Pre Board Examination - 2023-24

Class : XII

Paper : Maths

Time : 3 hrs.

M.M. : 80

General Instructions :

i) There are 5 sections A, B, C, D & E. Each section is compulsory.

ii) Section-A has 18 MCQs and 2 Assertion-Reason based questions of 1 marks each.

iii) Section-B has 5 (VSA type) questions of 2 marks each.

iv) Section-C has 6 (SA type) questions of 3 marks each.

v) Section-D has 4 (LA type) questions of 5 marks each.

vi) Section-E has 3 (case-based study) questions of 4 marks each with sub parts.

(Section-A)

1. The value of $x - y + z$ from the following equation is

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

a) -3 b) -1 c) 1 d) 3

2. If A be a 3 x 3 square matrix such that $A\,(\text{Adj } A) = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ then the value of $|\text{Adj } A|$ is

a) 5 b) 25

c) 125 d) 625

3. If A and B are symmetric matrices of same order, then (AB' - 2BA') is a

a) Skew symmetric matrix b) Symmetric matrix

c) Neither symmetric matrix nor skew-symmetric d) none

4. In the interval (1, 2) the function $f(x) = 2|x-1| + 3|x-2|$ is

a) Strictly increasing b) Strictly decreasing

c) Neither increasing & decreasing d) Remains constant

5. If the set A contains 5 elements and the set B contains 6 elements then the number of both one-one and onto mapping from A to B is

a) 720 b) 120 c) 30 d) 0

6. The sum of order & degree of the differential equation :

$$\frac{d^3y}{dx^3} = \left(1 + \frac{dy}{dx}\right)^5 \text{ is} \qquad \frac{d^3y}{dx^3}$$

a) 3 b) 4 c) 5 d) 8

7. The solution set of the inequation $3x + 2y > 3$ is

a) half plane containing the origin b) half plane not containing the origin

c) The point being on the line $3x + 2y = 3$ d) None of these

8. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two sides AB and AC, respectively of ΔABC. The length of median through A is

a) $\sqrt{34}/2$ b) $\sqrt{48}/2$ c) $\sqrt{18}$ d) $\sqrt{52}$

9. The value of $\int_{-\pi/3}^{\pi/3} x^3 \sin^4 x \, dx$ is

a) 0 b) $\pi/2$ c) π d) $\pi/4$

10. $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = KA$, then K is equal to

a) 19 b) 1/19 c) -1/19 d) -19

11. The projection of $\lambda\hat{i} + \hat{j} + 4\hat{k}$ on $2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units, then the value of λ is equal to

a) -9 b) -5 c) 5 d) 9

12. The corner points of the feasible regions for the LPP are (0, 2), (3, 0) (6, 0) (6, 8) and (0, 5). Let the objective function is $Z = 4x + 6y$, then the minimum value of the objective function occurs at

$\frac{2\lambda + 6 + 12}{\sqrt{4+36+9}}$

a) (0, 2) only b) (3, 0) only

c) The mid point on the line joining (0, 2) & (3, 0)

d) Any point on the line joining (0, 2) & (3, 0)

13. $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ & $B^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$ and $(AB)^{-1}$ is equal to

a) $\begin{bmatrix} 15 & -19 \\ -26 & 33 \end{bmatrix}$ b) $\begin{bmatrix} 11 & -14 \\ -29 & 37 \end{bmatrix}$ c) $\begin{bmatrix} 37 & 14 \\ 29 & 11 \end{bmatrix}$ d) $\begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

14. The value of λ for which the vectors $3\hat{i} - 6\hat{i} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is

a) 2/3 b) 3/2 c) 5/2 d) 2/5

15. The integrating factor of the differential equation $\frac{dy}{dx}(x \log x) + y = 2 \log x$ is

a) e^x b) $\log x$ c) $\log(\log x)$ d) x

16. The function $f(x) = x^x$ has a stationary point at

a) $x = e$ b) $x = \frac{1}{e}$ c) $x = 1$ d) $x = \sqrt{e}$

17. The direction ratios of the line $3x + 1 = 6y - 2 = 1 - z$ are

a) 3, 6, 1 b) 3, 6, -1 c) 2, 1, 6 d) 2, 1, -6

18. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. If the event of their solving the problem are independent then the probability that problem will be solved is :

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$

Assertion-Reason Based Question

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

a) Both (A) & (R) are true and (R) is the correct explanation of the (A).

b) Both (A) & (R) are true but (R) is not the correct explanation of the (A).

c) (A) is true and (R) is false.

d) (A) is false and (R) is true.

19. Let f(x) be a polynomial function of degree 6 such that

$\frac{d}{dx}f(x) = (x - 1)^3 (x - 3)^2$, then

Assertion (A) : f (x) has a minimum at $x = 1$

Reason (R) : When $\frac{d}{dx}(f(x) < 0$, for all $x \in (a-h, a)$ and $\frac{d}{dx}(f(x) > 0$, for all $x \in (a + h, a)$;

where 'h' is an infinitesimally small positive quantity then f(x) has a minimum at $x = a$, provided f(x) is cont. at $x = 4$.

20. Assertion (A) : The relation $f : \{1, 2, 3, 4\} \rightarrow \{x, y, z, p\}$ defined by $f = \{(1, x), (2, y), (3, z)\}$ is a bijective function.

