St. Paul's School

Class XII – Pre-Board Examination (2023-24)

Mathematics

Time – 3 hrs. **Max. Marks – 80**

General instructions:

(i) This question paper contains five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
(ii) Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
(iii) Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
(iv) Section C has 6 Short Answer (SA) type questions of 3 marks each.
(v) Section D has 4 Long Answer (LA) type questions of 5 marks each.
(vi) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A (1mk each)

1. The value of k, so that the function $f(x) = \begin{cases} kx^2 + 5, x \leq 1 \\ 2, x > 1 \end{cases}$ is continuous at $x = 1$ is
   a. -2
   b. 2
   c. -3
   d. 3

2. If $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$, then the value of $x + y$ is
   a. 1
   b. 2
   c. 3
   d. 0

3. The coordinates of the foot of the perpendicular drawn from the point$(2, -3, 4)$ on the y-axis is
   a. $(2,3,4)$
   b. $(-2,-3,-4)$
   c. $(0,-3,0)$
   d. $(2,0,4)$

4. A 2×2 matrix whose elements are given by $a_{ij} = |i^2 - j|$ is
   a. $\begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$
   b. $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$
   c. $\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$
   d. $\begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix}$

5. The principal value of $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$ is

a. $\frac{2\pi}{5}$
b. $\frac{-2\pi}{5}$
c. $\frac{3\pi}{5}$
d. $\frac{-3\pi}{5}$

6. An urn contains two red and four black balls. Two balls are drawn at random. Probability that they are of the different colors is

a. 2/5
b. 1/15
c. 8/15
d. 4/15

7. If $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then the value of $A(adjA)$ is

a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
b. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
c. $\begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$
d. $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 7 \\ 0 & 0 & 2 \end{bmatrix}$

8. The graph of the inequality $2x + 3y > 6$ is

a. A half – plane that contains the origin
b. Half – plane that neither contains the origin nor the points on the line $2x + 3y = 6$
c. Whole XOY – plane excluding the points on the line $2x + 3y = 6$
d. Entire XOY plane

9. The magnitude of the projection of $2\hat{\imath} - \hat{\jmath} + \hat{k}$ on $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ is

a. 1/2 units
b. 2 units
c. 3 units
d. 1/3 units

10. For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, value of A^{-1} is

a. $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$

b. $\begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$

c. $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

d. $\begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix}$

11. Vector of magnitude 5 units and in the direction opposite to $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ is

a. $5(2\hat{\imath} + 3\hat{\jmath} - 6\hat{k})$

b. $-5(2\hat{\imath} + 3\hat{\jmath} - 6\hat{k})$

c. $\frac{5}{7}(2\hat{\imath} + 3\hat{\jmath} - 6\hat{k})$

d. $\frac{5}{7}(-2\hat{\imath} - 3\hat{\jmath} + 6\hat{k})$

12. Corner points of the feasible region determined by the system of linear constraints are $(0,3), (1,1)$ and $(3,0)$. Let $Z = px + qy$, where $p, q > 0$. Condition on p and q, so that the minimum value of Z occurs at $(3,0)$ and $(1,1)$ is

a. $p = 2q$

b. $p = q$

c. $p = 3q$

d. $q = 2p$

13. If $|\vec{a}| = 4$ and $-3 \leq \lambda \leq 3$, then $|\lambda\vec{a}|$ lies in

a. [0,12]

b. [2,3]

c. [8,12]

d. [−12,8]

14. The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 = x$ is

a. 3

b. 2

c. 1

d. None of these

15. The interval in which the function f is given by $f(x) = x^2e^{-x}$ is strictly increasing is

a. $(-\infty, \infty)$

b. $(-\infty, 0)$

c. $(2, \infty)$

d. $(0,2)$

16. The area of a triangle formed by the vertices O,A and B where $\overrightarrow{OA} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ is

a. $3\sqrt{5}$ sq units
b. $5\sqrt{5}$ sq units
c. $6\sqrt{5}$ sq units
d. 4 sq units

17. If $y = f(x)$ and $f'(x) = e^{\sqrt{x}}$, then $\frac{dy}{dx}$ is

a. $2xe^{2x}$
b. $2xe^{x}$
c. $2xe^{x^2}$
d. xe^{x}

18. $\int_0^{\frac{\pi}{8}} tan^2(2x)dx$ is equal to

a. $\frac{4-\pi}{8}$
b. $\frac{4+\pi}{8}$
c. $\frac{4-\pi}{4}$
d. $\frac{4-\pi}{2}$

19. Assertion (A): the matrix $A = \begin{bmatrix} 5 & 0 \\ 0 & 3 \\ 11 & 0 \end{bmatrix}$ is a diagonal matrix

Reason (R): if $A = [a_{ij}]_{m\times m}$, where $a_{ij} = 0$ if $i \neq j$, then A is called a diagonal matrix

20. Assertion (A): if $e^{-xy} + \log(xy) + sin^2(xy) = 0$, then $\frac{dy}{dx} = -\frac{y}{x}$

Reason (R): $\frac{d}{dx}(xy) = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

Section B (2 mks each)

21. How many equivalence relations on the set $\{1,2,3\}$ containing $(1,2)$ and $(2,1)$ are there in all? Justify your answer

22. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$

23. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find k so that $A^2 = 5A + kI$

24. If $f(x) = \sqrt{\frac{\sec x-1}{\sec x+1}}$, then find $f'\left(\frac{\pi}{3}\right)$

25. Find $\int \frac{\log x}{(1+\log x)^2}dx$

Section C (3 mks each)

26. Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

27. If $y = \log(1 + 2t^2 + t^4)$, $x = \tan^{-1} t$, find $\frac{d^2y}{dx^2}$

28. Find $\int e^{2x} \sin(3x + 1)dx$

29. Find the solution of $x^2ydx - (x^3 + y^3)dy = 0$

30. Solve the following LPP graphically
Minimize $Z = 5x + 7y$
Subject to the constraints

$$2x + y \geq 8$$

$$x + 2y \geq 10$$
$$x, y \geq 0$$

31. Three machines E_1, E_2, E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of the machines E_1, E_2 are defective and that 5% produced by machine $E_3$are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective.

Section D (5 mks each)

32. Find the points at which the function $f(x) = (x - 2)^4(x + 1)^3$ has
   a. Local maxima
   b. Local minima
   c. Point of inflexion

33. Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$

34. Solve the differential equation: $(\cot^{-1} y + x)dy = (1 + y^2)dx$

35. Find the equation of the line through the point $(1, -1, 1)$ and perpendicular to the lines joining the points $(4,3,2), (1, -1, 0)$ and $(1, 2, -1), (2, 1, 1)$

Section E (Case study – 4 mks each)

36. A carpenter designs a window in the form of a rectangle surmounted by a semi-circle. The total perimeter of the window is 10 m.

    a. Write the perimeter in terms of x and y

    b. Express y in terms of x and π

    c. Find the value of x for maximum light

37. In a class, 40% students study mathematics, 25% study biology and 15% study both mathematics and biology. One student is selected at random

    M = event of studying mathematics

    B = event of studying biology

    a. What is the value of P(M)

    b. What is the value of P(B)

    c. Find P(M/B)

38. (a) Evaluate $\int_1^4 \frac{\sqrt{5-x}}{\sqrt{5-x}+\sqrt{x}}dx$

    (b) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\cot x}}dx$