No. Of Printed Pages: 6

# CLASS XII
## POST MID - TERM EXAM (2023 – 24)
## MATHEMATICS (041)
## Set A2

M.M.80

Time :3hrs.

**General Instructions:**

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 case-based assessment (4 marks) with sub parts.
7. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

## SECTION A

Q1) A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". The value of $P(A/B)$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) 0

Q2) The straight line $\frac{x-5}{2} = \frac{y-3}{1} = \frac{z+1}{0}$ is:

(a) parallel to x-axis (b) parallel to y-axis

(c) parallel to z-axis (d) perpendicular to z-axis

Q3) If $(\hat{\imath} + 3\hat{\jmath} + 8\hat{k}) \times (3\hat{\imath} - \lambda\hat{\jmath} + \mu\hat{k}) = \vec{0}$, then the value of $\lambda$ is

(a) 27 (b) 9 (c) –9 (d) – 1

Q4) If m and n are the order and degree, respectively of the differential equation

$5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$, then the value of m + n is

(a) 1 (b) 2 (c) 3 (d) 4

Q5) The number of corner points of the feasible region determined by the constraints $x - y \geq 0, 2y \leq x + 2, x \geq 0, y \geq 0$ *is*

(a)2 (b) 3 (c) 4 (d) 5

Q6) The value of $\alpha$ when the projection of $\alpha\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units is

(a) 5 (b) – 7 (c) – 5 (d) 7

Q7) A line makes equal angles with the three co-ordinate axes. The direction cosines of this line are

(a) $\pm(1,1,1)$ (b) $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (c) ) $\pm\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (d) ) $\pm\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

Q8) The corner points of the feasible region determined by the system of linear constraints are (2, 4), (6, 7), (0, 8). Let $Z = 3x - 5y$ be the objective function. The difference between maximum and minimum value of Z is

(a) 26 (b) 14 (c) 17 (d) 23

Q9) The general solution of the differential equation $y\,dx - x\,dy = 0$ (where $x, y > 0$), is of the form

(a) $xy = c$ (b) $x = cy^2$ (c) $y = cx$ (d) $y = cx^2$

(where "c" is an arbitrary positive constant of integration)

Q10) For any integer n, the value of $\int_0^{\pi} e^{\sin^2 x} \cos^3 (2n+1)x \, dx$ is

(a) $-1$ (b) 0 (c) 1 (d) 2

Q11) The value of $\hat{\imath}.(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{\imath} \times \hat{k}) + \hat{k}.(\hat{\imath} \times \hat{\jmath})$ is

(a) 1 (b) $-1$ (c) 0 (d) 3

Q12) Given that A is a square matrix of order 3 and $|A| = -2$, then $|Adj(2A)|$ is equal to

(a) $-2^6$ (b) 4 (c) $-2^8$ (d) $2^8$

Q13) The value of 'k' for which the function $f(x) = \begin{cases} kx^2, & if\ x < 2 \\ (3x+5), & if\ x \geq 2 \end{cases}$ is continuous at $x = 2$ is

(a) $\frac{-11}{4}$ (b) 11 (c) $\frac{4}{11}$ (d) $\frac{11}{4}$

Q14) If $y = \sqrt{a^2 - x^2}$, then $y\frac{dy}{dx}$ is:

(a) 0 (b) x (c) $-x$ (d) 1

Q15) If for a square matrix A, $A^2 - A + I = 0$, then $A^{-1}$ equals

(a) A (b) $A + I$ (c) $I - A$ (d) $A - I$

Q16) If the area of the triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 square units, then the values/s of k will be

(a) $\pm 3$ (b) 9 (c) $-9$ (d) 6

Q17) Find the cofactor of $a_{12}$ in the following: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

(a) $-46$ (b) 46 (c) 0 (d) 1

Q18) If $\begin{bmatrix} 0 & a & 1 \\ -2 & b & 1 \\ -1 & c & 0 \end{bmatrix}$ is a skew symmetric matrix, the value of $(a + b + c)^3$ is

(a) 0 (b) 1 (c) 2 (d) 3

**ASSERTION-REASON BASED QUESTION:**

In the following question, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

(2, 4), minimum v.

Q19) **Assertion(A):** The function $f(x) = \begin{cases} 3 - x^2, & if\ x > 1 \\ x^2 + 1, & if\ x \leq 1 \end{cases}$ is not differentiable at x = 1.

