - 0.2 = 0.8$

$\therefore$ Probabilities of two hits

$= P(A).P(B).P(\bar{C}) + P(A).P(\bar{B}).P(C) + P(\bar{A}).P(B).P(C)$

$= 0.4 \times 0.3 \times 0.8 + 0.4 \times 0.7 \times 0.2 + 0.6 \times 0.3 \times 0.2$

$= 0.096 + 0.056 + 0.036 = 0.188$

Hence, the correct option is (*b*).

Q77. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has atleast one girl is

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{4}{7}$

Sol. Let G denotes the girl and B denotes the boy of the given family.

So, n(S) = {(BGG), (GBG), (GGB), (GBB), (BGB), (BBG), (BBB), (GGG)}

Let E_1 be the event that the family has alteast one girl.

$\therefore$ E_1 = {(BGG), (GBG), (GGB), (GBB), (BGB), (BBG), (GGG)}

Let E_2 be the event that the eldest child is a girl.

$\therefore$ E_2 = {(GBG), (GGB), (GBB), (GGG)}

$(E_1 \cap E_2)$ = {(GBB), (GGB), (GBG), (GGG)}

$$\therefore P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{4/8}{7/8} = \frac{4}{7}$$

Hence, the correct option is (*d*).

Q78. If a die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number on the die and a spade card is

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{1}{8}$ (*d*) $\frac{3}{4}$

Sol. Let E_1 be the event of getting even number on the die. E_2 be the event of selecting a spade card.

$$\therefore P(E_1) = \frac{3}{6} = \frac{1}{2} \text{ and } P(E_2) = \frac{13}{52} = \frac{1}{4}$$

So $$P(E_1 \cap E_2) = P(E_1).P(E_2) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

Hence, the correction option is (c).

Q79. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is

(*a*) $\frac{3}{28}$ (*b*) $\frac{2}{21}$ (*c*) $\frac{1}{28}$ (*d*) $\frac{167}{168}$

Sol. Probability of drawing 2 green and 1 blue balls

$= P(G) . P(G) . P(B) + P(G) . P(B) . P(G) + P(B) . P(G) . P(G)$

$$= \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{2}{6} + \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{2}{6} + \frac{2}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} = \frac{12}{336} + \frac{12}{336} + \frac{12}{336} = \frac{36}{336} = \frac{3}{28}$$

Hence, the correct option is (*a*).

Q80. A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested then the probability that both are dead is

(*a*) $\frac{33}{56}$ (*b*) $\frac{9}{64}$ (*c*) $\frac{1}{14}$ (*d*) $\frac{3}{28}$

Sol. Required probability = P(dead).P(dead)

$$= \frac{3}{8}.\frac{2}{7} = \frac{3}{28}$$

Hence, the correct option is (*d*).

Q81. If eight coins are tossed together, then the probability of getting exactly 3 heads is

(*a*) $\frac{1}{256}$ (*b*) $\frac{7}{32}$ (*c*) $\frac{5}{32}$ (*d*) $\frac{3}{32}$

Sol. Here, $n = 8$, $p = \frac{1}{2}$, $q = 1 - \frac{1}{2} = \frac{1}{2}$ and $r = 3$

We know that $P(x = r) = {}^nC_r\, p^r.q^{n-r}$

$$\therefore \quad P(x = 3) = {}^8C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{8-3}$$

$$= \frac{8!}{3!.5!}.\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^5 = 56.\left(\frac{1}{2}\right)^8 = 56.\frac{1}{256} = \frac{7}{32}$$

Hence, the correct option is (*b*).

Q82. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3, is

(*a*) $\frac{1}{18}$ (*b*) $\frac{5}{18}$ (*c*) $\frac{1}{5}$ (*d*) $\frac{2}{5}$

Sol. Let E_1 be the event showing the sum of the numbers on the two dice was less than 6 and E_2 be the event that the sum of the numbers is 3.

$$\therefore \quad E_1 = \{(1, 1), (1, 2), (2, 1), (1, 3), (3, 1), (1, 4), (4, 1), (2, 2), (2, 3), (3, 2)\}$$

$$n(E_1) = 10$$

and $E_2 = \{(1, 2), (2, 1)\} \Rightarrow n(E_2) = 2$ and $n(E_1 \cap E_2) = 2$

$\therefore$ Required probability

$$P(E_2/E_1) = \frac{n(E_1 \cap E_2)}{n(E_1)} = \frac{2}{10} = \frac{1}{5}$$

Hence, the correct option is (*c*).

Q83. Which one is not a requirement of a Binomial distribution?

(*a*) There are two outcomes for each trial.

(*b*) There is a fixed number of trials.
(*c*) The outcomes must be dependent on each other.
(*d*) The probability of success must be the same for all trials.

