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FIRST TERM EXAMINATION, 2015–16

Subject : Mathematics

Time : 3 Hrs. **CLASS : XII** Gobind Agarwal **M.M. : 100**

Note :

(1) All questions of section–A carry 1 mark each.

(2) All questions of section–B carry 4 marks each.

(3) All questions of section–C carry 6 marks each.

Section–(A)

1. If A is any square matrix of order 3 and $|A.adj.A| = 8$, Find $|A|$

2. Find $(AB)^1$ if $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$, $B^1 = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$

3. Evaluate :

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$$

4. Evaluate :

$$\int \frac{(e^{x-1} + x^{e-1})}{e^x + x^e} dx$$

5. Differentiate :

$$e^{\sqrt{a\cos x + 3}}$$

6. Evaluate :

$$\int e^{3\log x} \cdot x^4 dx$$

Section–(B)

7. Evalaute :

$$\int \frac{dx}{\cos(x-a)\cos(x-b)}$$

8. Find the ara of the region :

$\{(x.y) : y^2 \geq ax,\ x^2 + y^2 \leq 2ax,\ x \geq 0,\ y \geq 0\}$

9. Let $f : N \to R$ be a function defined as

$$f(x) = 4x^2 + 12x + 15.$$

Show that $f : N \to S$ where S is the range of f, is invertible. Find the inverse of f.

10. Prove that :

$$\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$$

11. If a, b, c are all positive and are the pth, qth and rth terms of G.P. then show that :

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

OR

By using elementary row operations find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

12. The volume of a cube is increasing at the rate of 7 cm^3/sec. How fast is the surface area increasing when the length of an edge is 12 cm.

13. Evaluate :

$$\int \left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$$

14. Evaluate :

$$\int_{-1}^{3/2} |x \sin \pi x|\ dx$$

15. Evaluate :

$\int_0^1 (3x^2 + 2x + 1)\ dx$ as limit of sums.

16. Let * be the binary operation on the set Q. of rational numbrs as $a * b = \frac{ab}{4}$, check the operation for commutative and associative property.

17. Prove that :

$$\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}\frac{3}{5}$$

18. The sum of the length of the hypotenuse and a side of a right triangle is given show that the area of triangle is maximum when the angle between them is $\frac{\pi}{3}$.

19. Verify rolle's theorem for

$f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$

Section–(C)

20. An open box with a square base is to be made out of a given card board of area c^2 sq. units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

21. Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$

OR

Find $\frac{dy}{dx}$ if $x=\frac{\sin^3 t}{\sqrt{\cos 2t}}$, $y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$

22. Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations :

$$x - y + 2z = 1$$

$$2y - 3z = 1$$

$$3x - 2y + 4z = 2$$

23. Water is dripping out from a conical funnel of semi vertical angle $\frac{\pi}{6}$ at the uniform rate of 6 cm^2/sec, in its curved surface area through a tiny hole at vertex in the bottom when slant height of water is 4 cm, Find the rate of decrease of slant height of water. Water is a natural resource. Write the importance of water in our daily life.

24. Using integration find the area of the triangle formed by positive x-axis and tangent and normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$

25. Evaluate :

(a) $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 \, dx$

(b) $\int_{1/3}^{1} \frac{(x - x^3)^{1/3}}{x^4} dx$

26. (a) Check the continuity of :

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

at $x = 0$

(b) Differentiate :

$\tan^{-1} \left\{ \frac{\sqrt{1+x^2} - 1}{x} \right\}$ w.r.t. x.