**Reason (R):** A real function continuous at x = c is always differentiable as well at x = c. (c)

Q20) **Assertion(A):** The relation R = {(a, b): a ≤ $b^2$ } on the set R of real numbers is reflexive.
**Reason (R):** A relation R on a set A is said to be reflexive if a R a for all a ∈ $A$. (d)

## SECTION B

Q21) Evaluate $\int e^x \left(\frac{\sin 4x - 4}{1 - \cos 4x}\right) dx$

Q22) Find the value of: $\sin^{-1}(cos\left(\frac{33\pi}{5}\right)$

OR

Draw the graph of $y = \cos^{-1} x, where\ x \in [-1, 1]. Also, find\ its\ range$

Q23) The volume of a cube is increasing at the rate of 9 $cm^3$/s. How fast is its surface area increasing when the length of an edge is 10 cm?

Q24) Find the maximum profit that a company can make, if the profit function is given by $P(x) = 72 + 42x - x^2$, where x is the number of units and P is the profit in rupees.

Q25) Find the values of x for which the function $y = [x(x-2)]^2$ *is an increasing function.*

OR

Show that the function f defined by $f(x) = (x - 1)e^x + 1$ is an increasing function for all x > 0.

## SECTION C

Q26) Solve the following LPP graphically:
Minimize Z = 10x + 4y subject to the constraints
$4x + y \geq 80\ , 2x + y \geq 60\ , x \geq 0\ and\ y \geq 0.$

OR

Maximize $Z = 80x + 120y$ subject to the constraints
$3x + 4y \leq 60, x + 3y \leq 30, x \geq 0\ and\ y \geq 0.$

Q27) Probability of solving a specific problem independently by A and B are $\frac{1}{4}$ and $\frac{1}{2}$ respectively. If both try to solve the problem independently, find the probability that
(i) the problem is solved (ii) exactly one of them solves the problem

Q28) Evaluate $\int \frac{3x+1}{(x-1)^2(x+3)} dx$

Q29) Evaluate: $\int_0^{\pi/4} \frac{(\sin x + \cos x)\, dx}{9 + 16 \sin 2x}$

OR

Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + cos^2 x} dx$

Q30) If $y = 3\cos(logx) + 4\sin(logx), prove\ that\ x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$

Q31) Solve $\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\} y\,dx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\} x\,dy.$

OR

Find the general solution of the differential equation $\frac{dy}{dx} + y\cot x = 2x + x^2\cot x$

## SECTION D

Q32) Draw a rough sketch and using integration, find the area of the following region: $\{(x,y): \frac{x^2}{9} + \frac{y^2}{4} \le 1 \le \frac{x}{3} + \frac{y}{2}\}$.

Q33) Let R be a relation on N x N defined by (a,b) R (c,d) $\Leftrightarrow$ ad = bc for all (a, b), (c, d) $\in$ **N x N**. Show that R is an equivalence relation on N x N, where **N** is the set of natural numbers. Also, find the equivalence class of (1,3).

OR

Consider $f: R_0^+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$, where $R_0^+$ is the set of all non-negative real numbers. Show that $f$ is a bijective function.

Q34) $If\ A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, find\ A^{-1}$. Hence, solve the system of equations:

$2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3$

Q35) Find the vector equation of the line passing through the point (1, 3, – 7) and perpendicular to the two lines

$$\frac{x-6}{3} = \frac{y+23}{-16} = \frac{z-0}{7}\ and\ \frac{x-5}{3} = \frac{y+9}{8} = \frac{z+5}{-5}$$

OR

Find the shortest distance between the lines

$\vec{r} = 4\hat{\imath} - \hat{\jmath} + \lambda(\hat{\imath} + 2\hat{\jmath} - 3\hat{k})\ and\ \vec{r} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \mu(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$

Hence, obtain the acute angle between the lines.

## SECTION E

**This section comprises 3 case study/passage-based questions of 4 marks each. First two case study questions have 3 subparts (I), (II), & (III) of 1, 1 & 2 marks respectively. The third case study question has two sub parts (I) & (II) of 2 marks each respectively.**

### Case Study – 1

Q36) A student has to go to a particular place to give an online competitive examination comprising of Multiple-Choice questions with four choices each. For this purpose, the organization conducting the examination has to make perfect arrangements and students are expected to be well prepared. However, in the exam, a student might guess or copy or know the answer to each question. Let the probability that a student makes a guess be 1/3 and that he copies the answer be 1/6. Also, assume that for a student, who copies the answer, will be correct, the probability is $\frac{3}{4}$.

