ST.ANTHONY'S SR.SEC.SCHOOL
PRE BOARD EXAMINATION I
2023-24
CLASS XII
MATHEMATICS (SET B)

TIME : 3 HOURS **MAX.MARKS: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D, and E. Each section is compulsory.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer** (VSA)-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer** (SA)-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer** (LA)-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.

Section A
(Multiple Choice Questions)
Each question carries 1 mark

1. If $A = \begin{bmatrix} 2 & k \\ 3 & -3 \end{bmatrix}$ is a non-singular matrix, then
   (a) $k = 2$ (b) $k = -2$ (c) $k \neq 2$ (d) $k \neq -2$
2. Direction ratios of a line joining the points A(0,1,6) and B(2,3,4) so directed that line AB makes acute angle with the x-axis are
   (a) 1,1,1 (b) 1,1,-1 (c) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$ (d) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$
3. The point which does not lie in the half plane of $3x - y < 4$ is
   (a) (0, 0) (b) (1, 1) (c) (1,-2) (d) (1, 2)
4. If points A, B, C are collinear, then
   (a) $|\overrightarrow{AB}| = |\overrightarrow{AC}|$ (b) $\overrightarrow{AB} = \overrightarrow{AC}$ (c) $\overrightarrow{AB}.\overrightarrow{AC} = 0$ (d) $\overrightarrow{AB} \times \overrightarrow{AC} = 0$
5. The value of k for which the function $f(x) = \begin{Bmatrix} kx.\,cosec\ 3x, x \neq 0 \\ 2\,, x = 0 \end{Bmatrix}$ is continuous at $x = 0$ is
   (a) 3 (b) 0 (c) 6 (d) $\frac{3}{2}$
6. If $f(x)$ is derivative of $g(x)$, then antiderivative of $f(x)$ is
   (a) $g(0)$ (b) $g(x)$ (c) $g(x) + C$ (d) $2g(x)$
7. If m and n respectively, are the order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} = x$, then value of $3n - m$ is
   (a) 3 (b) 6 (c) 2 (d) 4
8. If $A = [a_{ij}]$ is a symmetric matrix of order n, then
   (a) $a_{ij} = \frac{1}{a_{ij}}\ \forall\, i, j$ (b) $a_{ij} \neq 0\ \forall\, i, j$ (c) $a_{ij} = 0\ for\ i \neq j$ (d) $a_{ij} = a_{ji}\ \forall i, j$

9. Vectors $\vec{a}$ and $\vec{b}$ represent diagonals of a rhombus, then

(a) $\vec{a}.\vec{b} = 0$ (b) $\vec{a} \times \vec{b} = \vec{0}$ (c) $|\vec{a}| = |\vec{b}|$ (d) $\vec{a} = \vec{b}$

10. The value of $\int_0^2 \frac{1}{x^2+4} dx$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{16}$

11. Corner points of the feasible region of an LPP whose objective function is Z = px +qy are A(3, 2), B(0,4), C(2, 1) and D(1,6). If value of Z is same at the points A and D, then relation between p and q is

(a) p = 2q (b) 3p = 2q (c) 2p = q (d) p +2q = 0

12. If $A = \begin{bmatrix} 2x & 4 \\ 3 & x \end{bmatrix}$ and $|B| = 12$. The value(s) of x if $|A| = |B|$ is (are):

(a) 12 (b) ± 12 (c) $\pm 2\sqrt{3}$ (d) $\sqrt{12}$

13. If A is a square matrix of order 3 such that $|A.Adj\ A| = 64$, then $|A|$ is

(a) ± 4 (b) 4 (c) ± 8 (d) 8

14. Given two independent events A and B such that P(A) = 0.4 and $P(\bar{B}) = 0.7$ then $P(\bar{A} \cap B) =$

(a) 2.8 (b) 0.28 (c) 0.18 (d) 0.42

15. The general solution of the differential equation $\frac{dy}{dx} = \log x$ is

(a) $x \log x - x + C$ (b) $-\frac{1}{x} + C$ (c) $\frac{1}{x} + C$ (d) $x \log x + C$

16. If A is a square matrix of order 3 and $|A| = -4$, then $|A^2|$ is

(a) -4 (b) 16 (c) -16 (d) -8

17. If $y = \sin^{-1} x$, then $\sqrt{1-x^2}\, y_1$ is equal to

(a) 1 (b) $\sin^{-1} x$ (c) $\frac{1}{\sqrt{1-x^2}}$ (d) $\frac{1}{1-x^2}$

18. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}, \vec{a} \neq \vec{b}, \vec{b} \neq \vec{0}, \vec{c} \neq \vec{0}$ then

(a) $\vec{a}.\vec{b} = \vec{b}.\vec{c}$ (b) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c}$ (c) $|\vec{a}| + |\vec{b}| + |\vec{c}| = 0$ (d) $|\vec{a} + \vec{b}| = |\vec{c}|$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A)** : $\sin^{-1}(1.01)$ exists.

