HALF-YEARLY EXAMINATION

(SESSION : 2019-2020)

CLASS : XII

SUBJECT : MATHS

SUBJECT CODE : 041

TIME: 3 HRS. **MAXIMUM MARKS : 80**

GENERAL INSTRUCTIONS

(i) The question paper is divided into four sections. Section A, section B, section C and section D.

(ii) This question paper contains **36** questions. All questions are compulsory

(iii) Question nos. **1** to **20** in **SECTION-A** are **very short answer questions** carrying **1 mark** each.

(iv) Question nos. **21-26** in **SECTION-B** are **short answer questions** carrying **2 marks** each.

(v) Question nos. **27-32** in **SECTION-C** are **long answer-I type questions** carrying **4 marks** each.

(vi) Question nos. **33-36** in **SECTION-D** are **long answer-II type questions** carrying **6 marks** each

(vii) Use of calculator is not permitted. You may ask for logarithmic tables if required.

SECTION—A

1. If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, then $\cot^{-1} x + \cot^{-1} y$ equals to:

a. $\frac{\pi}{5}$ **b.** $\frac{2\pi}{5}$ **c.** $\frac{3\pi}{5}$ **d.** π

2. If $f(x) = \begin{cases} mx + 1, & \text{if } x \le \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then

a. m = 1, n = 0 **b.** $n = \frac{m\pi}{2} + 1$ **c.** $n = \frac{m\pi}{2}$ **d.** $m = n = \frac{\pi}{2}$

3. If $\frac{d}{dx} f(x) = x\cos x + \sin x$ and $f(0) = 2$, then $f(x) =$

a. x sin x **b.** x cos x + sin x + 2 **c.** x sin x + 2 **d.** x cos x + 2

4. Let the f : R $\to$ R be defined by f (x) = 2x + cosx, then f :

a. has a minimum at $x = \pi$ **b.** has a maximum, at $x = 0$

c. is a decreasing function **d.** is an increasing function

5. An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Probability that they are of the different colours is

a. 2/5 **b.** 1/15 **c.** 8/15 **d.** 4/15

6. The value of $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

a. $\frac{\pi}{6}$ **b.** $\frac{\pi}{12}$ **c.** 0 **d.** not possible to find

7. If $f(x) = x^{2/3}$ on [–1, 1] then is Rolle's theorem applicable to f(x) in [–1, 1].

8. The determinant $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$ equals to

a. abc **b.** 0 **c.** (a + b + c) (b – c) (c – a) (a – b) **d.** None of these

9. The slope of tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point (2, –1) is:

a. $\frac{22}{7}$ **b.** $\frac{6}{7}$ **c.** $-\frac{6}{7}$ **d.** – 6

10. Evaluate : $\int_{-2}^{3} \frac{dx}{1-x}$

a. 1 **b.** 0 **c.** 2 **d.** None of these

11. If $a_1, a_2, a_3, ..., a_n$ is an A.P. with common difference d, then

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+...+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right] = ...$$

12. Let A and B be symmetric matrices of the same order. Then AB – BA is _____ Matrix.

OR

If A is square matrix of order 3 and $|A| = 4$, then find the value of $|-5A|$ is _____.

13. If P (A) = 0.54, P (B) = 0.69 and $P(A \cap B) = 0.35$ then $P(A \cap \overline{B})$ is equal to _____ .

14. If tangent to the curve $y^2 + 3x - 7 = 0$ at the point (*h*, *k*) is parallel to line x –y = 4, then value of *k* is ___?

OR

For the curve $y = 5x - 2x^3$, if *x* increases at the rate of 2 units/sec, then at *x* = 3 the slope of the curve is changing at_____

15. The condition for any square matrix A to be singular is _____.

16. A die is thrown repeatedly until a six comes up. What is the sample space for this experiment?

17. Antiderivative of $\sqrt{x} - \frac{1}{\sqrt{x}}$

OR

Evaluate : $e^{\int \frac{-x\,dx}{1-x^2}}$

18. For what value of *k*, the matrix $\begin{bmatrix} 2-k & 4 \\ -5 & 1 \end{bmatrix}$ is singular?

19. The values of x and y for which the given matrices equal : $A = \begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix}$, $B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$.

