APEEJAY COMMON PRE-BOARD EXAMINATION

Class XII

Session 2023-24

Mathematics (Code-041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(Multiple Choice Questions)

Each question carries 1 mark

1. For any square matrix A, AA^T is a ______ matrix

   (a) Unit (b) Symmetric (c) Skew symmetric (d) Diagonal

2. The position vector of a point R which divides the line joining P(6,3,−2) & Q(3,1,−4) in the ratio 2:1 externally is

   (a) $\hat{i}+3\hat{j}-2\hat{k}$ (b) $3\hat{i}-\hat{k}$ (c) $-\hat{j}-6\hat{k}$ (d) $2\hat{i}+\hat{j}$

3. If A is a unit matrix of order n, then A(adj A) is

   (a) Zero matrix (b) Scalar matrix (c) Unit matrix (d) None of these

4. Let $f(x)=\dfrac{\sqrt{4+x}-2}{x}, x\neq 0$ be continuous at x = 0, then f(0) =

   (a) 1/4 (b) 1/2 (c) 2 (d) 3/2

5. Derivative of e^{x^3} with respect to $\log x$ is

   (a) $e^x e^3$ (b) $3x^2 e^{x^3}$ (c) $3x^2 e^{x^3}+3x^2$ (d) $3x^3 e^{x^3}$

6. If a and b are the degree and order of the differential equation $y=x\dfrac{dy}{dx}+\left(\dfrac{dy}{dx}\right)^{-2}$.

   The value of 2a - 3b is

   (a) 7 (b) -7 (c) 3 (d) -3

7. Solution to LPP, Minimize Z = - 3x + 2y, subject to $0\le x\le 4, 1\le y\le 6, x+y\le 5$ is

   (a) -10 (b) 0 (c) 2 (d) 10

8. If $\vec{a}=2\hat{i}+5\hat{j}+\hat{k}$ and $\vec{b}=4\hat{i}+m\hat{j}+n\hat{k}$ are colinear vectors, then m+ n =

   (a) 12 (b) 15 (c) 20 (d) 60

9. The value of the integral $\int_0^{\pi^2/4} \frac{\sin\sqrt{x}}{\sqrt{x}}dx$ is

(a) 2 (b) 1 (c) π (d) $\pi/2$

10. The area of triangle with vertices (-3,0), (3,0) and (0,k) is 9 sq units. The value of k is

(a) 9 (b) ± 3 (c) -3 (d) 6

11. The objective function of a Linear Programming Problem is

(a) a constant function

(b) a linear function

(c) a quadratic function

(d) a relation between x and y.

12. If A and B are square matrices of order 3 such that $|A| = -1, |B| = 3$, then $|3AB| =$

(a) – 9 (b) – 81 (c) – 27 (d) 81

13. If value of a third order determinant is 11, then the value of the square of the determinant formed by the cofactors will be

(a) 11 (b) 121 (c) 1331 (d) 14641

14. The probability that a leap year will have 53 Fridays or 53 Saturdays

(a) 3/7 (b) 2/7 (c) 4/7 (d) 1/7

15. The general solution of the differential equation $xy\frac{dy}{dx} - 1 = 0$ is

(a) $xy = \log x + c$ (b) $\frac{x^2}{2} = \log y + c$ (c) $\frac{y^2}{2} = \log x + c$ (d) $x^2 = \log y + c$

16. If $\sec\left(\frac{x-y}{x+y}\right) = a$, then $\frac{dy}{dx} =$

(a) -y/x (b) x/y (c) -x/y (d) y/x

17. If $\vec{a}\cdot\vec{b} = \sqrt{3}|\vec{a}\times\vec{b}|$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

18. How many lines through the origin make equal angles with the coordinate axes in three dimensional space?

(a) 1 (b) 4 (c) 2 (d) 8

ASSERTION – REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** Function $f: R \to R$, given by $f(x) = x^3$ is injective.

Reason (R): A function $f: A \to B$ is injective, if $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$

20. **Assertion(A):** The pair of lines given below intersect each other.
$\vec{r} = \hat{i} - \hat{j} + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = 2\hat{i} - \hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$
Reason(R): Two lines intersect each other, if they are not parallel and shortest distance between them is 0.

