APEEJAY SCHOOL, PANCHSHEEL PARK

Class - XII
Subject - Maths
PREBOARD II EXAMINATION (2023-2024)

Name of the student: | Date:
Time Allowed: | M.M.: 80

General Instructions:
i. This question paper contains- five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
ii. Section A has 18 MCQ's and 02 Assertion- Reason based questions of 1 mark each.
iii. Section B has 5 very short answer type questions (VSA) of 2 marks each.
iv. Section C has 6 short answer type questions (SA) of 3 marks each.
v. Section D has 4 long answer type questions (LA) of 5 marks each.
vi. Section E has 3 source base / case based/passage based / integrated units of assessment 4 marks each with sub parts.

Q.No. **SECTION A** Marks
(MULTIPLE CHOICE QUESTIONS)

Q1. If $x = \sqrt{a^{t}}$ and $y = \sqrt{a^{t}}$ then — 1

(a) $x\frac{dy}{dx} + y = 0$ (b) $x\frac{dy}{dx} = y$

(c) $y\frac{dy}{dx} = x$ (d) None of the above

Q2. $\int \frac{3x^2}{x^6+1}\, dx$ *is equal to* — 1

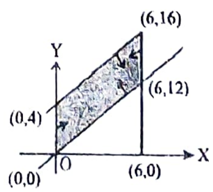
(a) $\log(x^6+1) + c$ (b) $\tan^{-1} x^3 + c$
(c) $3\tan^{-1} x^3 + c$ (d) $\log x^2 + c$

Q3. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $A^2 + 2A$ is equal to — 1

(a) 4A (b) 3A (c) 2A (d) A

Q4. A square matrix $A = [a_{ij}]_{n \times n}$ is called a diagonal matrix, if $a_{ij} = 0$ for — 1
(a) $i = j$ (b) $i < j$ (c) $i > j$ (d) $i \neq j$

Q5. The feasible region for LPP is shown shaded in the figure. — 1
Let $Z = 3x - 4y$ be the objective function, then maximum value of Z is
(a) 12 (b) 8 (c) 0 (d) -18



Q6. The area of the feasible region for the following constraints $3y + x \geq 3$, $x \geq 0$, $y \geq 0$ will be — 1
(a) Bounded (b) Unbounded (c) Convex (d) Concave

Q7. The direction cosines of the line which makes equal angles with the coordinate axes are 1

(a) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (b) $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

(c) $\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$ (d) none of the above

Q8. If $|\vec{a}| = \frac{\sqrt{3}}{2}$, $|\vec{b}| = 4$ and angle between $\vec{a}$ *and* $\vec{b}$ is 60^0 then the value of $\vec{a} \cdot \vec{b}$ is equal to 1

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $-\sqrt{3}$ (d) none of the above

Q9. Order and degree of differential equation is 1

$$\frac{d^2y}{dx^2} = \left[y + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{4}}$$

(a) 4 and 2 (b) 1 and 2 (c) 1 and 4 (d) 2 and 4

Q10. The number of all possible matrices of order 3x3 with each entry -1 or 1 is 1

(a) 512 (b) 81 (c) 27 (d) 18

Q11. If the lines $\frac{x-1}{k} = \frac{y-3}{1} = \frac{z+6}{-2}$ *and* $\frac{x-1}{1} = \frac{y-3}{-2} = \frac{z+6}{k}$ are perpendicular, then k is equal to 1

(a) 2 (b) 1 (c) -2 (d) 3

Q12. Integrating factor of differential equation $x\frac{dy}{dx} + 2y = x^2$ is 1

(a) $\frac{1}{x^2}$ (b) x^2 (c) x (d) $\frac{1}{x}$

Q13. The solution of differential equation $2x\frac{dy}{dx} - y = 3$ represents: 1

(a) straight lines (b) circles (c) parabola (d) ellipse

Q14. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 9 \\ 4 & 6 \end{vmatrix}$, then x is equal to 1

(a) 6 (b) -6 (c) ± 6 (d) none of the above

Q15. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is 1

(a) 0 (b) $\frac{1}{3}$ (c) $\frac{1}{12}$ (d) $\frac{1}{36}$

Q16. A unit vector perpendicular to both the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ *and* $\hat{i} + 2\hat{j} - \hat{k}$ is 1

(a) $\pm\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (b) $\pm\frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} + \hat{k})$

(c) $\pm\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ (d) $\pm\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

Q17. If A = $\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then find the value of $|2A|$ 1

(a) -6 (b) -24 (c) 12 (d) -12

Q18. If α is the angle between any two vectors $\vec{a}$ *and* $\vec{b}$, *then* $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$ when α is equal to 1

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

ASSERTION – REASON BASED QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

Q19. **Assertion**: A relation R = {(a,b) : |a-b|<2} defined on the set A = {1, 2, 3, 4, 5} is reflexive. 1

Reason : A relation R on the set A is said to be reflexive if (a,b) $\in R$ *and* $(b, c) \in R$ *for all* $a, b \in A$.

Q20. **Assertion**: The intervals in which f(x) = log sin x , $0 \le x \le \pi$ is Increasing is $\left(0, \frac{\pi}{2}\right)$. 1

Reason: A function is increasing in (a,b) if f′(x) > 0 for each $x \in (a, b)$.

SECTION B

Q21. Find the value of 2

$$\cos^{-1}\frac{1}{2} + 2\sin^{-1}\left(\frac{1}{2}\right)$$

OR

$$\left(\tan^{-1}\tan\frac{7\pi}{6}\right)$$

Q22. Find $\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$. 2

Q23. If function 2

$$f(x) = \begin{cases} x + k & , if\ x < 3 \\ 4 & , if\ x = 3 \\ 3x - 5 & , if\ x > 3 \end{cases}$$

is continuous function at x=3, then find the value of k.

