The Mother's International School
Pre - Board Examination 2023 – 2024
Class 12
Mathematics
December 08, 2023

M.M : 80 | Time : 3 hours

Note :

1) The question paper has five sections A, B, C, D and E.
2) Each section is compulsory. However, there are internal choices in some questions.
3) Section A has 18 MCQs and 2 Assertion – Reason based questions of 1 mark each.
4) Section B has 5 very short answer type questions of 2 marks each.
5) Section C has 6 short answer type question of 3 marks each.
6) Section D has 4 long answer type questions of 5 marks each.
7) Section E has 3 source based/ case based/integrated units of assessment (4 marks each) with sub – parts.
8) All questions have to be done in order.

SECTION A

Q1: Let A and B be two symmetric matrices of the same order.
Which of the following statements is true ?

(a) A + B is a skew – symmetric matrix. (b) AB + BA is a symmetric matrix.
(c) 2A – 3B is a skew – symmetric matrix. (d) 2AB + 3BA is a symmetric matrix.

Q2: The total number of 2 x 2 matrices with entry either 0 or 1 are:

(a) 8 (b) 16 (c) 4 (d) ~~16~~ 32

Q3: Let f be a function defined as $f : \{\, \mathbb{R} - 3 \,\} \rightarrow Y$ such that $f(x) = \dfrac{2x+1}{x-3}$ is onto,

then the range of f (x) is given by :

(a) $\mathbb{R} - \{2\}$ (b) $(2, \infty)$ (c) $(-\infty, 2)$ (d) $\left[\dfrac{-1}{3}, 2\right)$

Q4: Let R = { (1, 1), (2, 2), (1, 2), (2, 1), (3, 3), (1,3), (3,1) } be a relation defined on the set A = { 1, 2, 3 }. Then R is :

(a) reflexive and transitive but not symmetric.
(b) symmetric and transitive but not reflexive.
(c) reflexive and symmetric but not transitive.
(d) reflexive, symmetric and transitive.

$yx - 3y = 2x + 1$
$x(y-2) = 1+3y$

Q5: The total number of onto functions from A = { a, b, c } to B = { 1, 5, 25, 125 } is:

(a) 0 (b) 3 (c) 81 (d) 64

Q6: Given $f(x) = \begin{cases} 3ax + b, & x > 1 \\ 11, & x = 1 \\ 5ax - 2b, & x < 1 \end{cases}$ is continuous at x = 1, then :

(a) a = 3, b = –2 (b) a = –3, b = 2 (c) a = – 3, b = – 2 (d) a = 3, b = 2

Q7: The derivative of y = $\sec^2 \sqrt{5+3x}$ wrt x is given by :

(a) $\frac{3 \sec\sqrt{5+3x}.\ \tan\sqrt{5+3x}}{\sqrt{5+3x}}$ (b) $\frac{3 \sec^2\sqrt{5+3x}.\ \tan\sqrt{5+3x}}{\sqrt{5+3x}}$

(c) $\frac{5 \sec\sqrt{5+3x}.\ \tan^4\sqrt{5+3x}}{\sqrt{5+3x}}$ (d) $\frac{2\sqrt{3} \sec\sqrt{5+3x}.\ \tan^4\sqrt{5+3x}}{\sqrt{5+3x}}$

Q8: The derivative of y = $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ wrt $z = \sqrt{1-x^2}$ at $x = \frac{1}{2}$ is given by:

(a) $-\frac{1}{2}$ (b) 2 (c) 4 (d) 0

Q9: The interval in which f (x) = $2x^3 + 9x^2 + 12x - 1$ is decreasing is:

(a) $(-\infty, -2)$ (b) $(-1, \infty)$ (c) $[-2, -1]$ (d) $[-1, 1]$

Q10: Which of the following statement is NOT true?

