MANAVA BHARATI
INDIA INTERNATIONAL SCHOOL

MID-TERM EXAMINATION 2014
CLASS XII – MATHEMATICS

Time: 3 Hours. M.M.100

General Instructions:

i) Question No. 1 to 6 carry one mark each.
ii) Question No. 7 to 19 carry 4 marks each.
iii) Question No. 20 – 26 carry 6 marks each.

1. For what value of K, the matrix is a skew symmetric matrix.

$$\begin{pmatrix} 2K+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2K-3 \end{pmatrix}$$

2. If $\begin{vmatrix} \sin\alpha & \cos\beta \\ \cos\alpha & \sin\beta \end{vmatrix} = \frac{1}{2}$, where α, β are acute angles, then write the value of $\alpha+\beta$

3. Write the principal value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

4. Find the derivative of $\log_{10}x$ w.r.t. x.

5. Evaluate $\int_0^{1/\sqrt{2}} \frac{dx}{\sqrt{1-x^2}}$

6. Evaluate $\int \frac{x^2}{1+x^3}\,dx$

7. Using properties of determinants show that

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

8. Differentiate with respect to x

$$(\log x)^x + x^{\log x}$$

OR

If $x = a\sin t$ and
$y = a\left(\cos t + \log\tan\frac{t}{2}\right)$ Find $\frac{d^2y}{dx^2}$

9. Evaluate $\int \frac{5x}{(x+1)(x^2+9)}\,dx$

OR

Evaluate $\int \frac{x^2}{(x^2+4)(x^2+9)}\,dx$

10. Find the interval in which the function f given by $f(x) = \sin x - \cos x$ $0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

11. If $x^p y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

12. Show that the function $f(x) = |x-3|$, $x \in R$ is continuous but not differentiable at $x=3$.

13. Evaluate $\int \frac{Sinx+Cosx}{\sqrt{sinx\ cosx}}$

OR

Evaluate $\int \frac{x^2+4}{x^4+x^2+16}\,dx$

14. Find all points of discontinuity of f, where

$$f(x) = \begin{cases} \frac{Sin\,x}{x}, & \text{if } x < 0 \\ x+1, & \text{if } x \geq 0 \end{cases}$$

15. Prove that

$$tan^{-1}\left(\frac{1}{3}\right) + tan^{-1}\left(\frac{1}{5}\right) + tan^{-1}\left(\frac{1}{7}\right) + tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$$

OR

Prove that $tan^{-1}\left\{\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right\} = \frac{\pi}{4} - \frac{1}{2}\,cos^{-1}\,x;\ -\frac{1}{\sqrt{2}} \leq x \leq 1$

16. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

17. Solve for x

$$tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$$

18. Evaluate $\int_1^3 (x^2+1)dx$ as limit of sums.

19. If $F(x) = \begin{vmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{vmatrix}$ show that $F(x)\,F(y) = F(x+y)$

20. If $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$

Verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1}.

OR

Obtain the inverse of the matrix using elementary row transformations.

$$A = \begin{pmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix}$$

21. Using matrix, solve the following system of equations

$x + y + z = 6;\ \ x + 2z = 7;\ 3x + y + z = 12$

22. Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$

OR

Find the area of the region in the first quadrant enclosed by x – axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

23. Show that the height of the cylinder of maximum volume, that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

24. Evaluate $\int_0^{\pi/2} \log \sin x \ dx$

25. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

26. a) Evaluate $\int x^2 \tan^{-1} x \, dx$

b) Using differential, find the approximate value of f (2.01) where $f(x) = 4x^3 + 5x^2 + 2$