Navyasha Chadha
SS2-B
11

GYAN BHARATI SCHOOL
Pre Board Examination (2023-24)
Mathematics (041)
Class – SS2
Set A

Time : 3 Hrs. Marks : 80

General Instructions:

1. There are five printed pages in this question paper.
2. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
3. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
4. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
5. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
6. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
7. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section – A

Q1 The Cartesian equation of a line is given by $\frac{2x-6}{3} = \frac{3y-7}{2} = \frac{4z-7}{-2}$. Its direction ratios are (1)

(a) 3,2,–2
(b) 9,–4,3
(c) 9,4,–3
(d) 3,–2,2

Q2 Value of $\int_0^1 \frac{dx}{9-4x^2}$ is (1)

(a) $\frac{1}{2}ln5$
(b) $-\frac{1}{12}ln5$
(c) $\frac{1}{3}ln5$
(d) $\frac{1}{12}ln5$

Q3 If $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then value of $A^2 - 2A$ is (1)

(a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
(d) $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$

Q4 Direction cosines of x, y and z-axis are respectively (1)

(a) < a,0,0 >, < 0,b,0 >, < 0,0,c >
(b) < 1,0,0 >, < 0,1,0 >, < 0,0,1 >
(c) < –1,0,0 >, < 0,–1,0 >, < 0,0,1 >
(d) < 1,0,0 >, < 0,1,0 >, < 0,0,–1 >

Q5 Value of 'k' so that $f(x) = \begin{cases} \frac{x}{|x| + 2x^2} & \text{when } x \neq 0 \\ k & \text{when } x = 0 \end{cases}$ may be continuous at x = 0, is (1)

(a) 2
(b) −2
(c) does not exist
(d) can't be decided

Q6 Value of $sec^{-1} sec\left(-\frac{11\pi}{3}\right)$ is (1)

(a) 2π/3
(b) −π/3
(c) π/2
(d) π/3

Q7 Let A(1,2), B(2,1), C(3,4) and D(3,2) be feasible points of a bounded region and we need to maximize z = − x + y. Maximum value occurs at (chose best possible answer) : (1)

(a) A(1,2)
(b) C(3,4)
(c) all points on the line segment AC
(d) A and C only

Q8 If A and B are independent events, then A' and B' are (1)

(a) independent events
(b) not necessarily independent events
(c) can't be decided
(d) none of these

Q9 Derivative of $2\sqrt{\cot x^2}$ w.r.t. 'x' at $x = \sqrt{\frac{\pi}{4}}$ is (1)

(a) $-\sqrt{\pi}$
(b) $-2\sqrt{\pi}$
(c) $2\sqrt{\pi}$
(d) $\sqrt{\pi}$

Q10 If $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, then a + b + c + d is (1)

(a) 8
(b) 9
(c) 10
(d) 11

Q11 If $|\vec{a}| = |\vec{b}|$, then $\vec{a}$ and $\vec{b}$ are (1)

(a) equal vectors
(b) different vectors
(c) can't be decided
(d) not necessarily equal vectors

Q12 If $x^3 + xy^2 + x^2y + y^3 = 10$, then dy/dx is (1)

(a) $\frac{3x^2+2xy+y^2}{x^2+2xy+3y^2}$
(b) $-\frac{x^2+2xy+3y^2}{3x^2+2xy+y^2}$
(c) $\frac{x^2+2xy+3y^2}{3x^2+2xy+y^2}$
(d) $-\frac{3x^2+2xy+y^2}{x^2+2xy+3y^2}$

Q13 If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is (1)

(a) 6
(b) −6

(c) ± 6
(d) 0

Q14 Unit vector in the direction of sum of the vectors $2\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 6\hat{j} + \hat{k}$ is (1)
(a) $\frac{5\hat{i}-8\hat{j}+4\hat{k}}{\sqrt{105}}$
(b) $\frac{2\hat{i}-6\hat{j}+4\hat{k}}{\sqrt{56}}$
(c) $\frac{5\hat{i}-8\hat{j}-4\hat{k}}{\sqrt{105}}$
(d) $\frac{5\hat{i}+8\hat{j}-4\hat{k}}{\sqrt{105}}$

Q15 Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. (1)
The restriction on n, k and p so that PY + WY will be defined are
(a) k = 3, n = 3
(b) k is arbitrary, p = 2
(c) p is arbitrary, k = 3
(d) k = 2, p = 3

