DELHI PUBLIC SCHOOL, GBN

SESSION 2023-24

PRE- BOARD- II

CLASS XII- MATHEMATICS(041)

DURATION: 3 Hours **MM:80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

i. This question paper contains 38 questions. All questions are compulsory.

ii. This question paper is divided into five Sections A, B, C, D and E.

iii. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.

iv. In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.

v. In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.

vi. In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.

vii. In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.

viii. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.

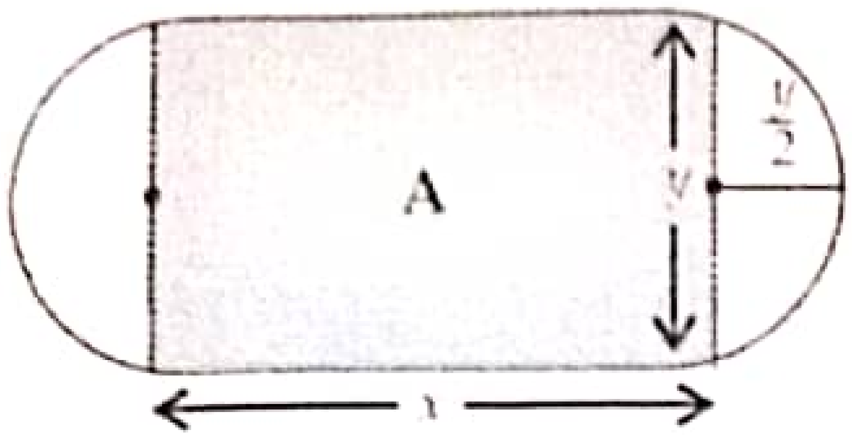
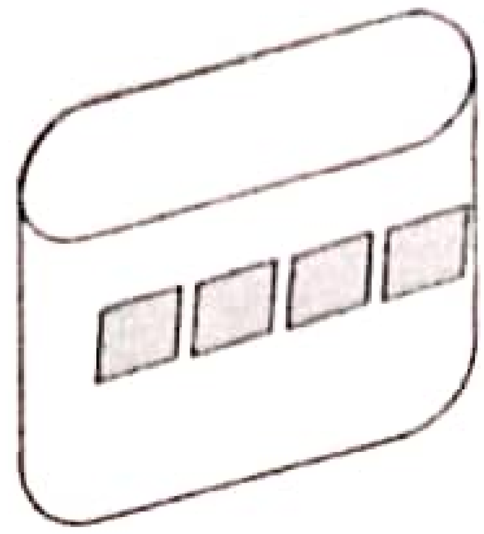
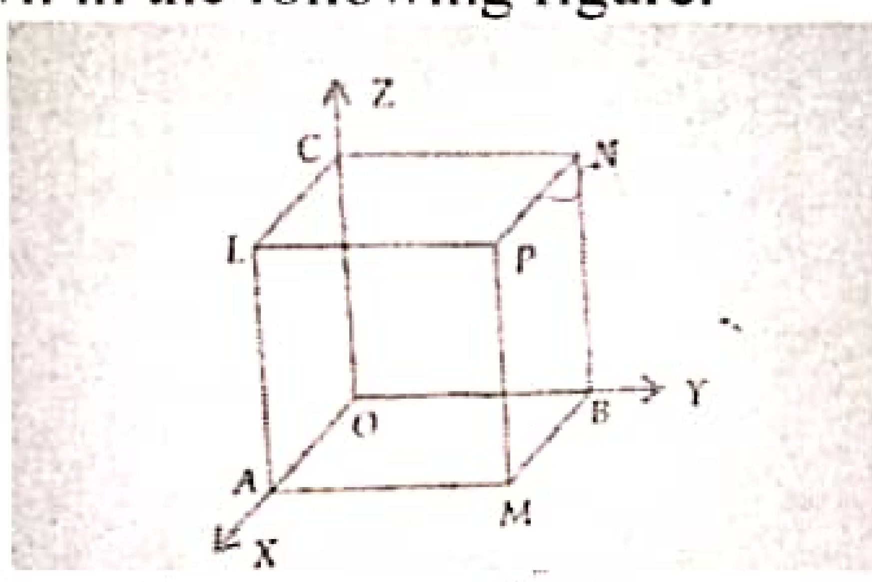
ix. Use of calculators is not allowed.