SECTION B

(This section comprises of very short answer type-questions (VSA) of 2 marks each)

21. Find the value of $\tan(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2})$.

OR

Find the domain of the function $y = \cos^{-1}|x-1|$.

22. If $y = \cos^{-1} x$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

23. Find a vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes an angle $\frac{\pi}{4}, \frac{\pi}{2}$ with y & z axes respectively.

OR

Find the value of p so that the lines, $\frac{4-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.

24. If $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$ then prove that $\frac{dy}{dx} = -\frac{y}{x}$

25. Find a unit vector perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Evaluate $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$

27. There are three coins. One is a two headed coin, another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows head, what is the probability that it is the two headed coin?

OR

Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

28. Evaluate $\int_{-1}^{2} |x^3 - x| dx$.

OR

Evaluate $\int_0^{\pi/4} \log(1+\tan x)dx$

29. Solve the differential equation $(\tan^{-1} y - x)dy = (1+y^2)dx$.

OR

Solve the differential equation: $(3xy + y^2)dx + (x^2 + xy)dy = 0$

30. Solve the following Linear Programming Problem graphically

Maximize $z = 8x+y$

Subject to constraints:

$x + y \le 40,$

$2x + y \le 60,$

$x \ge 0, y \ge 0.$

31. Evaluate $\int \frac{5x}{(1+x)(x^2+9)}dx$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. (i) Find the area of the region in the first quadrant enclosed by x-axis, x= 3 and the curve $y^2 = 4x$.

(ii) Find the area of the region in the first quadrant enclosed by y-axis, y= 3 and the curve $y^2 = 4x$.

33. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

Hence solve the system of equations

$x + 2y - 3z = -4, \quad 2x + 3y + 2z = 2, \quad 3x - 3y - 4z = 11.$

34. Find the shortest distance between the lines whose vector equations are

$\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and $\vec{r} = (-4\hat{i} - 2\hat{k}) + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$

OR

Find the vector and cartesian equation of the line which is perpendicular to the lines, $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1,1,1). Also find the angle between the given lines.

35. Give an example of a relation which is

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

OR

Show that the function $f: R \to (-1,1)$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$ is one-one and onto function.

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub of marks 1, 1, 2 respectively. The third case study question has two sub marks each.)

36. **Case-Study 1:** Read the following passage and answer the questions given below:

An owner of an electric bike rental company have determined that if they charge customers ₹ x per day to rent a bike, where $50 \le x \le 200$, then number of bikes 'n', they rent per day can be shown by linear function $n(x) = 2000 - 10x$. If they charge ₹ 50 per day or less, they will rent all their bikes. If they charge ₹200 or more they will not rent any bike.

(i) Express total revenue R as a function of x. [1]

(ii) Find the number of bikes rented per day if $x = 105$. [1]

(iii) For what value of x, maximum revenue is collected? How much is the maximum revenue? [2]

37. **Case-Study 2:** One day Shweta's Mathematics teacher was explaining the topic 'Increasing and Decreasing functions' in the class. He explained about different terms like stationary points, turning points etc. He also explained about the conditions for which, a function will be increasing or decreasing. He took examples of different functions to make it more clear to the students. He then took the function $f(x) = (x+1)^3(x-3)^3$ and asked the students to answer the following questions. With Shweta, you can also test your knowledge by answering the questions

(i) Find the derivative of the function. [1]

(ii) Find the stationary points on the curve. [1]

(iii) Find the intervals where the function is increasing and decreasing. [2]

38. **Case-Study 3:**

ICAR grows vegetables and grades each one as either good or bad for its taste, good or bad for its size, and good or bad for its appearance.

Over all 78% of the vegetables have a good taste. However, only 69% of the vegetables have both a good taste and a good size. Also, 5% of the vegetables have both a good taste and a good appearance, but a bad size. Finally, 84% of the vegetables have either a good size or a good appearance.

(i) If a vegetable has a good taste, what is the probability that it also has a good size? [2]

(ii) If a vegetable has a bad size and a bad appearance, what is the probability that it has a good taste? [2]

———————————— ALL THE BEST ————————————