Sol. We know that for a Binomial distribution, the outcomes must not be dependent on each other.
Hence, the correct option is (*c*).

Q84. If two cards are drawn from a well shuffled deck of 52 playing cards with replacement, then the probability that both cards are queens is

(*a*) $\frac{1}{13}\times\frac{1}{13}$ (*b*) $\frac{1}{13}+\frac{1}{13}$ (*c*) $\frac{1}{13}\times\frac{1}{17}$ (*d*) $\frac{1}{13}\times\frac{4}{51}$

Sol. Probability of getting Queen = $\frac{4}{52}$
So, the required probability

$$= \text{P(Queen).P(Queen)}$$
$$= \frac{4}{52}.\frac{4}{52} = \frac{1}{13}.\frac{1}{13} \quad \text{(with replacement)}$$

Hence, the correct option is (*a*).

Q85. The probability of guessing correctly atleast 8 out of 10 answers on a true false type examination is

(*a*) $\frac{7}{64}$ (*b*) $\frac{7}{128}$ (*c*) $\frac{45}{1024}$ (*d*) $\frac{7}{41}$

Sol. Here, $n = 10$, $p = \frac{1}{2}$ and $q = \frac{1}{2}$ (for true/false questions)
and $r \geq 8$ i.e. 8, 9, 10

$$\therefore \ P(X \geq 8) = P(x = 8) + P(x = 9) + P(x = 10)$$
$$= {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right) + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0$$
$$= 45.\left(\frac{1}{2}\right)^{10} + 10.\left(\frac{1}{2}\right)^{10} + \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{10}(45+10+1)$$
$$= 56\times\frac{1}{1024} = \frac{7}{128}$$

Hence, the correct option is (*b*).

Q86. The probability that a person is not a swimmer is 0.3. The probability that out of 5 persons 4 are swimmers is

(*a*) ${}^5C_4\,(0.7)^4\,(0.3)$ (*b*) ${}^5C_1\,(0.7)\,(0.3)^4$
(*c*) ${}^5C_4\,(0.7)\,(0.3)^4$ (*d*) $(0.7)^4\,(0.3)$

Sol. Given that: $\overline{P} = 0.3 \ \therefore \ p = 0.7$ and $q = 1 - 0.7 = 0.3$

$n = 5$ and $r = 4$

We know that

$$P(X = r) = {}^nC_r\,(p)^r.(q)^{n-r}$$

$\therefore \quad P(x = 4) = {}^5C_4\,(0.7)^4\,(0.3)^{5-4} = {}^5C_4\,(0.7)^4\,(0.3)$

Hence, the correct option is (*a*).

Q87. The probability distribution of a discrete random variable X is given below:

X	2	3	4	5
P(X)	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The value of k is

(*a*) 8 (*b*) 16

(*c*) 32 (*d*) 48

Sol. We know that $\sum_{i=1}^{n} P(X_i) = 1$

$$\therefore \quad \frac{5}{k} + \frac{7}{k} + \frac{9}{k} + \frac{11}{k} = 1$$

$$\frac{32}{k} = 1 \Rightarrow k = 32.$$

Hence, the correct option is (*c*).

Q88. For the following probability distribution:

X	– 4	– 3	– 2	– 1	0
P(X)	0.1	0.2	0.3	0.2	0.2

E(X) is equal to:

(*a*) 0 (*b*) – 1

(*c*) – 2 (*d*) – 1.8

Sol. We know that:

$$E(X) = \sum_{i=1}^{n} X_i P_i$$

$$= (-4)(0.1) + (-3)(0.2) + (-2)(0.3) + (-1)(0.2) + 0(0.2)$$

$$= -0.4 - 0.6 - 0.6 - 0.2 = -1.8$$

Hence, the correct option is (*d*).

Q89. For the following probability distribution

X	1	2	3	4
P(X)	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{3}{10}$	$\frac{2}{5}$

$E(X^2)$ is equal to

(*a*) 3 (*b*) 5

(*c*) 7 (*d*) 10

Sol. We know that

$$E(X^2) = \sum_{i=1}^{n} P_i X_i^2$$

$$= 1 \times \frac{1}{10} + 4 \times \frac{1}{5} + 9 \times \frac{3}{10} + 16 \times \frac{2}{5}$$

$$= \frac{1}{10} + \frac{4}{5} + \frac{27}{10} + \frac{32}{5} = \frac{28}{10} + \frac{36}{5} = \frac{100}{10} = 10$$

Hence, the correct option is (*d*).

