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CLASS XII
PRE BOARD EXAMINATION 2023-24
MATHEMATICS (041)
SET A2

Time:3hrs.

General Instructions: **M.M.80**

1.This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A (Multiple Choice Questions)
Each question carries 1 mark

1.Two events A and B will be independent, if

(a) A and B are mutually exclusive (b) $P(A) = P(B)$
(c) $P(A'B') = [1 - P(A)][1 - P(B)]$ (d) $P(A) + P(B) = 1$

2. The number of corner points of the feasible region determined by the constraints
True $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is
True
(a) 2 (b) 3 (c) 4 (d) 5

3. Which of the following points satisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$?
(a) (2, 4) (b) (3, 2) (c) (5, 6) (d) (4, 2)

4.The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is

(a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $\log x - \log y = C$
(c) $xy = C$ (d) $x + y = C$

5.In which of the following differential equations is its degree equal to its order?

(a) $x^3\left(\frac{dy}{dx}\right) - \frac{d^3y}{dx^3} = 0$ (b) $\left(\frac{d^3y}{dx^3}\right)^3 + \sin\left(\frac{dy}{dx}\right) = 0$
(c) $x^2\left(\frac{dy}{dx}\right)^4 + \sin y - \left(\frac{d^2y}{dx^2}\right)^2 = 0$ (d) $\left(\frac{dy}{dx}\right)^3 + x\left(\frac{d^2y}{dx^2}\right) - y^3\left(\frac{d^3y}{dx^3}\right) + y = 0$

6. $\int e^{(x\log 5)} e^x dx =$
(a) $\frac{(5e)^x}{\log 5e} + c$ (b) $\log 5^x + x + c$ (c) $5^x e^x + c$ (d) $(5e)^x \log x + c$

$\int 5^x e^x dx$

$5^x e^x$

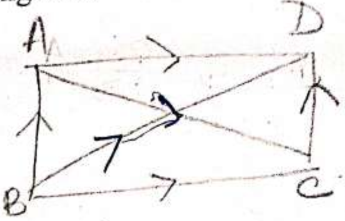
$4a+4=6$
$4a=2$ or $1/2$

7. If the point P(a, b, 0) lies on the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$ then (a,b) is

(a) (1,2) (b) $\left(\frac{1}{2},\frac{2}{3}\right)$ (c) $\left(\frac{1}{2},\frac{1}{4}\right)$ ✓ (d) (0,0)

8. The value of k for which the lines $\frac{x-5}{7}=\frac{2-y}{5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{2y-1}{k}=\frac{z}{3}$ are at right angles is

(a) 2 (b) 4 (c) -4 (d) -2 ✓



9. If ABCD is a parallelogram and AC and BD are its diagonals, then $\overrightarrow{AC}+\overrightarrow{BD}$ is :

(a) $2\overrightarrow{DA}$ (b) $2\overrightarrow{AB}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{BD}$

10. The position vectors of two points A and B are $(2\vec{a}+\vec{b})$ and $(\vec{a}-3\vec{b})$ respectively. The position vector of a point C which divides AB externally in the ratio 1: 2 is

(a) $-3\vec{a}-5\vec{b}$ ✓ (b) $-7\vec{b}$ (c) $\frac{1}{3}(5\vec{a}-\vec{b})$ (d) $(3\vec{a}+5\vec{b})$

$\overrightarrow{AD}+\overrightarrow{BA}+\overrightarrow{BC}+$
$\overrightarrow{CD}$
$2\overrightarrow{BC}+2\overrightarrow{AB}$

11. If $(2\hat{\imath}+6\hat{\jmath}-22\hat{k})\times(\hat{\imath}+\lambda\hat{\jmath}+\mu\hat{k})=\vec{0}$ then $\lambda-\mu$ is equal to

(a) -8 (b) -14 (c) 14 ✓ (d) 8

12. If $y=\sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x=1$ is equal to

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\frac{1}{2}$ ✓

13. The value of k for which the function f given by $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & \text{if } x\neq\frac{\pi}{2}\\ 5, & \text{if } x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$, is

(a) 6 (b) 5 (c) $\frac{5}{2}$ (d) 10

14. A and B are skew-symmetric matrices of same order. AB is symmetric if

(a) AB = O (b) AB = −BA (c) AB = BA ✓ (d) BA = O

15. For which value of x, are the determinants $\begin{vmatrix}2x & -3\\ 5 & x\end{vmatrix}$ and $\begin{vmatrix}10 & 1\\ -3 & 2\end{vmatrix}$ equal ?