(a) An unbounded feasible region cannot provide the optimal value
(b) Optimal solution is at the corner points of the feasible region
(c) If an optimal value is at more than one point, then there are infinite optimal solutions
(d) A feasible solution must satisfy all linear constraints

Q11: The corner points of the bounded feasible region determined by a system of linear constraints are A (0, 3), B (1, 1) and C (3, 0).
Let z = px + qy, where p, q > 0.
The condition on p and q such that minimum of z occurs at B and C is :

(a) p = 2q (b) p = q (c) p = 3q (d) 2p = q

Q12: The probability distribution of a random variable X is given as :

$$P(X = x) = \begin{cases} kx^2, & \text{if } x = 1, 2 \text{ or } 3 \\ 2kx, & \text{if } x = 4, 5 \text{ or } 6 \\ 0, & \text{otherwise} \end{cases}$$

Then P (X = atleast 2) is given by:

(a) $\frac{1}{44}$ (b) $\frac{43}{44}$ (c) $\frac{39}{44}$ (d) $\frac{5}{44}$

Q13: A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that the probability that exactly one of them is selected is 0.6. The probability that B is selected is :

(a) 0.25 (b) 0.33 (c) 0.1 (d) 0.45

Q14: The angle between the lines $2x = 3y = -z$ and $\frac{1-x}{3} = \frac{z}{4}$, $y = 5$ is :

(a) $\cos^{-1}\left(\frac{33}{35}\right)$ (b) $\pi - \cos^{-1}\left(\frac{33}{35}\right)$

(c) $\pi - \cos^{-1}\left(\frac{6}{7}\right)$ (d) $\cos^{-1}\left(\frac{6}{7}\right)$

Q15: If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46$, then $|\vec{a}|$ is:

(a) 20 (b) 24 (c) 22 (d) 26

Q16: On evaluating, $\int \sec^{-1} x \, dx$ equals:

(a) $\sec^{-1} x . \tan^{-1} x + c$ (b) $x \sec^{-1} x + \log |x - \sqrt{x^2 - 1}| + c$

(c) $x \sec^{-1} x - \tan^{-1} x + c$ (d) $x \sec^{-1} x - \log |x + \sqrt{x^2 - 1}| + c$

Q17: On evaluating, $\int \frac{dx}{x\sqrt{9(\log x)^2 - 25}}$ equals :

(a) $\frac{1}{9} \sin^{-1}\left(\frac{5\log x}{3}\right) + c$ (b) $\frac{1}{10} \log \left|\frac{3\log x - 5}{3\log x + 5}\right| + c$

(c) $\frac{1}{3} \sin^{-1}\left(\frac{3\log x}{5}\right) + c$ (d) $\frac{1}{3} \log \left|\log x + \sqrt{9(\log x)^2 - 25}\right| + c$

Q18: $\int_{-1}^{1} \sin^5 x . \cos^6 x \, dx$ equals:

(a) 0 (b) 1 (c) 2 (d) π

ASSERTION – REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true

Q19: Assertion (A): If A is a 3 x 3 matrix and | A | = 4, then | adj A | = 64
Reason (R): For any square matrix P of order n, | adj P | = $|P|^{n-1}$

Q20: Assertion (A) : If $\vec{p} = 5\hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{q} = \hat{i} + 3\hat{j} - 5\hat{k}$ are two vectors such that $(\vec{p} + \vec{q})$ and $(\vec{p} - \vec{q})$ are perpendicular, then $\lambda = \pm 1$.
Reason (R): If $\vec{a}$ and $\vec{b}$ are two non – zero vectors, then $(\vec{a} + \vec{b}) . (\vec{a} - \vec{b}) = |\vec{a} + \vec{b}|^2$

SECTION B

Q21: Evaluate $y = \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) + 2\tan^{-1}(-1)$

OR

Evaluate $y = \sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$.