Reason (R) : Domain of $\sin^{-1} x$ is $x \in [-1,1]$

20. **Assertion (A)** : Line $\vec{r} = (\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ passes through a fixed point (1, -1, -1)
Reason (R) : In vector equation of a line $\vec{r} = \vec{a} + \lambda\vec{b}$, $\vec{a}$ represents position vector of fixed point through which line passes.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21. Given $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = p\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ and $\vec{c} = 5\hat{\imath} - \hat{k}$, find the value of p such that $\vec{a} + 2\vec{c}$ is perpendicular to $\vec{b}$
22. Evaluate $\tan^{-1}\left[tan\left(\frac{3\pi}{4}\right)\right]$

OR

Show that the function $f: R - \{1\} \rightarrow R - \{1\}$ *defined as* $f(x) = \frac{x}{x-1}$ *is injective*
23. If $y = Ae^{7x} + Be^{-7x}$, prove that $y_2 - 49y = 0$
24. Show that $|\vec{b}|\vec{a} + |\vec{a}|\vec{b}$ *and* $|\vec{b}|\vec{a} - |\vec{a}|\vec{b}$ are orthogonal vectors.
25. The circular waves are moving at the rate of 0.7 cm/s. At what rate the area is changing when radius of circular wave is 7cm?

SECTION C

This section comprises of short answer type questions (SA) of 3 marks each

26. Evaluate $\int \frac{1}{x^3-x^2-x+1} dx$
27. Solve the differential equation $\tan y \, sec^2x \; dx + tanx \; sec^2y \; dy = 0$
28. Two defective bulbs are accidently mixed with 6 good ones. If three bulbs are drawn at random, find the mean number of defective bulbs drawn.
29. Evaluate $\int_3^8 \frac{\sqrt{11-x}}{\sqrt{x}+\sqrt{11-x}} dx$

OR

Evaluate $\int_{-1}^{3} |2x + 1| dx$
30. Solve the following linear programming problem:
Maximize $Z = x + y$
Subject to the constraints $2x + y \leq 50, x + 2y \leq 40, x \geq 0, y \geq 0$
31. Evaluate $\int \frac{dx}{\sqrt{x^2+x+1}}$

SECTION D

This section comprises of long answer-type questions (LA) of 5 marks each

32. Let $A = \{1, 2, 3, 4, 5, 6\}$ and R be a relation in $A \times A$ defined as (a,b) R (c,d) iff a + d = b + c for $(a, b), (c, d) \in A \times A$. Prove that R is an equivalence relation, also obtain equivalence class $[(1, 3)]$.
33. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}; z = 2$. Are the line intersecting?
34. Solve the system of equations:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4; \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1; \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

35. Find the area bounded by the x-axis, circle $x^2 + y^2 = 32$ and the line $y = x$ in the first quadrant.

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with sub-parts.)

Read the texts carefully and answer the following questions:

36. A house is being constructed and a lot of planning is put into it. Now a person is confused about the window. He wants window in the form of rectangle surmounted by a semicircle and also perimeter of 10 m is suggested. If radius of semicircular portion is r m and height of window is x m, then

(i) Write a relation between x and r.
(ii) Represent the area in terms of r.
(iii) Find the critical point, with respect to area, in terms of r.

37. As board examinations are approaching near, students are working hard to score well and they all study together but independently. For one particular problem the probability of solving it correctly by A, B and C are $\frac{1}{2}, \frac{1}{3}$ *and* $\frac{1}{5}$ *respectively*. If all the three try, then

(i) What is the probability that exactly two will solve the problem?

(ii) What is the probability that the problem will be solved?

38. A company has two plants to manufacture TVs. The first plant- plant A manufactures 70% of the TVs and the rest are manufactured by second plant. 80% of the TVs manufactured by the first plant are rated of standard quality, while that of the second plant 90% are of standard quality. One TV is selected at random.

(i) Find the probability that the TV is of standard quality, given that it was made by first plant.
(ii) It is given that the TV is not manufactured by first plant. Then what is the probability that it is rated of standard quality?