20. If y = sin x°, find dy/dx.

SECTION—B

21. Evaluate : $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$.

OR

Find : $\int \frac{(x^2 + \sin^2 x)\sec^2 x}{1+x^2}\ dx$

22. If each element of a second order determinant is either zero or one, what is the probability that the value of he determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability ½).

23. Prove $\tan^{-1}\frac{yz}{xr} + \tan^{-1}\frac{zx}{yr} + \tan^{-1}\frac{xy}{zr} = \frac{\pi}{2}$, if $x^2 + y^2 + z^2 = r^2$.

OR

Find the value of $\left[\frac{1}{2}\left(\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right)\right]$, $|x| < 1$, $y > 0$ and $xy < 1$.

24. At what point on the ellipse $16x^2 + 9y^2 = 400$ does the ordinate decrease at the same rate at which the abscissa increases.

25. Find the intervals for which $f(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing and decreasing on R.

OR

Find the equation of normal to the curve $x = a\sin^3\theta$ and $y = a\cos^3\theta$ at $\theta = \pi/4$.

26. Find the value of x such that $[1\ x\ 1]\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$.

SECTION—C

27. Evaluate : $\int_{-\pi/2}^{\pi/2} \sqrt{1-\cos^2 t}\, dt$

OR

Evaluate using limit as a sum $\int_{2}^{3} \left(x^2 + 1\right) dx$.

28. A toy manufacturer produces two types of dolls: a basic version - doll A and a deluxe version - doll B. Each doll of type B takes twice as tong to produce as one doll of type A. The company has time to make a maximum of 2,000 dolls of type A per day. The supply of piastic is sufficient to produce 1,500 dolls per day and each type requires equal amount of it.

The deluxe version, i.e., type B requires a fancy dress of which there are 600 per day available. If the company makes a profit of Rs. 3 and Rs. 5 per *doll,* respectively, on doll A and B, how many of each should be produced per day in order to maximize profit? Solve it by graphical method.

29. A problem on mathematics is given to 3 students whose chances of solving it are 1/2,1/3,1/4.What is the probability that the problem will be solved by exactly one of them, if all the three try to solve the problem simultaneously.

30. Find a and b if $\begin{cases} \dfrac{\tan x - \sin x}{x^3}, & \text{when x } < 0 \\ a, & \text{when x } = 0 \\ \dfrac{\sqrt{2+x} - \sqrt{2-x}}{bx}, & \text{when x } > 0 \end{cases}$ is continuous at x = 0

31. Differentiate $\tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$ w.r.t. $\sin^{-1}\dfrac{1-x^2}{1+x^2}$

32. If x = a sin2t (1 + cos 2t) and y = b cos2 t (1 – cos2 t), then show that $\left(\dfrac{dy}{dx}\right)_{t=\pi/4} = \dfrac{b}{a}$.

OR

If $\sqrt{1-x^4} + \sqrt{1-y^4} = k\left(x^2 - y^2\right)$. Prove that $\dfrac{dy}{dx} = \dfrac{x\sqrt{1-y^4}}{y\sqrt{1-x^4}}$.

SECTION—D

33. Using elementary transformations, find the inverse of the matrix A = $\begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ and use it to solve the following system of linear equations :

$$8x + 4y + 3z = 19 \quad ; \quad 2x + y + z = 5 \quad ; \quad x + 2y + 2z = 7$$

OR

Prove that $\begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix}$ is divisible by (x + y + z) and hence find the quotient.

34. An open box with square base and vertical sides is to be made to hold given quantity of water c^3 units. Show that the area of the material will be least if the depth of the box will be half the width.Also prove that the area of the box will be $3\sqrt[3]{4c^2}$.

35. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a mediation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities.

a. Find the probability that the person suffers from heart attack given that he chose one of the two options.

b. If it is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that patient followed a course of mediation and yoga?

36. Evaluate : $\int_0^{\pi/2} \log \sin x dx$.

OR

Evaluate : $\int \frac{x^2 - 3x + 1}{x^4 + x^2 + 1} dx$