AMITY, PV

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First Terminal Examination 2015 - 2016

Class – XII

Subject – Mathematics

Time : 3 Hours **Max. Marks : 100**

General Instructions **:**

1. All questions are compulsory.
2. The question paper consists of 26 questions, divided into three Sections – A, B and C. Section A comprises of 6 questions of 1 mark each. Section B comprises of 13 questions of 4 marks each and Section C comprises of 7 questions of 6 marks each.
3. All question in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, an internal choice has been provided in 4 questions of 4 marks each and 2 questions of 6 marks each.
5. Use of calculator is not permitted. You may ask for logarithms tables if requires.

SECTION – 'A' (1×6=6)

1. If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, find x, $0 < x < \frac{\pi}{2}$, when $A + A' = I$.

2. Write the value of $\int e^{3 \log x} . (x^4)\, dx$

3. Evaluate : $\sin^{-1}\left(-\frac{1}{2}\right) + 2\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

4. Find the cofactor of a_{23} in the following :

$$\begin{bmatrix} 3 & 4 & 5 \\ -6 & 2 & -3 \\ 8 & 1 & 7 \end{bmatrix}$$

5. Let $*$ be a binary operation defined by $a * b = 2a + b - 3$, find $3 * 4$.

6. For what value of k, the matrix

$$\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$$ is not invertible ?

SECTION – 'B' (4×13=52)

7. Show that the relation R on the set $A = \{x \in z : 0 \leq x \leq 12\}$, given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

OR

Let $*$ be the binary operation and N given by $a * b$ = LCM of a and b. Find :

(a) Is $*$ commutative ?

(b) Is $*$ associate ?

(c) Find the identity element of $*$ in N.

8. Prove that :

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$$

9. Prove that :

$$\tan^{-1}\left[\frac{3\sin 2\alpha}{5 + 3\cos 2\alpha}\right] + \tan^{-1}\left(\frac{1}{4}\tan\alpha\right) = \alpha$$

10. Evaluate : $\int \frac{4x+5}{\sqrt{2x^2+x-3}}\,dx$

OR

Evaluate : $\int \frac{3x-2}{(x+3)(x+1)^2}$

11. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

12. If f(x), defined by the following, is continuous at x = 0, find the values of a, b and c.

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & , \text{ if } x < 0 \\ c & , \text{ if } x = 0 \\ \dfrac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}} & , \text{ if } x > 0 \end{cases}$$

13. Find the equation of the tangent and normal to the curve $x = a\sin^3\theta$ and $y = a\cos^3\theta$ at $\theta = \frac{\pi}{4}$.

14. If $y = \sqrt{x^2 + 1} - \log\left(\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}}\right)$, find $\frac{dy}{dx}$.

OR

If $y = \left(x + \sqrt{x^2 - 1}\right)^m$, then show that

$$(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$$

15. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, find x and y such that $(xI + yA)^2 = A$.

16. Find the intervals in which the function $f(x) = \sin^4 x + \cos^4 x$, $0 \le x \le \frac{\pi}{2}$ is increasing or decreasing.

17. Consider $f: R_+ \longrightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$.

Prove that f(x) is one-one and onto and show that f is invertible with :

$$f^{-1}(y) = \left(\frac{\left(\sqrt{y+6}\right) - 1}{3}\right)$$

18. Prove that :

$$\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$$

OR

Prove that :

$$\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$$

19. Find the area of the region enclosed between

$x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

SECTION – 'C' (6×7=42)

20. Evaluate the following definite integral as limit of sum :

$$\int_1^4 (x^2 - x)\,dx$$

21. Using integration, find the area of the region bounded by the triangle where vertices are (–1, 2), (1, 5) and (3, 4).

22. A total amount of ₹ 7000 is deposited in three different savings bank accounts with annual interest rate of 5%, 8% and $8\frac{1}{2}$% respectively. The total annual interest from these three accounts is ₹ 550. Equal amounts have been deposited in the 5% and 8% saving accounts. Find the amount deposited in each of the three accounts, with the help of matrices.

23. If length of three sides of a trapezium other than base are equal to 10 cm, then find the area of the trapezium when it is maximum.

OR

A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

24. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$,

prove that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$

25. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

Show that $A^3 - 6A^2 + 5A + 11I = 0$. Hence find A^{-1}.

26. Using the properties of definite integrals, evaluate :

$$\int_0^{2\pi} \frac{1}{1 + e^{\sin x}}\, dx$$

OR

$$\int_0^{\pi} \frac{4x \sin x}{1 + \cos^2 x}\, dx$$

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