Based on the above information, answer the following questions.

(I) What is the probability that he knows the answer?

(II) If it is given that he guesses the answer, what is the probability that he answered it correctly? Justify your answer.

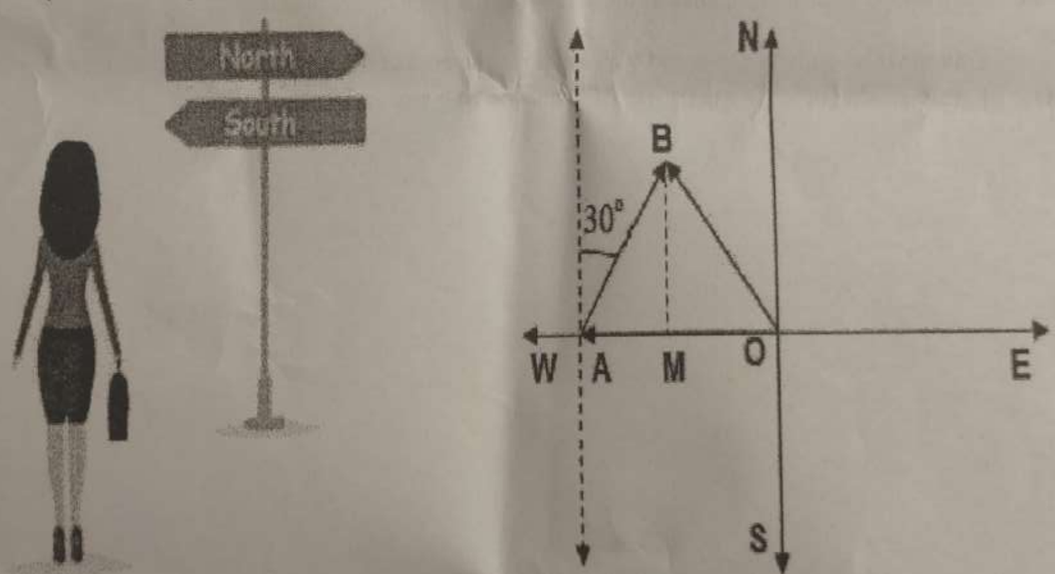
(III) What is the probability of answering the question correctly?

**OR**

What is the probability that he copied the answer given that his answer is correct?

**Case Study – 2**

Q37) Suhani walks 4 km towards west, then 3 km in a direction $30^0$ east of north and then she stops. The situation above has been depicted in the diagram as shown below, assuming that the girl starts her walk from the point O (taken as origin).




In the diagram, ON represents positive y-axis and North direction, OE represents positive x-axis and East direction. Similarly, OW is representing negative x-axis and West direction, whereas OS represents negative y-axis and South direction. Let OA = 4 km and AB = 3 km.
Based on the above information, answer the following questions.

(I) What is the vector $\overrightarrow{OM}$?

(II) Evaluate the dot product $\overrightarrow{OM}.\overrightarrow{MB}$

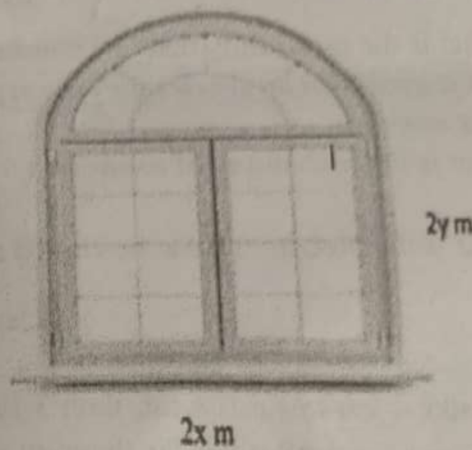
(III) What is the position vector of point B?

**OR**

Calculate the area of triangle OAB using vectors.

## Case Study – 3

Q38) The windows of a newly constructed building are in the form of a rectangle surmounted by a semi-circle. The perimeter of each window is 40m. Assume that $2x$ and $2y$ are length and breadth of rectangular part of the window (in metres) as shown in the figure.




Based on the above information, answer the following questions.

(I) Find the relation between $x$ and $y$ and express the area of the window in terms of $x$.

(II) Find the value of $x$ for which area of window will be maximum?