Utkarsh Verma
12 - B
(25)

GYAN BHARATI SCHOOL

Second Pre Board Examination (2023-2024)

Mathematics (041)

SS2

Time Allowed: 3 Hr. Maximum Marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION – A

Q1 The Cartesian equation of a line is given by $6x - 2 = 3y + 1 = 2z - 2$. Its direction ratios are (1)

(a) 2,3,1 (b) 3,1,2 (c) 1,2,3 (d) 1,3,2

Q2 Value of $\int_0^3 \frac{dx}{9 + x^2}$ is (1)

(a) $\pi/3$ (b) $\pi/6$ (c) $\pi/12$ (d) $\pi/15$

Q3 If $A = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$, then value of $3A^2 + 2I$ is (1)

(a) $\begin{bmatrix} 12 & 11 \\ 35 & 24 \end{bmatrix}$ (b) $\begin{bmatrix} 35 & 24 \\ 12 & 11 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 8 \\ 12 & 9 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 4 \\ 12 & 35 \end{bmatrix}$

Q4 Direction cosines of x, y and z-axis are (1)

(a) $< 1,0,0 >, < 0,1,0 >, < 0,0,1 >$ (b) $< a,0,0 >, < 0,b,0 >, < 0,0,c >$
(c) $< -1,0,0 >, < 0,-1,0 >, < 0,0,1 >$ (d) $< 1,0,0 >, < 0,1,0 >, < 0,0,-1 >$

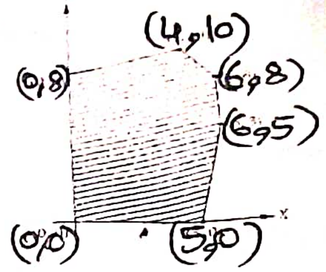
Q5 Value of 'a' so that $f(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3} & \text{when } x \neq 3 \\ a & \text{when } x = 3 \end{cases}$, may be continuous at x = 3, is (1)

(a) 4 (b) 5 (c) –4 (d) –1

Q6 Value of $\cos^{-1} \cos\left(\frac{5\pi}{3}\right)$ is (1)

(a) $\pi/3$ (b) $\pi/6$ (c) $\pi/2$ (d) $\pi/4$

Q7 The feasible region of a LPP is shown in the adjacent figure. If $z = 3x - 4y$ is the objective function, then minimum value of z occurs at
(A) (0,0)
(B) (0,8)
(C) (5,0)
(D) (4,10)



Q8 If P(A) = 0.8, P (B) = 0.5 and P(B|A) = 0.4, then value of P(A|B) is (1)
(a) 1/16 (b) 1/25 (c) 16/25 (d) 5/16

Q9 Derivative of $\sin[\cos(xy^2 + x^2)] = 1/3$ w.r.t. 'x' is (1)
(a) $\frac{x^2 + y^2}{2xy}$ (b) $\frac{-x^2 + y^2}{xy}$ (c) $\frac{x^2 - y^2}{-2xy}$ (d) $\frac{-2x - y^2}{2xy}$

Q10 If A = $\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$, then matrix B such that AB = I, is (1)

(a) $\begin{bmatrix} \frac{1}{2} & 1 \\ \frac{3}{2} & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

Q11 If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46$, then $|\vec{a}|$ is (1)
(a) 43 (b) 44 (c) 21 (d) 22

OR

If $|\vec{a}| = 5$, $|\vec{b}| = 13$, $|\vec{a} \times \vec{b}| = 25$, then $\vec{a}.\vec{b}$ is
(a) 60 (b) –56 (c) 56 (d) ± 60

Q12 Derivative of 10^{10^x} w.r.t. 'x' is (1)
(a) $10^{10^x}10^x \log 10$ (b) $10^{10^x} (\log 10)^2$
(c) $10^{10^x}10^x (\log 10)^2$ (d) $10^x\, 10^{10^x - 1} (\log 10)^2$

Q13 If A is a 3x3 invertible matrix, then value of k if $|A^{-1}| = |A|^k$ is (1)
(a) 0 (b) 1 (c) –1 (d) 2

OR

If A = diag.(d_1 d_2 d_3), then A^{-1} will be
(a) diag.(d_1 d_2 d_3) (b) diag.($1/d_1$ $1/d_2$ $1/d_3$) (c) O (d) 2I

Q14 If $\vec{a} \times \vec{b} = \vec{0}$ as well as $\vec{a} \cdot \vec{b} = 0$, then (1)
(a) both are null vectors (b) they are perpendicular as well as parallel vectors
(c) at least one is null vector (d) insufficient information

Q15 Values of x and y if $2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 14 \\ 15 & 14 \end{pmatrix}$ are (1)

(a) 1,2 (b) 9,1 (c) 1,3 (d) 2,9

Q16 Solution of $\frac{dy}{dx} = x + y + xy + 1$ is (1)

(a) $(x + 1)^2 = \ln|y + 1| + c$ (b) $x^2/2 + x = \ln|y + 1| + c$

(c) $x/2 + x^2 = \ln|y| + c$ (d) $(x + 1)^2 = \ln|y| + c$

Q17 If A is an invertible matrix of order 3 and $|A| = 5$, then $|\text{adj}.A|$ is (1)

(a) 25 (b) ± 25 (c) 125 (d) 5

Q18 Area of the parallelogram whose adjacent sides are determined by the vectors $\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $2\hat{\imath} - 7\hat{\jmath} + \hat{k}$, is (in square units) (1)

(a) 45 (b) 50 (c) $15\sqrt{2}$ (d) $\sqrt{500}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.
(e) Both A and R are false.