Q16 Solution of $\frac{dy}{dx} = \frac{x+1}{2-y}$; $y \neq 2$, is (1)
(a) $2x^2 + y^2 + x - 4y = c$
(b) $x^2 + 2y^2 + 2x - y = c$
(c) $x^2 + y^2 + 2x - 4y = c$
(d) $2x^2 + 2y^2 + x - 3y = c$

Q17 If A is an invertible matrix of order 4 and $|adjA| = 125$, then |A| is (1)
(a) 25
(b) ± 25
(c) 125
(d) 5

Q18 The points $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ and $C(3\hat{i} - 4\hat{j} - 4\hat{k})$ are the vertices of (1)
(a) equilateral triangle
(b) isosceles triangle
(c) right angled triangle
(d) right angled isosceles triangle

ASSERTION-REASON BASED QUESTIONS

In the following 2 questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19 Assertion : Order and degree of differential equation $2x\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$ are 2 and not defined, respectively. (1)
Reason : Order of a differential equation is highest derivative present and degree is highest power of derivatives present.

Q20 Assertion : $x^4 e^{-x}$ is strictly increasing in the interval (0, 2). (1)
Reason : f(x) is strictly increasing in an interval if $f'(x) > 0$.

Section – B

Q21 If $x = a\left(cost + \log\left(\tan\frac{t}{2}\right)\right)$, $y = a\sin t$, then find $\frac{dy}{dx}$ in simplified form. (2)

OR

If $y = 3e^{2x} + 2e^{3x}$, then prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

Q22 If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, then find $|\vec{a} - \vec{b}|$. (2)

Q23 A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ? (2)

OR

The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 centimetres ?

Q24 Write two branches of $\sec^{-1} x$ other than PVB. (2)

OR

Find domain of $\cos^{-1}\sqrt{3 - x^2}$.

Q25 Using vectors, find area of a triangle with vertices A(1,1,2), B(2,3,5) and C(1,5,5). (2)

Section – C

Q26 Solve : $\frac{dy}{dx} - 3y\cot x = \sin 2x$; $y = 2$ when $x = \pi/2$. (3)

OR

Show that the family of curves for which the slope of the tangent at any point (x, y) on it is $\frac{x^2 + y^2}{2xy}$ is given by $x^2 - y^2 = cx$.

Q27 Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the probability that the die shows a number greater than 4 given that there is at least one tail. (3)

OR

A pair of dice is thrown thrice. Find the probability distribution of the number of doublets. Also find mean of doublets

Q28 Evaluate : $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx$. (3)

Q29 Evaluate : $\int \frac{dx}{\sqrt{\sin^3 x \sin(x + a)}}$. (3)

Q30 Evaluate : $\int_{-1}^{2} |x^3 - x|\, dx$. (3)

OR

Evaluate : $\int_{-1}^{3/2} |x \sin(\pi x)|\, dx$.

Q31 Evaluate : $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$. (3)

Section – D

Q32 Using integration, find the area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$. (5)

OR

Using integration, find the area of the region $|x| + |y| = 2$.

Q33 Check the function $f : R \to R$ defined by $f(x) = x^3$ for (i) one-one, and (ii) onto. (5)

Q34 Find the image of the point A(2,3,0) in the mirror which is in the form of a line whose equation is given by $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z}{4}$. (5)

OR

Find the equation of the line passing through A(0,−2,3) and which is perpendicular to the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{4}$.

Q35 The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method. (5)

Section – E

Q36 Cost function for running a business is given by $C = 50x + 20y$ where x and y are variables subject to following constraints : $2x - y \geq -5$; $3x + y \geq 3$, $2x - 3y \leq 12$, $x \geq 0$; $y \geq 0$
Based on above information, answer following questions, graphically :
(i) Find all feasible points which can contribute to minimum cost. (2)
(ii) Find minimum cost. (2)

Q37 A company is making toys in the shape of a right circular cone in which a right circular cylinder having maximum curved surface area, is inscribed. Take radius and height of cone as r and h respectively. Based on the above information, answer following questions :

(i) Find expression for CSA of cylinder in terms of radius of cylinder. (1)
(ii) Prove that radius of cylinder is half of the radius of the cone. (2)
(iii) Find the maximum CSA of cylinder. (1)

Q38 Two groups are competing for the position in Board of directors of a corporation. They both want to introduce a new product for increasing profit of company. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Based on above information, answer following questions :
(i) What is the probability that the new product is introduced ? (2)
(ii) Find the probability that the new product introduced was by the second group. (2)

OR

(ii) Find the probability that the new product introduced was by the first group.