Q90. Suppose a random variable X follows the Binomial distribution with parameters n and p, where $0 < p < 1$. If $\frac{P(x = r)}{P(x = n - r)}$ is independent of n and r, then p equals

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{5}$ (*d*) $\frac{1}{7}$

Sol. $$P(X = r) = {}^nC_r\, p^r q^{n-r} = \frac{n!}{r!(n-r)!} p^r .(1-p)^{n-r}$$

$$P(X = n - r) = {}^nC_{n-r}\, p^{n-r} . (q)^{n-(n-r)} = {}^nC_{n-r}\, p^{n-r} . q^r$$

$$= \frac{n!}{(n-r)!(n-n+r)!} p^{n-r} q^r = \frac{n!}{(n-r)!r!} . p^{n-r} . q^r$$

Now $$\frac{P(x = r)}{P(x = n - r)} = \frac{\frac{n!}{r!(n-r)!} . p^r .(1-p)^{n-r}}{\frac{n!}{r!(n-r)!} . p^{n-r} .(1-p)^r} = \frac{\left(\frac{1-p}{p}\right)^{n-r}}{\left(\frac{1-p}{p}\right)^r}$$

The above expression will be independent of n and r if

$$\left(\frac{1-p}{p}\right) = 1 \Rightarrow \frac{1}{p} = 2 \Rightarrow p = \frac{1}{2}$$

Hence, the correct option is (*a*).

Q91. In a college, 30% students fail in Physics, 25% fail in Mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in Physics if she has failed in Mathematics is

(*a*) $\frac{1}{10}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{9}{20}$ (*d*) $\frac{1}{3}$

Sol. Let E_1 be the event that the student fails in Physics and E_2 be the event that she fails in Mathematics.

$$\therefore \quad P(E_1) = \frac{30}{100},\ P(E_2) = \frac{25}{100} \text{ and } P(E_1 \cap E_2) = \frac{10}{100}$$

$$\therefore P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{10/100}{25/100} = \frac{2}{5}$$

Hence, the correct option is (*b*).

Q92. A and B are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If the probability of their making a common error is $\frac{1}{20}$ and they obtain the same answer, then the probability of their answer to be correct is

(*a*) $\frac{1}{12}$ (*b*) $\frac{1}{40}$ (*c*) $\frac{13}{120}$ (*d*) $\frac{10}{13}$

Sol. Let E_1 be the event that both of them solve the problem.

$\therefore \quad P(E_1) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

and E_2 be the event that both of them same incorrectly the problem.

$\therefore \quad P(E_2) = \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right) = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

Let H be the event that both of them get the same answer.

Here, $P(H/E_1) = 1,\ P(H/E_2) = \frac{1}{20}$

$$\therefore P(E_1/H) = \frac{P(E_1).P(H/E_1)}{P(E_1).P(H/E_1) + P(E_2).P(H/E_2)}$$

$$= \frac{\frac{1}{12} \times 1}{\frac{1}{12} \times 1 + \frac{1}{2} \times \frac{1}{20}} = \frac{\frac{1}{12}}{\frac{1}{12} + \frac{1}{40}} = \frac{\frac{1}{12}}{\frac{10+3}{120}} = \frac{1/12}{13/120} = \frac{10}{13}$$

Hence, the correct option is (*d*).

Q93. A box has 100 pens of which 10 are defective. What is the probability that out of a sample of 5 pens drawn one by one with replacement atmost one is defective?

(*a*) $\left(\frac{9}{10}\right)^5$ (*b*) $\frac{1}{2}\left(\frac{9}{10}\right)^4$

(*c*) $\frac{1}{2}\left(\frac{9}{10}\right)^5$ (*d*) $\left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$

Sol. Here, $n = 5$, $p = \frac{10}{100} = \frac{1}{10}$ and $q = 1 - \frac{1}{10} = \frac{9}{10}$ and $r \le 1$

We know that

$P(X = r) = {}^nC_r\, p^r\, q^{n-r}$

So $\quad P(x \le 1) = P(x = 0) + P(x = 1)$

$$= {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 + {}^5C_1 \left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^4$$

$$= \left(\frac{9}{10}\right)^5 + \frac{5}{10}\left(\frac{9}{10}\right)^4 = \left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$$

Hence, the correct option is (*d*).

State True or False for the statements in each of the Exercises 94 to 103.

Q94. Let P(A) > 0 and P(B) > 0. Then A and B can be both mutually exclusive and independent.

Sol. False

Q95. If A and B are independent events, then A′ and B′ are also independent.

Sol. True

Q96. If A and B are mutually exclusive events, then they will be independent also.

Sol. False

Q97. Two independent events are always mutually exclusive.

Sol. False

Q98. If A and B are two independent events, then P(A and B) = P(A).P(B).