$2x^2+15=20+3$
$2x^2+15=23$
$2x^2=8$
$x^2=4$

(a) ±3 (b) -3 (c) ±2 ✓ (d) 2

16. In the determinant $\begin{vmatrix}2 & -3 & 5\\ 6 & 0 & 4\\ 1 & 5 & -7\end{vmatrix}$, M_{23} is : (where M_{ij} denotes the minor of element a_{ij})

$\begin{vmatrix}2 & -3\\ 1 & 5\end{vmatrix}=10+3$

(a) 7 (b) -13 (c) 13 ✓ (d) -7

17. If A is a symmetric matrix then which of the following is not Symmetric matrix

(a) $A-A'$ ✓ (b) $A.A'$ (c) $A+A'$ (d) A'

18. Suppose P, Q and R are different matrices of order 3×5, $\underline{a} \times \underline{b}$ and $\underline{c} \times \underline{d}$ respectively, then value of $ac + bd$ is, if matrix $2P + 3Q - 4R$ is defined

(a) 9 (b) 34 (c) 30 (d) 15

Question number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19. Assertion(A) :Minimum value of the function $f(x) = (2x-1)^2 + 3$ is 3.
Reason(R): $f(x) < 3$ for all real values of x.

20. Assertion (A):Relation R on set A={1,2} defined as R={(1,1),(2,2),(1,2)} is not an identity relation.
Reason (R) : A relation R on a set A is identity relation iff R= {(a, b) : a∈A, b ∈A and a =b} .

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Evaluate $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} \, dx$.

22. Find critical point/s for $f(x) = x^x$.

Or

Find the point/s of local maximum/local minimum for $f(x) = \frac{\log x}{x}$.

23. Find the intervals in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$ is strictly increasing.

24. The radius of an air bubble is increasing at the rate of 0.5 cm/s. At what rate is the surface area of the bubble increasing when the radius is 1.5 cm ?

25. Draw the graph of $f(x) = \sin^{-1} x, x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).

$0 \quad \frac{1}{2} \quad \frac{1}{\sqrt{2}}$

Or

Check the injectivity and surjectivity of the function f : N⟶ N, given by $f(x) = x^3$.

1+4+4+5

SECTION C

This section comprises of short answer (SA) type questions of 3 marks each.

26. Solve the following Linear Programming Problem graphically:
Maximise $P = 70x + 40y$
subject to the constraints : $3x + y \leq 9$, $3x + 2y \leq 9$, $x \geq 0, y \geq 0$.

Or

Solve the following linear programming problem graphically:
Minimise $Z = 200x + 500y$
subject to the constraints: $x + 2y \geq 10$, $3x + 4y \leq 24$, $x \geq 0$, $y \geq 0$

27. Solve the differential equation $x\,dy - y\,dx = \sqrt{x^2 + y^2}dx$.

Or

Find the particular solution of the differential equation

$\frac{dy}{dx} + y\cot x = 4x\ \mathrm{cosec}\,x\ (x \neq 0)$ given $y\left(\frac{\pi}{2}\right) = 0$.

28. Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

29. Evaluate $\int \frac{1}{1+\cot x} dx$.

Or

Evaluate $\int_{\frac{\pi}{2}}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx$.

30. Evaluate $\int \frac{2x^2+1}{x^2(x^2+4)} dx$.

31. If $y = \log\left(\frac{x}{a+bx}\right)^x$, prove that $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

SECTION D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ find AB and hence solve the system of equations $x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$.

164
13
182

33.Find the shortest distance between the pair of lines
$\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}; z = 2.$

Or

Find the foot of the perpendicular drawn from the point $(3, -1, 11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.
Also, find the equation of the line perpendicular to the line from the given point.

