Name Kirti Nagpal Class & Section _____ Roll No. _____

FIRST TERMINAL EXAMINATION-2015-2016

Class-XII

Subject-Mathematics

Time Allowed : 3 Hrs. **M.M. : 100**

Please check the total marks

Do not write any answers on the question paper.

Instructions :

- *Section-A consists of 6 questions carrying 1 mark each.*
- *Section-B consists of 13 questions carrying 4 marks each.*
- *Section-C consists of 7 questions carrying 6 marks each.*
- *All the questions are compulsory however internal choice has been given in some of the questions.*

Section-A

1. Find the number of binary operatios on $\{1,2\}$ having 1 as identity and having 2 as inverse of itself.

2. Find the value of $\tan^{-1}\left(\tan\left(\frac{-7\pi}{6}\right)\right)$.

3. If A is a square matrix such that $A^2 = A$, then the value of $(I+A)^3 - 7A$.

4. If A is an invertible matrix of order n, then find the value of $\det\left(A^{-1}\right)$.

5. Find the differentition of $\sin^{-1}\left(\sec\left(x^3\right)\right)$.

6. Find the minimum and maximum value if any of the function $f(x) = -|x+1| - 3$.

IS→ $3A^2 + 3A + I$

$= 2I + 6A - 7$

$-\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$

0-7/3

Section-B

7. Show that the function $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \dfrac{x}{1+|x|}, x \in R$ in one one and onto function. Find the inverse function also.

8. Prove that :

$$\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$$

OR

Evaluate : $\tan\left(\sin^{-1}\dfrac{3}{5} + \cot^{-1}\dfrac{3}{2}\right)$.

9. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, and $f(x) = x^2 - 5x - 14$, find $f(A)$.

Hence obtain A^3.

10. Using properties of determinants, prove that :

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

$R_3 \to R_3 - R_1 - R_2$

OR

Using properties of determinants, prove that

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$$

11. Let

$$f(x) = \begin{cases} = \dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x < \dfrac{\pi}{2} \\ a, & \text{if } x = \dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2}, & \text{if } x > \dfrac{\pi}{2} \end{cases}$$

If $f(x)$ be a continuous function at $x = \dfrac{\pi}{2}$, Find a and b.

12. If $\sin y = x \sin(a+y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

OR

If $y = \sqrt{\frac{1-x}{1+x}}$ prove that $(1-x^2)\frac{dy}{dx} + y = 0$.

13. If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant independant of a and b.

14. A man is walking at the rate 6.5 km/hr towards the foot of a tower 120m high. At what rate is he approaching the top of the tower when he is 50m away from the tower?

15. Use differentials to find the approximate value of $\log_e(4.01)$, having given that $\log_e 4 = 1.3863$.

16. Find the integral of the function $\frac{1}{\cos(x-a)\cos(x-b)}$ with respect to x.

OR

Find the integral of the function, $\frac{x+3}{\sqrt{5-4x+x^2}}dx$

17. Find $\int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta} d\theta$.

18. Find $\int x\cos^{-1}x \; dx$.

19. Evaluate the integral $\int \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$.

Section-C

20. Solve the following system of equations, using matrix method.

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\quad \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\quad \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2.,\ x,y,z\neq 0$$

21. Differentiate $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ with respect to x.

Also find its domain.

22. Find the equation of tangents to the curve $y=\cos(x+y),\quad -2\pi\le x\le 2\pi$ that are parallel to the line $x+2y=0$.

OR

Find intervals in which the function given by

$f(x)=\frac{3}{10}x^4-\frac{4}{5}x^3-3x^2+\frac{36}{5}x+11$ is strictly increasing and strictly decreasing.

23. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$

24. Evaluate $\int\frac{1}{\sin x-\sin 2x}dx$

OR

Evaulate $\int_1^3(x^2-2x+e^x)dx$ using limit of a sum method.

25. Using properties of definite integral; evaluate

$$\int_0^{\pi}\log(1+\cos x)dx$$

26. Using integration, find the area lying above the x–axis and included between the circle $x^2+y^2=8x$ and the parabola $y^2=4x$.