Reason (R) : The function

$f : \{1, 2, 3\} \rightarrow \{x, y, z, P\}$ such that $f = \{(1, x) (2, y) (3, z)\}$ is one-one.

(Section-B)

21 Find the value of $\sin^{-1}\left(\cos \frac{33\pi}{5}\right)$

22. Find the maximum and minimum values of the function given by $f(x) = 5 + \sin 2x$

23. The matrix $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$ is the sum of symmetric matrix B and a skew-symmetric matrix c. Find C

24. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance 5 units from the point (1, 3, 3).

25. If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$, then find the angle between $\vec{a}$ & $\vec{b}$.

(Section-C)

26. Solve the following linear programming problem graphically :

Minimize $Z = 3x + 9y$

Subject to the constraints

$x + 3y \le 60$

$x + y \ge 10$

$x \le y$

$x \ge 0,\ y \ge 0$

27. Solve the integral :

$$I = \int \frac{3x+5}{x^2 + 4x + 7}\, dx$$

OR

Solve : $\int \frac{dx}{(1 + x^2)(2 + x^2)}$

28. Solve the differential equation

$2ye^{x/y}\, dx + (y - 2xe^{x/y})\, dy = 0$

OR

Solve the differential equation

$$\frac{dy}{dx} - 3y \cot x = \sin 2x$$

29. Draw a rough sketch of the curve $y = |x + 1|$, $x = -3$, $x = 3$ and find the area of the region bounded by them using integration.

30. If $x = ae^{\theta}(\sin\theta - \cos\theta)$ and $y = ae^{\theta}(\sin\theta + \cos\theta)$, then find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$

31. Suppose a girl throws a die. If she gets 1 ~~to~~ or 2, she tosses a coin three times and notes the no. of tails. If she gets 3, 4, 5 or 6 , she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one tail, what is probability that she threw 3, 4, 5 or 6.

(Section-D)

32. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -3 \\ -3 & 2 & -4 \end{bmatrix}$, find A^{-1} and hence solve the system of equations

$x + 2y - 3z = -4,\ 2x + 3y + 2z = 14\ \ 3x - 3y - 4z = -15$

33. Find the shortest distance between the following lines :

$\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda\,(\hat{i} - 2\hat{j} + \hat{k})$ and

$\vec{r} = (-1 + 7\mu)\,\hat{i} + (-1 - 6\mu)\,\hat{j} + (-1 + \mu)\,\hat{k}$

34. Evaluate : $\displaystyle\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$

OR

$\displaystyle\int_0^{5} (|x-1| + |x-5|)\, dx$

35. Show that the relation R on the set N x N defined by (a, b) R (c, d). If ad (b + c) = bc (a + d). Show that R is an equivalence relation.

(Section -E)

Case Study Based Questions :

This section comprises of 3 case-study based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each.)

36. Read the following passage and answer the questions given below:

In an Office three employees James, Sophia and Oliver process incoming copies of a certain form. James processes 50% of the forms, Sophia processes 20% and Oliver the remaining 30% of the forms James has an error rate of 0.06, Sophia has an error rate of 0.04 and Oliver has an error rate of 0.03

Based on the above information, answer the following questions.

(i) Find the probability that Sophia processed the form and committed an error. (1)

(ii) Find the total probability of committing an error in processing the form. (1)

(iii) The manager of the Company wants to do a quality check. During inspection, he select a form at random from the days output of processed form. If the form selected at random an error, find the probability that the form is not processed by James. (2)

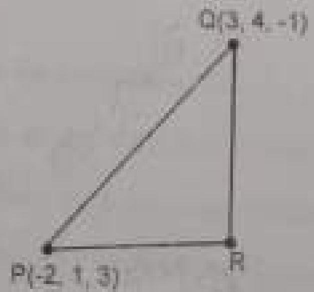
OR

(iii) Let E be the event of committing an error in processing the form and let E_1, E_2, and E_3 be the events that James, Sophia and Oliver processed the form. Find the value of

$\sum_{i=1}^{3} P(E_i|E)$ (2)

31. Answer the questions based on the given information.

The flight path of two airplanes in a flight simulator game are shown below. The coordinates of the airports P and Q are given.



Airplane 1 flies directly from P to Q.

Airplane 2 has a layover at R and then flies to Q.

The path of Airplane 2 from P to R can be represented by the vector $5\hat{i} + \hat{j} - 2\hat{k}$.

(Note : Assume that the flight path is straight and fuel is consumed uniformly throughout the flight).

i) Find the vector that represents the flight path of Airplane 1. Show your steps. 1

ii) Write the vector represents the path of Airplane 2 from R to Q. Show your steps. 1

iii) What is the angle between the flight paths of Airplane 1 and Airplane 2 just after takeoff ? Show your work. 2

OR

iii) Consider that Airplane 1 started the flight with a fuel tank. 2

Find the position vector of the point where a third of the fuel runs out if the entire fuel is required for the flight. Show your work.

38 Read the following passage and answer the questions given below:

The relation between the height of the plant ('y' in cm) with respect to its exposure to the sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to the sunlight, for $x \leq 3$

(i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight. (2)

(ii) Does the rate of growth of the plant increase or decrease in the first three days? What will be the height of the plant after 2 days? (2)