34.Let $A = R - \{3\}$ and $B = R - \{1\}$ be two sets. Prove that the function $f: A \rightarrow B$ given by $f(x) = \left(\frac{x-2}{x-3}\right)$ is onto. Is the function f one-one? Justify your answer.

Or

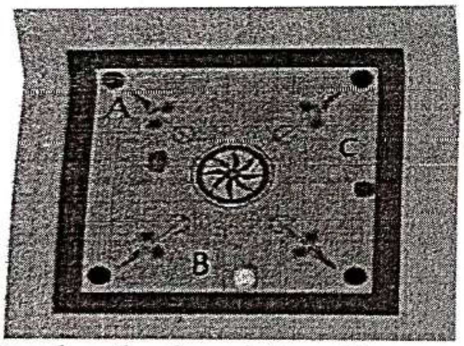
A relation R is defined on a set of real numbers **R** as R = {(x, y) : xy is an irrational number}.
Check whether R is reflexive, symmetric and transitive or not.

35.Using integration find the area of the region enclosed by the line $y = \sqrt{3}\,x$, semi circle $y = \sqrt{4 - x^2}$ and the x-axis in the first quadrant.

SECTION E

(This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

36.A player is playing carrom game.Suppose the striker is at point A(1,1,3) , a white coin is at the point B(2,3,5) and a black coin is at point C(4,5,7)



Based on the above information answer the following

(i)If the striker hits the white coin then find the vector representing its path.

(ii)Find the distance covered by the striker to hit the white coin.

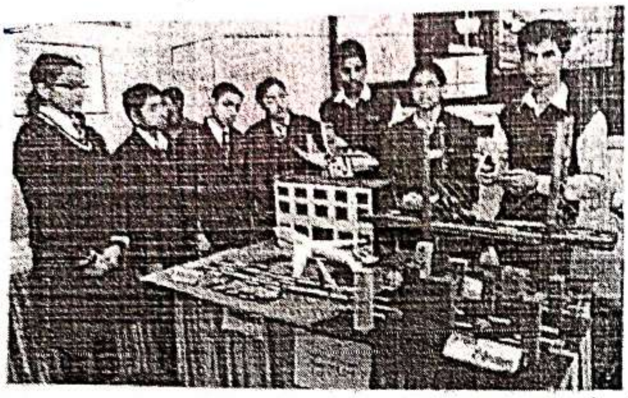
(iii)Find $\overrightarrow{AC} \times \overrightarrow{AB}$.

Or

Find a unit vector along the vector $\overrightarrow{AC} + \overrightarrow{AB}$.

37. In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

14 = 1
15 = 2
16 = 3
17 = 2
19 = 2



On the basis of the above information, answer the following questions :

(i) Find the probability that the age of the selected student is a composite number.

$\frac{\frac{2}{8}}{\frac{3}{5}}$

(ii) Let X be the age of the selected student. What can be the value of X ?

(iii) Find the probability distribution of random variable X and hence find the mean age.

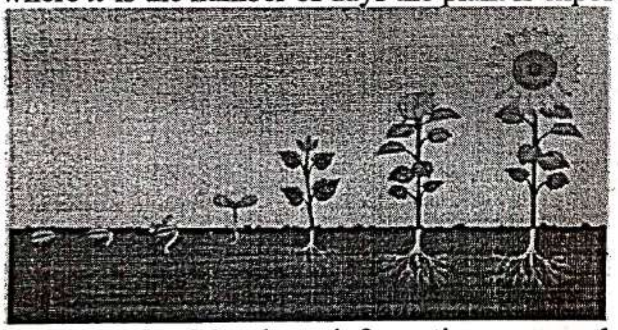
Or

A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number.

38. A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function

$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2\ ,\ 0 \le x \le 10$

where x is the number of days the plant is exposed to sunlight.



On the basis of the above information, answer the following questions :

(i) What are the critical points of the function $f(x)$?

(ii) Using second derivative test, find the minimum value of the function.