

## GURU HARKRISHAN PUBLIC SCHOOL

First Terminal Exam (2015-16)

CLASS -XII

SUBJECT - Mathematics

Time: - 3 hrs.

M.M. 100

General Instruction:

1. All questions are compulsory
2. The question paper consists of 26 question divided into three section A,B and C. Section A comprises of 6 question of one mark each, section B Comprises of 13 question of 4 marks each and section C comprises of 7 questions of 6 marks each.
3. All question in section A are to be answered in one word, one sentence or per the exact requirement of the questions
4. Use of calculators is not permitted.

SECTION A

1. Let \* be the binary operation on the set Q of rational numbers given as

$$a*b=3a-b+5 \text{ find } 3*5$$

2. What is the value of  $|3I_3|$  where  $I_3$  is the identity matrix of order 3?

3. Find x if  $\begin{vmatrix} 3 & 4 \\ -5 & 2 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ -5 & 3 \end{vmatrix}$

4. Write the principal value of  $\cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\left(\frac{-1}{2}\right)$

5. Evaluate  $\int \frac{dx}{\sqrt{4-x^2}}$

6. Evaluate  $\int_{-\pi/2}^{\pi/2} \sin^3 x \cos x \, dx$

SECTION B

7. Show that the relation R in the set

$$A = \{x: x \in \mathbb{Z}, 0 \leq x \leq 12\}$$

given by  $R = \{(a,b); |a-b| \text{ is divisible by } 4\}$  is an equivalence relation. Find the set of all elements related to 1.

8. Express  $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & +4 \\ 1 & -2 & -3 \end{bmatrix}$  as a sum of symmetric and skew symmetric matrix.

9. If x,y,z are different and

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0 \text{ show that } xyz+1=0$$

10. Using properties of determinates prove the following

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

11. Solve for x

$$2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x), 0 < x < \frac{\pi}{2}$$

Or

$$\text{Show that } \tan^{-1} \left[ \sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2} \right] = \frac{x+y}{1-xy} \quad |x| < 1, y > 0, xy < 1$$

12. Find the value of K, the given function is defined as

$$f(x) = \begin{cases} \frac{\sqrt{5x+2} - \sqrt{4x+4}}{x-2} & x \neq 2 \\ K & x = 2 \end{cases}$$

is continuous at  $x=2$

13. If  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

$$\text{Prove that } \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

Or

If  $y = (x + \sqrt{x^2 + 1})^m$  then show that

$$(x^2 + 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - m^2 y = 0$$

14. Differentiate the following function with respect to x :

$$(\log x)^x + x^{\log x}$$

15. Using differentiate, find the approximate value of  $\sqrt{25.2}$

Or

Two equal sides of an isosceles triangle with fixed base a are decreasing at the rate of 9cm/second. How fast is the area of the triangle decreasing when the two sides are equal to 'a'?

16. Find the interval in which the following function is strictly increasing or strictly decreasing  $f(x) = 2 - 9x + 6x^2 - x^3$

17. Evaluate  $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$

18. Evaluate  $\int_0^{\pi/4} \log(1 + \tan x) dx$

19. Evaluate  $\int \frac{(3 \sin - 2) \cos x dx}{5 - \cos^2 x - 4 \sin x}$

or

$$\int e^x \left( \frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

### SECTION - C

20. If  $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$  show that

$$A^2 - 5A - 14I = O \text{ Hence find } A^{-1}$$

21. Evaluate  $\int_{-1}^2 (x^2 + x + 2) dx$  as a limit of sums

OR

$$\text{Find } \int x \sqrt{1+x-x^2} dx$$

$$(n^2 + 4)(n-1)$$
$$n^2$$

Show that the volume of the greatest cylinder which can be inscribed in a cone of height  $h$  and semi vertical angle  $\alpha$  is  $\frac{4}{27} \pi h^3 \tan^2 \alpha$

Or

Show that the normal at any point  $\theta$  to the curve  $x = a \cos \theta + a \theta \sin \theta$  is at a constant distance from the origin.  $\therefore y = a \sin \theta - a \theta \cos \theta$

23. Three friends A, B and C visited the Reliance fresh to purchase fruits. A Purchased 2 Kg apples, 1 Kg grapes and 3 Kg of oranges, B purchased 3 kg apples, 2 kg grapes and 1 kg oranges, C purchased 4 kg apples 3 kg grapes and 2 kg oranges, the amount paid by them are Rs 440, Rs 410 and Rs. 620. Find the Cost of 1 Kg of each fruit using matrix multiplication. Why we take fruits in our diet?

24. If  $a, b, c$  are real numbers, find factors of the determinant

$$A = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}, \text{ Show that if } \Delta=0, \text{ then either } a+b+c=0 \text{ or } a=b=c.$$

25. Find the area of the region  $\{(x, y): x^2 + y^2 \leq 1 \leq x+y\}$ .

26. Prove that  $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$