Q19 **Assertion :** Order and degree of differential equation $2x\left(\frac{dy}{dx}\right)^{3/2} + \frac{d^2y}{dx^2} = 3x^2$, are 2 and 2. (1)

Reason : Order of a differential equation is highest derivative present and degree is highest power present.

Q20 **Assertion :** $(x + 2)\, e^{-x}$ is increasing in the interval $(-\infty, 1]$. (1)

Reason : f(x) is increasing in an interval if $f'(x) \geq 0$.

SECTION – B

Q21 If $x = \frac{2t}{1+t^2}$, $y = \frac{1-t^2}{1+t^2}$; prove that $\frac{dy}{dx} = -\frac{x}{y}$. (2)

OR

If $y = \log\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right)$, show that $\frac{dy}{dx} - \sec x = 0$.

Q22 Find vector(s) of magnitude 9 units which is perpendicular to both the vectors $4\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $-2\hat{\imath} + \hat{\jmath} - 2\hat{k}$. (2)

Q23 Two equal sides of an isosceles triangle with fixed base 'a' are decreasing at the rate of 9 cm/second. How fast is the area of the triangle decreasing when the two equal sides are equal to 'a' ?

OR

Sand is pouring from a pipe at the rate of $12 cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm.

Q24 Find domain of $\sin^{-1}\sqrt{x-1}$. (2)

OR

Find domain of $\cos^{-1}(1-x^2)$.

Q25 A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure. (2)

SECTION – C

Q26 Find particular solution of the differential equation $\frac{dy}{dx} - y\tan x = e^x \sec x$; $y(0) = 0$. (3)

OR

Solve : $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$.

Q27 Ram and shyam throw a dice at the stake of Rs. 22, which will be won by the player who throws 6 before other player. If Ram starts, what are their expected amounts ? (3)

OR

The probability that A hits a target is 1/3 and the probability that B hits it is 2/5. If each one of A and B shoots at the target, what is the probability (i) the target is hit?, (ii) exactly one of them hits the target?

Q28 Evaluate : $\int_{-1}^{2}(|x+1| + |x| + |x-1|)\,dx$. (3)

Q29 Evaluate : $\int \frac{x^2}{x^4 - x^2 - 2}\,dx$. (3)

OR

Evaluate : $\int \frac{2x}{(x^2+1)(x^2+2)^2}\,dx$.

Q30 Evaluate : $\int_0^{\pi/2} \frac{x\sin x\cos x}{\sin^4 x + \cos^4 x}\,dx$. (3)

Q31 Evaluate : $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$. (3)

SECTION – D

Q32 Find the area of the region in the first quadrant enclosed by x-axis, $x = \sqrt{3}y$ and $x^2 + y^2 = 4$. (5)

OR

Find the area of the region bounded by $x = \sqrt{y-1}$, y = 2, y = 5 and y-axis.

Q33 Show that the function $f : R \rightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$ is one-one and onto function. (5)

Q34 Find the length of the perpendicular from the point (1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ (5)

OR

Check whether the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{4}$ are intersecting or not. Also, find the point of intersection, if existing.

Q35 Use product $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}\begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$ to solve the system of equations (5)

$$x + 3z = 9,\ -x + 2y - 2z = 4 \text{ and } 2x - 3y + 4z = -3.$$

SECTION – E

Q36 A factory produces two kinds of garments : Shirts and Trousers. Let x shirts and y trousers be produced per day. Let profit function of the factory be P = 3x + 5y. Suppose x and y are subjected to following constraints :

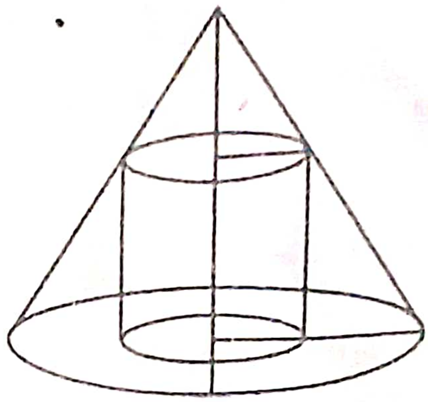
$x + 3y \leq 60$; $x + y \geq 10$; $x \leq y$; $x \geq 0$; $y \geq 0$

Based on above information, answer following questions :

(i) Find all feasible points which can contribute to maximum profit. (2)

(ii) Find maximum profit that can be earned per day. (2)

Q37 A company is making toys in the shape of a right circular cone of height 30cm and radius of base 10cm, in which a largest cylinder is inscribed. Based on the above information, answer following questions :



(i) Find the radius of the cylinder. (2)

(ii) Find the height of the cylinder. (1)

(iii) Find the maximum volume of the cylinder (1)

Q38 The reliability of a COVID PCR test is specified as follows:

Of people having COVID, 90% of the test detects the disease but 10% goes undetected. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as COVID positive. From a large population of which only 0.1% are COVID positive, one person is selected at random, given the COVID PCR test and the pathologist reports him/her as COVID positive. Based on above information, answer following questions :

(i) What is the probability that the person is COVID positive. (2)

(ii) What is the probability that the person is actually COVID positive when he is already declared to be positive. (2)

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