GYAN BHARATI SCHOOL
Pre Board Examination (2023-24)
Mathematics (041)
Class – SS2
Set B

Time : 3 Hrs. Marks : 80

General Instructions:

1. There are five printed pages in this question paper.
2. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
3. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
4. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
5. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
6. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
7. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section – A

Q1 If a line makes angles 90°, 135°, 45° with the x, y and z-axes respectively, then its direction cosines are (1)

(a) $0, -\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}$

(b) $0, \frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{2}}$

(c) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

(d) $0, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}$

Q2 Value of $\int_0^1 \frac{dx}{25+9x^2}$ is (1)

(a) $\frac{1}{5}\tan^{-1}\left(\frac{3}{5}\right)$

(b) $\frac{1}{10}\tan^{-1}\left(\frac{3}{5}\right)$

(c) $\frac{1}{30}\tan^{-1}\left(\frac{3}{5}\right)$

(d) $\frac{1}{15}\tan^{-1}\left(\frac{3}{5}\right)$

Q3 If $A = (a_{ij})_{2\times2}$, such that $a_{ij} = \frac{1}{2}|-3i+j|$, then A is (1)

(a) $\begin{bmatrix} 1 & 1/2 \\ 5/2 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 \\ 3/2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 1/2 \\ -5/2 & -2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

Q4 Angle between the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{4}$ is (1)

(a) $\cos^{-1}\left(\frac{18}{\sqrt{29}\sqrt{38}}\right)$

(b) $\cos^{-1}\left(\frac{13}{\sqrt{29}\sqrt{38}}\right)$

(c) $\cos^{-1}\left(\frac{-13}{\sqrt{29}\sqrt{38}}\right)$

(d) $\cos^{-1}\left(\frac{19}{\sqrt{29}\sqrt{38}}\right)$

Q5 Consider the function f(x) = 1/x at x = 0 (1)

(a) limit is existing but not continuous

(b) limit is not existing but it is continuous

(c) neither limit is existing nor the function is continuous

(d) limit is existing but value of the function is not defined

Q6 Value of $tan^{-1}\ tan\left(\frac{2\pi}{3}\right)$ is (1)

(a) $2\pi/3$

(b) $-2\pi/3$

(c) $-\pi/4$

(d) $-\pi/3$

Q7 Let A(1,2), B(2,1), C(3,4) and D(3,2) be feasible points of a bounded region and we need to maximize z = – x + y. Maximum value occurs at (choose best possible answer) : (1)

(a) A(1,2)

(b) C(3,4)

(c) all points on the line segment AC

(d) A and C only

Q8 Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, then the probability that it is an even number, is (1)

(a) 4/7

(b) 3/7

(c) 4/10

(d) 3/10

Q9 [x] (greatest integer function) is (1)

(a) continuous everywhere

(b) not continuous at integral values of x

(c) not continuous at rational values of x

(d) not continuous at irrational values of x

Q10 The number of all possible matrices of order 3 × 2 with each entry 0 or 1 is: (1)

(a) 16

(b) 128

(c) 64

(d) 512

Q11 The projection of $2\hat{i} - 2\hat{j} + 3\hat{k}$ along $3\hat{i} - 6\hat{j} + \hat{k}$ is (1)

(a) 21/46

(b) $21/\sqrt{17}$

(c) 21/17

(d) $21/\sqrt{46}$

Q12 If y + siny = cosx, then dy/dx at $(\pi/2, 0)$ is (1)

(a) 0

(b) –1

(c) 1/2

(d) –1/2

Q13 If $A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & -2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$, then value of $M_{12} + C_{23} + M_{31}$ is (1)

(a) 1

(b) –1

(c) –2

(d) 0

Q14 Unit vector(s) perpendicular to $\hat{i} - 2\hat{j} + 3\hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ is/are (1)

(a) $\frac{\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{6}}$

(b) $-\frac{\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{6}}$

(c) $\pm\frac{\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{6}}$

(d) $\pm\frac{\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{6}}$

Q15 If A is a 3x3 matrix and $|A| = 3$, then the value of $|4A|$ is (1)

(a) 192

(b) 12

(c) 48

(d) 3

Q16 Equation of a curve passing through the point (–2, 3), given that the slope of the tangent to the curve at any point (x, y) is $2x/y^2$, is (1)

(a) $y^3 = 3x^2 + c$

(b) $y^2 = 3x^2 + 15$

(c) $y^3 = 3x^2 + 15$

(d) $y^3 = 3x^3 + 15$

Q17 If area of triangle is 4 sq. units and vertices are (–2, 0), (0, 4), (0, k), then value(s) of k is/are (1)

(a) 0,–8

(b) 0

(c) 8

(d) 0,8

Q18 If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46$, then $|\vec{a}|$ is (1)

(a) 10

(b) 20

(c) 22

(d) 42

ASSERTION-REASON BASED QUESTIONS

In the following 2 questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Q19 Assertion : Order and degree of differential equation $2x\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$ are 1 and not defined, respectively. (1)

Reason : Order of a differential equation is highest derivative present and degree is highest power of derivatives present.

