MANAVA BHARATI
INDIA INTERNATIONAL SCHOOL

MID-TERM ASSESSMENT (2015-16)
CLASS XII- MATHEMATICS

Time: 3 Hours

M.M. 100

General Instructions:

   i)   Question No. 1 to 6 carry 1 mark each.
   ii)   Question No. 7 to 19 carry 4 marks each.
   iii)   Question No. 20 and 26 carry 6 marks each.

1.   If A is a square matrix of order 3 and $|3A| = K |A|$ then write the value of k.

2.   If $A = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$ Find the value of $|adj\ A|$

3.   Give examples of two non-zero matrices whose product is zero.

4.   Find the principal value of $\sin^{-1}\left[\sin \frac{4\pi}{3} \right]$

5.   Write the principal value of $\tan^{-1}\left[\sqrt{3} \right] - \cot^{-1}\left[-\sqrt{3} \right]$

6.   Find $\frac{dy}{dx}$ if $\sin^2 x + \cos^2 y = 1$

7.   Prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2\,(a+b+c)^3$$

8.   Solve

$$\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$$

OR

Prove that $\tan^{-1}\left\{ \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right\} = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$.

$$-\frac{1}{\sqrt{2}} \le x \le 1$$

9. Prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

10. For what value of K, the following function is continuous at x = 0

$$f(x) = \begin{cases} \dfrac{1-\cos 4x}{8x^2}, & x \neq 0 \\ K, & x = 0 \end{cases}$$

OR

For what value of K

$$f(x) = \begin{cases} \dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1 \leq x \leq 0 \\ \dfrac{2x+1}{x-1}, & \text{if } 0 \leq x \leq 1 \end{cases}$$

Is continuous at x = 0

11. Differentiate w.r.t. x

$$(\log x)^x + x^{\log x} + x^{\cos x} + x^x$$

12. $e^x + e^y = e^{x+y}$ then prove that $\dfrac{dy}{dx} + e^{y-x} = 0$

OR

$X = a\sin t$, $y = a(\cos t + \log \tan \frac{t}{2})$ then find $\dfrac{d^2y}{dx^2}$

13. Prove that the curves $x = y^2$ and $xy = k$ cut at right angle if $8k^2 = 1$

14. Evaluate $\displaystyle\int \frac{x^2+1}{(x+1)^2} e^x \, dx$

15. Evaluate $\displaystyle\int \frac{1-x^2}{x(1-2x)} \, dx$

OR

Evaluate $\displaystyle\int \frac{x^2}{(x^2+4)(x^2+9)} \, dx$

16. Evaluate $\displaystyle\int \sin^{-1}\sqrt{x} \, dx$

17. Evaluate $\displaystyle\int (2x+3)\sqrt{x^2+4x+3} \, dx$

18. Evaluate $\int_1^3 (x^2 + 1)\, dx$ as limit of sums

19. Verify mean value theorem if $f(x) = x^2 - 4x - 3$ is the interval [a, b], where a =1, b =4.

20. Using matrices solve the following system of linear equations

$$X - y + 2z = 7, \quad 3x + 4y - 5z = -5, \quad 2x - y + 3z = 12$$

21. Show that the height of the cylinder of maximum volume that can be inscribed in a cone of height h is $\frac{1}{3}h$.

OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.
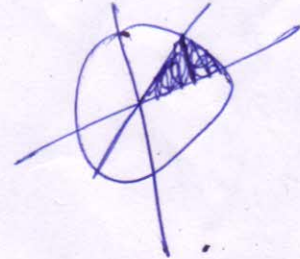
22. Find the area of the region in the quadrant enclosed by x-axis, the line $x = \sqrt{3}\, y$ and the circle $x^2 + y^2 = 4$.

23. Evaluate :

$$\int_0^{\pi/2} \frac{x \operatorname{Sin} x \cos x}{\operatorname{sin}^4 x + \operatorname{Cos}^4 x}\, dx$$

OR

Evaluate:

$$\int_0^{\pi/2} \log(\operatorname{sin} x)\, dx$$

24. Find the interval in which the function f given by $f(x) = \operatorname{sin} x + \cos x, \ 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

25. a) For well being of an orphanage, three trust A,B and C has 10%, 15% and 20% of their total fund Rs.2,00,000, Rs.3,00,000 and Rs.5,00,000 respectively. Using matrix multiplication, find the total amount of money received by orphanage by three trust. By such donation which value are generated?

    b) Prove that $\tan^{-1}\left[\frac{\operatorname{Cos} x}{1 + \operatorname{Sin} x}\right] = \frac{\pi}{4} - \frac{x}{2}$.

    $$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

26. a) Using differentiation find the approximate value of $\sqrt{49.5}$

    b) Evaluate $\int (\sqrt{\tan x} + \sqrt{\cot x})\, dx$