

Time: 3hrs

General Instructions:

- i) All questions are compulsory
- ii) The question paper consist of 26 questions divided into three sections: A, B and C. Section A comprises of 6 questions of one mark each. Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
- iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the questions.
- iv) There is no overall choice. However internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternative in all such questions.
- v) Use of calculator is not permitted. You may ask for logarithmic tables and graph paper if required.

SECTION A

(Question 1 to 6 carry 1 mark each)

1. If  $f: R \rightarrow R$  and  $g: R \rightarrow R$  and  $f(x) = x^2 + 1$  and  $g(x) = \sin x$  then find  $g \circ f$ .

2. Construct a  $2 \times 2$  matrix  $A = [a_{ij}]$  whose elements  $a_{ij}$  are given by  $a_{ij} = \frac{(i-j)^2}{2}$ .

3. The value of  $\sin\left(\frac{1}{4} \sin^{-1} \frac{\sqrt{63}}{8}\right)$  is:

- a)  $\frac{1}{\sqrt{2}}$
- b)  $\frac{1}{\sqrt{3}}$
- c)  $\frac{1}{2\sqrt{2}}$
- d) None of these

4. Show that all elements of main diagonal of skew - symmetric matrix are zeros?

5. Find an angle  $\theta$ , which increases twice as fast as its Sine.

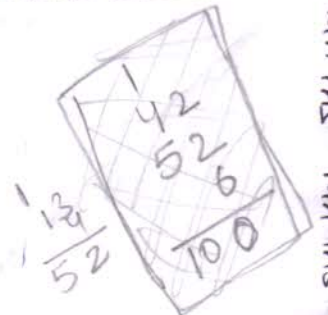
6. If  $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$  &  $B = [1 \ 0 \ 4]$ . Find  $(AB)^t$

SECTION B

(Question 11 to 22 carry '4' marks each)

7. If  $f(x) = \begin{cases} \frac{\sqrt{1+px} - \sqrt{1-px}}{x} & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-2} & \text{if } 0 \leq x \leq 1 \end{cases}$

is continuous, then find value of 'p'



BODMAS  
Bracket  
Division  
Multiplication  
Addition  
Subtraction  
 $26 - 8 + 5(7 - 8)$   
 $-26 - 8 - 5(-1)$   
 $= 13$

8. Show that the curves  $4x = y^2$  and  $4xy = k$  cut at right angles, if  $k^2 = 512$

OR

Prove that the function  $f(x) = x^2 - x + 1$  is neither increasing nor decreasing on  $(-1, 1)$

9. If  $x = \sin\left(\frac{1}{a} \log y\right)$  then show that  $(1-x^2)y_2 - xy_1 - a^2y = 0$

10. Solve the differential equation:

$$(1+x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}, \text{ given that}$$

$$y = 0 \text{ when } x = 1$$

OR

Integrate

$$\int \frac{x^2}{(x \sin x + \cos x)^2} dx$$

11. Prove that  $2 \tan^{-1}\left(\sqrt{\frac{a-b}{a+b}} \tan \frac{\theta}{2}\right) = \cos^{-1}\left(\frac{a \cos \theta + b}{a + b \cos \theta}\right)$

12. Integrate  $\int \frac{dx}{\sin x + \sec x}$

13. Show that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} \\ = (a-b)(b-c)(c-a)(ab+bc+ca)$$

14. If  $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  then prove that  $(aI + bA)^n = a^n I + na^{n-1} bA$  where  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

and  $n$  is a positive integer  $n$ .

OR

Solve initial value problem

$$(x+y+1)^2 dy = dx; \quad y(-1) = 0$$

15. Integrate

(i)  $\int \tan(x - \theta) \tan(x + \theta) \tan 2x \, dx$

(ii)  $\int \sqrt{1 + 2 \tan x (\tan x + \sec x)} \, dx$

16. Integrate  $\int_{-1}^1 e^x \, dx$  limits as a sum

OR

Integrate  $\int \frac{dx}{\sqrt{\sin^3 x \sin(x + \alpha)}}$

17. The total cost of providing x radio sets per day is Rs.  $\left(\frac{x^2}{4} + 35x + 25\right)$  and the price per set at which they may be sold is Rs  $\left(50 - \frac{x}{2}\right)$ . Find the daily output to maximize the total profit.

18. If  $x\sqrt{1+y} + y\sqrt{1+x} = 0$  then prove that

$$\frac{dy}{dx} = -\frac{1}{(1+x)^2}$$

19. Show that  $f(x) = x^2$  is differentiable at  $x = 1$  and find  $f'(1)$

### SECTION C

(Question 20 to 26 carry '6' marks each)

20. Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse having its vertex coincident with one end of the major axis.

21. Find the area of the region  $\{(x, y): y^2 \leq 4x; 4x^2 + 4y^2 \leq 9\}$

*Marks will be adjusted*

22. If  $x = \sqrt{a \sin^2 t}$ ;  $y = \sqrt{a \cos^2 t}$ ;  $a > 0$

and  $-1 < t < 1$  then show that  $\frac{dy}{dx} = -\frac{y}{x}$

23. If  $A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$  Find  $A^{-1}$  through elementary row transformation method.

OR

Solve the initial value problem

$$(1 + y^2) \, dx = (\tan^{-1} y - x) \, dy; \, y(0) = 0$$

*Handwritten notes:*  
Tanya  
2-1  
28+28

12

24. Show that the function  $f$  in  $A = \mathbb{R} - \left\{ \frac{2}{3} \right\}$  defined as  $f(x) = \frac{4x+3}{6x-4}$  is one - one and onto.

Hence find  $f^{-1}$ .

123

25. Integrate  $\int \frac{x^2+1}{(x-1)^2(x+3)} dx$

OR

Determine the product

$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$  and use it to solve the system of equation

$x - y + z = 4; \quad x - 2y - 2z = 0; \quad 2x + y + 3z = 1$

26. From differential equation representing the family of ellipses having centre at the origin and foci on x-axis.

OR

If  $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$  then show that  $A^2 - 7A + 10I_3 = 0$  and hence find  $A^{-1}$ .

40	30	20	10
39	29	19	9
38	28	18	8
37	27	17	7
36	26	16	6
35	25	15	5
34	24	14	4
33	23	13	3
32	22	12	2
31	21	11	1

$$\begin{array}{r} x - y + z = 4 \\ -x - 2y - 2z = 0 \\ \hline y + 3z = 4 \end{array}$$

$$\begin{array}{r} 4 \\ 27 \\ 6 \\ \hline 1620 \end{array} \quad \begin{array}{l} 2x + 4 = 1 \\ x = -\frac{3}{2} \end{array}$$