Q. No	Question	Marks
	SECTION A **(All questions are compulsory. No internal choice is provided in this section)**	
1.	Let $R = \{(a, a^3): a\ is\ a\ prime\ number\ less\ than\ 5\}$ be a relation. The range of R is: (a) (3, 2, 1) (b) (7, 8, 9) (c) not defined (d) none of these	1
2.	If $A = \begin{bmatrix} \propto & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, the values of $\propto$ for which $A^2 = B$. Will be: (a) $\propto = \pm 1$ (b) $\propto = 4$ (c) $\propto = 0$ (d) there is no value of $\propto$ for which $A^2 = B$ is true.	1
3.	The value of x such that: $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$ is: (a) 1 (b) 2 (c) -2 d) 14	1
4.	If $\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$, then values of x is: (a) 1, -3 (b) 2, $\frac{1}{2}$ (c) -3, 2 (d) -3, $\frac{1}{2}$	1
5.	If A is non- singular matrix such that $A^3 = I$, then $A^{-1} =$ (a) A (b) A^2 (c) A^3 (d) A^4	1
6.	If $A = \begin{bmatrix} x & 5 & 2 \\ 2 & y & 3 \\ 1 & 1 & z \end{bmatrix}, xyz = 80, 3x + 2y + 10z = 20$ and $A(adj\ A) = kI$, then $k =$ (a) 80 (b) 75 (c) 115 (d) 79	1
7.	The function $f(x) = \lvert x \rvert + \lvert x - 1 \rvert$ is: (a) Continuous at $x = 0$ as well as at $x = 1$ (b) Continuous at $x = 1$ as but not at $x = 0$ (c) discontinuous at $x = 0$ as well as at $x = 1$ (b) Continuous at $x = 0$ as but not at $x = 1$	1
8.	The function $f(x) = a^x$ is increasing on R, if: (a) $a > 0$ (b) $a > 1$ (c) $a < 1$ (d) $0 < a < 1$	1

9.	$\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$ is equal to: (a) $\frac{e^x}{1+x^2} + C$ (b) $-\frac{e^x}{1+x^2} + C$ (c) $\frac{e^x}{(1+x^2)^2} + C$ (d) $-\frac{e^x}{(1+x^2)^2} + C$	
10.	The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is: (a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $xy = C$ (c) $logxlogy = C$ (d) $x + y = C$	1
11.	Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals (a) -1 (b) 1 (c) log 2 (d) $-\log 2$	1
12.	The general solution of the differential equation $\frac{dy}{dx} = e^{\frac{x^2}{2}} + xy$ is: (a) $y = Ce^{\frac{-x^2}{2}}$ (b) $y = Ce^{\frac{x^2}{2}}$ (c) $y = (x + C)e^{\frac{x^2}{2}}$ (d) $y = (x - C)e^{\frac{x^2}{2}}$	1
13.	$\int_0^{1.5} [x^2]\, dx$ is equal to: (a) 2 (b) $2 - \sqrt{2}$ (c)) $2 + \sqrt{2}$ (d)) $\sqrt{2}$	1
14.	The least value of the function $f(x) = x^3 - 18x^2 + 96x$ in the interval $[0, 9]$ is: (a) 126 (b) 135 (c) 160 (d) 0	1
15.	The value of λ for which the vectors $\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ are orthogonal, is: (a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{5}{2}$	1
16.	Corner points of the feasible region for an LPP are: (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let $z = 4x + 6y$ the objective function. The minimum value of z occurs at (a) (0, 2) only (b) (3, 0) only (c) the mid-point of the line segment joining the points (0, 2) and (3, 0) only (d) any point on the line segment joining the points (0, 2) and (3, 0)	1
17.	The vector equation of z- axis is: (a) $\vec{r} = \hat{\imath}$ (b) $\vec{r} = \hat{\jmath}$ (c) $\vec{r} = \lambda\hat{k},\ \lambda$ *is a scalar* (d) $\vec{r}.\hat{k} = 0$	1
18.	Which of the following sets are convex? (a) $\{(x, y): x^2 + y^2 \geq 1\}$ (b) $\{(x, y): y^2 \geq x\}$ (c) $\{(x, y): 3x^2 + 4y^2 \geq 5\}$ (d) $\{(x, y): y \geq 2, y \leq 4\}$	1
	Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled **Assertion (A)** and the other labelled **Reason (R).** Select the correct answer from the codes (a), (b), (c) and (d) as given below. (i) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (ii) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (iii) Assertion (A) is true and Reason (R) is false. (iv) Assertion (A) is false and Reason (R) is true.	
19.	**Assertion (A):** If a is an integer, then the straight lines $\vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(a\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ and $\vec{r} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k} + \mu(3\hat{\imath} + a\hat{\jmath} + 2\hat{k})$ intersect at a point for $a = -5$. **Reason (R):** Two straight lines intersect if the shortest distance between them is zero. (a) i (b) ii (c) iii (d) iv	1