Q22: Find $\frac{dy}{dx}$ given that $(\cos x)^y = (\cos y)^x$

Q23: Evaluate $\int \frac{x.e^x}{(x+1)^2}\,dx$ **OR** $\int \frac{dx}{5+3\sin^2 x}\,dx$

Q24: There are two bags A and B. Bag A contains 3 white and 4 red balls whereas bag B contains 4 white and 3 red balls. Three balls are drawn at drawn at random (without replacement) from one of the bags and are found to be 2 white and one red. Find the probability that these were drawn from bag B.

OR

A black and a red dice are rolled together. Find the conditional probability of obtaining the sum 8, given that the red dice resulted in a number less than 4.

Q25: Find the shortest distance between the lines $\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\,(2\hat{i} + 3\hat{j} + 4\hat{k})$

and $\vec{r} = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) + \mu\,(4\hat{i} + 6\hat{j} + 8\hat{k})$

SECTION C

Q26: Let $\mathbb{N}$ be the set of all natural numbers and let R be a relation on $\mathbb{N} \times \mathbb{N}$ defined by (a,b) R (c,d) $\Leftrightarrow$ ad = bc, for all (a,b), (c,d) $\in \mathbb{N} \times \mathbb{N}$.

Show that R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$.

Also, find the equivalence class of [(2, 6)].

OR

Let f be a function defined as $f: \mathbb{R}^+ \to (-7, \infty)$ such that $f(x) = 2x^2 + 6x - 7$.

Show that f is one – one and onto.

Q27: If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 - xA + yI = 0$. Hence, find A^{-1}.

OR

Find the matrix P satisfying the equation $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} P \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$

Q28: Evaluate $\int \frac{2x}{(x^2+1)(x^2+4)}\,dx$

Q29: Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{\sin x + \cos x}$ **OR** $\int_0^{\pi} \log(1+\cos x)\,dx$

Q30: Solve $\frac{dy}{dx} + 2y = \cos 3x$ **OR** $(5xy - 2x^2)\,dy = (x^2 + 8y^2)\,dx$

Q31: Solve the following linear programming problem graphically :

Max $z = 12x + 16y$ subject to the linear constraints

$x + y \leq 1200$, $2y \leq x$, $x < 600 + 3y$, x and $y \geq 0$

SECTION D

Q32: An open box with a square base is to be made out of a quantity of cardboard of area c^2 sq. units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

Q33: In a box, 5 bad oranges are accidentally mixed with 20 good ones. If 3 oranges are drawn one by one successively with replacement.

(i) Find the probability distribution of number of bad oranges drawn.

(ii) Find the probability of drawing atmost one ~~orange.~~ bad orange

(iii) Find the mean of the probability distribution.

Q34: Two cars A and B are travelling along the straight roads $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ respectively.

A man standing at the point T (3, 2, − 1) wants to trace a path (L) perpendicular to both these roads.

Find the equation of this path L.

Also, find a point P on this path (L) whose distance from a flag – pole at Q (1, − 2, − 3) is $2\sqrt{2}$ units.

SECTION E

Q35: The sum of three numbers is – 1. If we multiply the second number by 2, third number by 3 and add them, we get 5. If we subtract third number from the sum of first and second number, we get – 1. Represent the information by a system of linear equations. Also, find the numbers, using Matrix Method.

Q36: **(i)** Find $\frac{dy}{dx}$ given $y = \cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

(ii) If $x = a \sin 2t\,(1 + \cos 2t)$ and $y = b \cos 2t\,(1 - \cos 2t)$, find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$

Q37: Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$

(i) Find a vector $\vec{d}$ which is perpendicular to both $\vec{b}$ and $\vec{c}$ and $\vec{a} \cdot \vec{d} = 21$

(ii) If $\vec{b}$ and $\vec{c}$ represent two adjacent sides of a triangle, then find its area.

Q38: Using integration, find the area of the region bounded by the curve $y^2 = 2x$ and the straight line $x - y = 4$.

OR

Using integration, find the area of the region bounded by $y = \sqrt{5 - x^2}$ and $y = |x - 1|$

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