Q24. The volume of the cube is increasing at the rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 centimeters? 2

OR

Find the maximum profit that a company can make, if the point function is given by

$$p(x) = 41\text{-}72x\text{-}18x^2$$

Q25. Find the intervals in which the function f given by 2

$$f(x) = 4x^3 - 6x^2 - 72x + 30$$

(a) Strictly increasing (b) strictly decreasing

SECTION C

Q26. Find $\frac{dy}{dx}$, $if\ y^x + x^y + x^x = a^b$ 3

OR

If x= a (cost +t sint) and y = a (sint-t cost), find $\frac{d^2y}{dx^2}$.

Q27. Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the expectation of X. 3

OR

A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?

Q28. Find a particular solution of the differential equation 3

$$\frac{dy}{dx} + y\cot x = 4x \operatorname{cosec} x, x \neq 0, \text{ given that}$$

$y = 0$ when $x = \frac{\pi}{2}$

OR

Find a general solution of the differential equation

$$e^x \tan y\, dx + (1-e^x) \sec^2 y\, dy = 0$$

Q29. Evaluate the definite integrals 3

$$\int_1^4 [|x-1| + |x-2| + |x-3|]\, dx$$

Q30. Evaluate the integrals 3

$$\int \frac{x+3}{\sqrt{5-4x+x^2}} dx$$

Q31. Solve the following Linear Programming Problems graphically 3

Maximize $Z = 5x + 3y$

Subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

SECTION D

Q32. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x=1$, $x=4$ and x-axis in the first quadrant. 5

Q33. Let A be the set of all the triangle in a plane and R be the relation defined on R as $R = \{(T_1, T_2): T_1 \text{ is similar to } T_2\}$

1. Show that the relation R is an equivalence relation. 4
2. Consider three right angle triangle T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6,8,10. Which triangle among T_1, T_2, and T_3 are related? 1

OR

Show that $f: R \rightarrow \{x \in R : -1 < x < 1\}$ 3

defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$ is one–one and onto function. 2

Q34. Two factories decided to award their employees three values of (a) adaptable to new situations, (b) careful and alert in difficult situations and (c) keeping calm in tense situations, at the rate of ₹ x, ₹ y and ₹ z per person respectively. The first factory decided to honor respectively 2, 4 and 3 employees with a total prize money of ₹ 29000. The second factory decided to honor respectively 5, 2 and 3 employees with a total prize money of ₹ 30500. If three prizes per person together cost ₹ 9500 then

(i) Represents the above situation by a matrix equation and form linear equations using matrix multiplication, 1

(ii) Solve these equations using matrices. 4

Q35. By computing the shortest distance, determine whether the lines intersect or not. If not then find the shortest distance between the lines. 5

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

OR

Find the vector equation of the line passing through the point (1, 2, – 4) and perpendicular to the two lines:

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

SECTION E

(This section comprises 3 case study questions of 4 marks having sub parts.)

Q36. A doctor is to visit a patient. From past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2, 0.1 and 0.4. The probabilities that he will be late are 0.25, 0.3, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively.

(i) What is the probability that the doctor is late by other means? 1

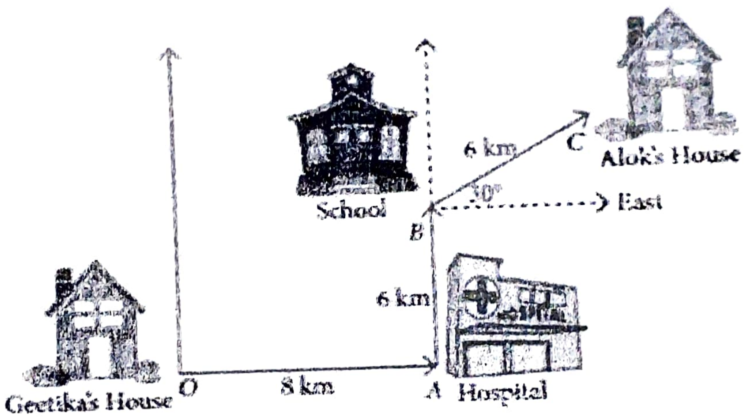
(ii) When the doctor arrives late, what is the probability that he comes by metro? 1

(iii) When the doctor arrives late, what is the probability that he comes by bike or other means? 2

OR

(iii) When the doctor arrives late, what is the probability that he comes by cab or metro? 2

Q37.



Gitika house is situated at Shalimar Bagh at O, going to Aloke's house she first travels 8 km in the east, here at point A a hospital is situated. From the hospital she takes an auto and goes 6 km in the north. Here at point B a school is situated. From school she travels by bus to reach Aloke's house which is 30^0 of east and 6 km from point B.

(i) What is the vector distance from Gitika's house to school? 1

(ii) What is the vector distance from school to Aloke's house? 1

(iii) What is the vector distance from Gitika's house to Aloke's house?2

OR

(iii) What is the total distance traveled by Gitika from her house to Aloke's house? 2

Q38. A telephone company in a town has 500 subscribers on its list and collects fixed charges of ₹ 300 per subscriber. The company proposes to increase the annual subscription and it is believed that every increase of ₹ 1, one subscriber will discontinue the service.

(i) Based on above information find out how much amount can be increased for maximum revenue. 2

(ii) Find out maximum revenue received by the telephone company. 2