20.	**Assertion (A):** 20 persons are sitting in a row. Two of these persons are selected at random. The probability that the two selected persons are not together is 0.9 **Reason (R):** If $\bar{A}$ denotes the negation of an event A, then $P(\bar{A}) = 1 - P(A)$. (a) i (b) ii (c) iii (d) iv	1
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21.	If P and Q are two points with position vectors $3\vec{a} - 2\vec{b}$ and $\vec{a} + \vec{b}$ respectively. Write the position vector of a point R which divides the line segment PQ externally in the ratio **2: 1**	2
22.	The volume of a cube is increasing at a constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.	2
23.	Find the intervals in which the function given by $f(x) = \sin x + \cos x,\ 0 \le x \le 2\pi$ is (i) Increasing (ii) Decreasing	2
24.	Evaluate: $\left[sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]\right]$	2
	or	
	What is the principal value of $\cos^{-1}\left(cos\frac{2\pi}{3}\right) + \sin^{-1}\left(sin\frac{2\pi}{3}\right)$?	2
25.	Evaluate: $\int_1^4 \{\vert x-1\vert + \vert x-2\vert + \vert x-4\vert\}\,dx$	
	or	
	Evaluate: $\int sin^3x\,dx$	
	SECTION C **This section comprises short answer (SA) type questions of 3 marks each.**	
26.	Evaluate: $\int (3x-2)\sqrt{x^2+x+1}dx$	3
	or	
	Evaluate: $\int_0^1 \frac{log\vert 1+x\vert}{1+x^2}dx$	
27.	A farmer has a field of shape bounded by $x = y^2$ and $x = 3$, he wants to divide this into his two sons equally by a straight line $x = c$. Can you find c?	3
28.	Find the particular solution of the following differential equation: $(x+1)\frac{dy}{dx} = 2e^{-y} - 1; y = 0$ when $x = 0$.	
29.	Solve the differential equation $(\tan^{-1}(x) - y)dx = (1+x^2)dy$	3
	or	
	Find the solution of the differential equation $(xdy - ydx)ysin\left(\frac{y}{x}\right) = (ydx + xdy)xcos\left(\frac{y}{x}\right)$.	
30.	Solve the following Linear Programming Problem graphically: Minimize: $Z = 18x + 10y$ Subject to constraints: $4x + y \ge 20$. $2x + 3y \ge 30$. $x, y \ge 0$.	
31.	There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards drawn at random without replacement. Let X denoted the sum of the numbers on the two drawn cards. Find the mean of X.	
	or	

	The random variable X has a probability distribution P(X) of the following form, where k is some number: $P(X = x) = \begin{cases} k, & if\ x = 0 \\ 2k, & if\ x = 1 \\ 3k, & if\ x = 2 \\ 0, & otherwise \end{cases}$ (i) Determine the value of k (ii) Find $P(\leq 2)$	3
	SECTION D **This section comprises long answer (LA) type questions of 5 marks each.**	
32.	Using the method of integration, find the area of the region bounded by the lines: $5x - 2y - 10 = 0; x + y - 9 = 0; and\ \ 2x - 5y - 4 = 0$	5
33.	Show that the function $f: \{x \in R - 1 < x < 1\}$ defined by $f(x) = \frac{x}{1+\|x\|}, x \in R$ is one-one and onto function.	5
34.	If $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$, find A^{-1}. Hence solve the following system of equations: $x + 2y + 5z = 10, x - y - z = -2, 2x + 3y - z = -11.$	5
	or	
	If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are square matrices, find A.B and hence solve the system of equations: $x - y = 3,\ 2x + 3y + 4z = 17,\ y + 2z = 7.$	5
35.	Using vectors, show that the points A(-2, 3, 5), B(7, 0, -1), C(-3, -2, -5) and D(3, 4, 7) are such that AB and CD intersect at the point P(1, 2, 3).	5
	or	
	Find the shortest distance between the following pair of lines: $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{4}; \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$	5
	SECTION E **This section comprises 3 case study based questions of 4 marks each.**	
	CASE STUDY- I	
36.	At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. Such a coin is unbiased with equal probabilities of getting head and tail.  Based on the above information, answer the following questions. Show steps to support your answer.	
(a)	Find the Random variable for the number of heads in two tosses of the coin.	1
(b)	If such a coin is tossed two times; find the probability distribution of number of tails	1
(c)	Find the probability of getting at least one head in three tosses of such a coin.	1

	CASE STUDY- II	
37.	Read the following passage and answer the questions given below. An architect designs a building for a multinational company. The floor consists of a rectangular region with semi-circular ends having a perimeter of 200 m as shown here:  *Design of Floor*  *Building*	
(a)	If x and y represents the length and breadth of the rectangular region, then find the relation between the variable.	1
(b)	Find the area of the rectangular region A expressed as a function of x.	1
(c)	Find the maximum value of area A. or The CEO of the multi-national company is interested in maximizing the area of the whole floor including the semi-circular ends. Find the value of x for this to happen	2
	CASE STUDY- III	
	A student made a cube of side 10cm with one vertex at the origin and edges along the coordinate axes as shown in the following figure.  Based on the above information, answer the following:	
(a)	Identify the vertex of P.	1
(b)	What are the direction cosines of diagonal OP?	1
(c)	What are the direction cosines of diagonal CM? or Find the acute angle between the two diagonals of a cube.	2