Q20 Assertion : The function $x^3 - 3x^2 + 4x$ is increasing on R. (1)

Reason : f(x) is increasing in an interval if $f'(x) \geq 0$.

Section – B

Q21 Find $\frac{dy}{dx}$ if $y = (\sin x)^x + \sin^{-1}\sqrt{x}$. (2)

OR

If $x = at^2$ and $y = 2at$, then find $\frac{d^2y}{dx^2}$ at $t = 2$.

Q22 Find the area of the parallelogram whose adjacent sides are determined by the vectors $3\hat{i} + 4\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$. (2)

Q23 The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2cm/minute. When x =10cm and y = 6cm, find the rates of change of area of the rectangle. (2)

OR

A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

Q24 Express $\tan^{-1}\left(\frac{cosx}{1-sinx}\right)$; $-3\pi/2 < x < \pi/2$ in the simplest form. (2)

OR

Find domain of $\sin^{-1}\sqrt{3-2x^2}$.

Q25 Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors of magnitudes 3, 4, 5 respectively. If each one is perpendicular to the sum of the other two vectors, then evaluate $|\vec{a}+\vec{b}+\vec{c}|$. (2)

Section – C

Q26 Solve : $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$. (3)

OR

Solve : $x\frac{dy}{dx} + 2y = x^2\log x$.

Q27 A coin is tossed 4 times. Find probability distribution and mean of number of heads. (3)

OR

A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even' then check whether E and F are independent or not ?

Q28 Evaluate : $\int_0^{\pi/2}\log(\sin x)dx$. (3)

Q29 Evaluate : $\int\frac{(x^4-x)^{1/4}}{x^5}dx$. (3)

Q30 Evaluate : $\int_{\pi/6}^{\pi/3}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$. (3)

OR

Evaluate : $\int_1^4(|x-1|+|x-2|+|x-3|)\,dx$.

Q31 Evaluate : $\int\frac{1}{\cos(x+a)\cos(x+b)}dx$. (3)

Section – D

Q32 Find the area of the region bounded by y = cos x, x axis, x = 0 and x = 2π, after plotting a rough sketch. (5)

OR

Find area of the region bounded by $y = x^3$, the x-axis and the ordinates x = – 2 and x = 1. Also, draw a rough sketch representing the region.

Q33 Show that the relation R on the set of integers given by R = {(a, b) : 2 divides a – b ; a, b ∈ Z} is an equivalence relation. (5)

Q34 Find perpendicular distance of the point A(–2,1,0) from the line $\frac{x-1}{3} = \frac{y+1}{-3} = \frac{z-2}{2}$. (5)

OR

Find the equation of the line passing through A(1,2,3) and which is perpendicular to the lines $\frac{2x+1}{2} = \frac{3y+1}{2} = \frac{z+1}{3}$ and $x-2 = \frac{y}{3} = \frac{z-1}{3}$.

Q35 The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is ₹70. Find cost of each item per kg by matrix method. (5)

5, 8, 8

Section – E

Q36 Cost function for running a business is given by $C = -50x + 20y$ where x and y are variables subject to following constraints : $2x - y \geq -5$; $3x + y \geq 3$, $2x - 3y \leq 12$, $x \geq 0$; $y \geq 0$
Based on above information, answer following questions, graphically :
(i) Find all feasible points which can contribute to minimum cost. (2)
(ii) Find minimum cost. (2)

Q37 An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Based on above information, answer following questions :
(i) Find abscissa of the position of the enemy plane when it is nearest to the soldier. (2)
(ii) Find the nearest distance. (2)

Q38 A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). In a city 0.1 percent of the population actually has the disease. Based on the above information, answer the following questions :
(i) What is the probability that the person has the disease ?
(ii) If the person is diagnosed to be positive, then what is the probability that he is actually suffering from the disease ? (2)
(2)

OR

(ii) If the person is diagnosed to be positive, then what is the probability that he is actually not suffering from the disease ?

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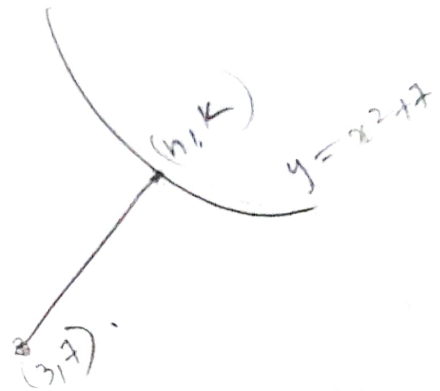
$k = h^2 + 7$.



$d = \sqrt{(h-3)^2 + (k-7)^2}$

$d^2 = (h-3)^2 + (h)^4$

$d(k) = (h-3)^2 + h^4$

$d'(k) = 2(h-3) + 4h^3$

$d'(k) = 0$

$4h^3 + 2h - 6 = 0$