Sol. True

Q99. Another name for the mean of a probability distribution is expected value.

Sol. True $\left[\because \ \mathrm{E(X)} = \sum \mathrm{X}_i \mathrm{P(X}_i)\right]$

Q100. If A and B′ are independent events, then $P(A' \cup B) = 1 - P(A).P(B')$

Sol. True $[\because \ P(A' \cup B) = 1 - P(A \cap B') = 1 - P(A).P(B')]$

Q101. If A and B are independent events, then P (exactly one of A, B occurs) = P(A).P(B′) + P(B).P(A′)

Sol. True

Q102. If A and B are two events such that P(A) > 0 and P(A) + P(B) > 1, then $P(B/A) \geq 1 - \frac{P(B')}{P(A)}$

Sol. False $\left[\because \ P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A) + P(B) - P(A \cup B)}{P(A)} > \frac{1 - P(A \cup B)}{P(A)}\right]$

Q103. If A, B and C are three independent events such that $P(A) = P(B) = P(C) = p$, then P (atleast two of A, B and C occur) $= 3p^2 - 2p^3$

Sol. True

Since P (atleast two of A, B and C occur)

$$= p \times p \times (1 - p) + (1 - p) . p . p + p(1 - p).p + p . p . p$$
$$= 3p^2 (1 - p) + p^3 = 3p^2 - 3p^3 + p^3 = 3p^2 - 2p^3$$

Fill in the blanks in each of the following questions.

Q104. If A and B are two events such that $P(A/B) = p$, $P(A) = p$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{5}{9}$, then p is equal to ....

Sol. Given that: $P(A) = p$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{5}{9}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = p \Rightarrow P(A \cap B) = p . P(B) = p.\frac{1}{3}$$

and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{5}{9} = p + \frac{1}{3} - \frac{p}{3} \Rightarrow \frac{5}{9} - \frac{1}{3} = \frac{2p}{3} \Rightarrow \frac{2}{9} = \frac{2p}{3} \quad \Rightarrow p = \frac{1}{3}$$

Hence, p is equal to $\frac{1}{3}$.

Q105. If A and B are such that $P(A' \cup B') = \frac{2}{3}$ and $P(A \cup B) = \frac{5}{9}$, then $P(A') + P(B')$ is equal to ....

Sol. Here, $P(A' \cup B') = \frac{2}{3}$ and $P(A \cup B) = \frac{5}{9}$

$\therefore \quad 1 - P(A \cap B) = \frac{2}{3}$

$\Rightarrow \quad P(A \cap B) = 1 - \frac{2}{3} = \frac{1}{3}$

Now $\quad P(A') + P(B') = 1 - P(A) + 1 - P(B) = 2 - [P(A) + P(B)]$

$$= 2 - [P(A \cup B) + P(A \cap B)]$$
$$= 2 - \left[\frac{5}{9} + \frac{1}{3}\right] = 2 - \frac{8}{9} = \frac{10}{9}$$

Hence, the value of the filler is $\frac{10}{9}$.

Q106. If X follows Binomial distribution with parameters $n = 5$, p and $P(X = 2) = 9P(X = 3)$, then p is equal to ....

Sol. Given that: $P(X = 2) = 9P(X = 3)$

$\Rightarrow \quad {}^5C_2\, p^2 q^3 = 9.\, {}^5C_3\, p^3 q^2$

$$\Rightarrow \quad \frac{1}{9} = \frac{{}^5C_3 p^3 q^2}{{}^5C_2 p^2 q^3} \Rightarrow \frac{1}{9} = \frac{p}{q} \qquad [\because {}^5C_3 = {}^5C_2]$$

$$\Rightarrow \quad 9p = q$$

$$\Rightarrow \quad 9p = 1 - p \Rightarrow 9p + p = 1 \Rightarrow 10p = 1 \therefore p = \frac{1}{10}$$

Hence, the value of the filler is $\frac{1}{10}$.

Q107. Let X be a random variable taking values $x_1, x_2, x_3, \ldots, x_n$ with probabilities $p_1, p_2, p_3, \ldots, p_n$ respectively. Then Var(X) is equal to ....

Sol. $\text{Var}(X) = E(X^2) - [E(X)]^2$

$$= \sum X^2 P(X) - \left[\sum X.P(X)\right]^2 = \sum p_i x_i^2 - \left(\sum p_i x_i\right)^2$$

Hence, Var (X) is equal to $\sum p_i x_i^2 - \left(\sum p_i x_i\right)^2$.

Q108. Let A and B be two events. If $P(A/B) = P(A)$, then A is .... of B.

Sol. $\because \quad P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow \quad P(A) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \quad P(A \cap B) = P(A).P(B)$$

So, A is **independent** of B.