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SET - B

GF-1-400

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NEW GREEN FIELDS SCHOOL
HALF YEARLY EXAMINATION, 2015-16

MATHEMATICS

Time : 3 hrs.

Class - XII

M.M. : 100

General Instructions—

- (1) All questions are compulsory, though internal choices have been provided in 4 questions of 4 marks and two questions of 6 marks.
- (2) Questions have been subdivided in three sections. Section A has six questions of 1 mark each. Section B has thirteen questions (Q. 7 to Q. 19) of 4 marks each. Section C has 7 questions (Q. 20 to Q. 26) of 6 marks each.
- (3) Calculators are not allowed. Mathematical tables may, however, be asked for, if needed.
- (4) Any rough work needs to be done in a column on the right side of the answer sheet.
- (5) Only Blue/Blue black pen should be used to write the answers.
- (6) Please write the correct serial number of the question which you are attempting.

SECTION—A

1 If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in \mathbb{R}$, find $(f \circ g)(7)$

2 If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of a_{32} .

3 If $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -7 & 2 \\ 8x & 4 \end{pmatrix}$, find the value of x .

4 Write the principal value of :

$$\cos^{-1}\left(\frac{1}{2}\right) - 2 \sin^{-1}\left(-\frac{1}{2}\right)$$

[P.T.O.]

5 Find the derivative of $\tan^{-1} \sqrt{2}$ with respect to $\cos^{-1} x$.

6 Simplify :

$$\cos a \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix} + \sin a \begin{bmatrix} \sin a & -\cos a \\ \cos a & \sin a \end{bmatrix}$$

SECTION—B

7 Prove :

$$\cot^{-1} \left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right) = \frac{x}{2}, \quad x \in \left(0, \frac{\pi}{4} \right)$$

8 $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2} & \text{when } x < 0 \\ a & \text{when } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} & \text{when } x > 0 \end{cases}$ for what value of a , f is continuous at $x = 0$?

9 Solve the equation :

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, \quad a \neq 0$$

OR

10 If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & -3 \end{pmatrix}$, verify that $(AB)^T = B^T A^T$.

11 If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$.

12 If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2) \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} - y = 0$

13 Find the intervals in which the following function is (a) strictly increasing (b) strictly decreasing :

$f(x) = -2x^3 - 9x^2 - 12x + 1$

13. Find the condition that the curves $2x = y^2$ and $2xy = k$ intersect orthogonally. K is any constant.

OR

Find the equation of the normal lines to the curve $3x^2 - y^2 = 8$ which are parallel to the line $x + 3y = 4$.

14. If $x = a \sec^3 \theta$ and $y = a \tan^3 \theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$.

15. Prove that :

$\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$

16. Let R be the relation on the set of integers Z defined by $R = \{(a, b) : 2 \text{ divides } a - b\}$. Show that R is an equivalence relation. Also write the equivalence class [0].

OR

Let $A = \mathbb{N} \times \mathbb{N}$ and * be a binary operation on set A, defined as $(a, b) * (c, d) = (a + c, b + d)$. Show that * is commutative and associative. Also find the identity element of * on A (if it exists).

18. The probability of a man hitting a target is 0.5. He shoots 7 times. What is the probability of his hitting at least twice ?

19. Four defective bulbs are accidentally mixed with six good ones. If it is not possible to just look at a bulb and tell whether or not it is defective, find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.

OR

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X) and variance of X.

17. verify Rolle's Theorem for the fun, $f(x) = x(x-4), x \in [0, 4]$

Handwritten calculations and scribbles on the right margin, including a table with numbers like 625, 162, 131, and 137.

SECTION—C

- 20 Examine the consistency of the following system of simultaneous linear equations. If consistent solve it :

$$2x + 3y + 3z = 5; \quad x - 2y + z = -4; \quad 3x - y - 2z = 3$$

OR

Find the inverse of the following matrix using elementary row operations :

$$\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$$

- 21 A rectangular sheet of tin 45 cm and 24 cm is to be made into a box without a top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum.
- 22 Show that the semi-vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1} \sqrt{2}$.

OR

Prove that the perimeter of a right angled triangle of given hypotenuse is maximum when the triangle is isosceles.

- 23 Consider $f : \mathbb{R}_+ \rightarrow (5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible and hence find f^{-1} .
- 24 Using properties of determinants prove that :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

- 25 An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver ?

Mr. Menon goes walking to his office, even though it is 4 km away, and he is wealthy enough to buy any vehicle. But he is afraid of having an accident. What advice would you give him and why ?

26 A merchant plans to sell two types of personal computers— a desk top model MD1 and a portable model MP1 that will cost ₹ 25,000 and ₹ 40,000 respectively. He estimates that the total monthly demand of the computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get a maximum profit if he does not want to invest more than ₹ 70 lakhs and if his profit per MD1 is ₹ 4,500 and that on MP1 is ₹ 5,000. Assuming that he is able to sell all that he stocks. Also what will be his maximum profit ?

SECTION - 3

If $f(x) = 2x^2 + 3x - 4$ and $g(x) = x - 7$, find $(f \circ g)(7)$

If $a \in \mathbb{R}$, write the inverse of a .

If $\begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ 8x & 4 \end{pmatrix}$, find the value of x .

Write the principal value of:

$\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(\frac{1}{2}\right)$

[P.T.O.]