DPS RKP

First Term Examination
MATHEMATICS
FT-2016-12(B)

80

Time : 3 hrs. **M. Marks : 100**

General Instructions :

(i) *All questions are compulsory.*
(ii) *This question paper contains 29 questions.*
(iii) *Questions 1-4 in Section A are very short-answer type questions carrying 1 mark each.*
(iv) *Questions 5-12 in Section B are short-answer type questions carrying 2 marks each.*
(v) *Questions 13-23 in Section C are long-answer I type questions carrying 4 marks each.*
(vi) *Questions 24-29 in Section D are long answer II type questions carrying 6 marks each.*
(vii) *Write down the serial number of the question before attempting it.*

SECTION - A (4).

1. If a matrix A is a zero of the polynomial $f(x) = 2x^2 - 3x + 5$. Write an expression to find A^{-1}.

2. Construct a 2×1 matrix $A = [a_{ij}]$ given by $a_{ij} = \frac{i-j}{i+j}$.

3. Differentiate $(\log x)^{\log x}$ $w.r.t.\ x : x > 1$.

4. Find the order and degree (if defined) of

$$\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) = 0$$

SECTION - B

5. Differentiate w.r.t. x : $\sin^{-1}\frac{2^{x+1}}{1+4^x}$. (2)

6. Using differentials, find the approximate value of $(1.999)^5$. → 31.92. (2)

7. Using elementary row transformation, find the inverse of $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$. (2)

8. Find the value of $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5}$ in terms of inverse of tan. (2)

9. Using determinants, find the equation of the line joining (3, 1) an (9, 3). (2)

10. Evaluate : $\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ (4)

11. Evaluate : $\int \frac{e^{2x}-1}{e^{2x}+1}\,dx$ (2)

12. Using properties of integrals, evaluate :

$$\int_0^1 x(1-x)^n\,dx.$$

(2)

SECTION - C

13. Evaluate : $\int \sqrt{\tan x}+\sqrt{\cot x}\,dx$ (4)

OR

Evaluate : $\int \frac{\tan\theta+\tan^3\theta}{1+\tan^3\theta}\,d\theta$

14. Evaluate :

$$\int_0^2 \frac{dx}{\sqrt{(x-1)(2-x)}}$$

$\frac{dy}{dx}=\frac{x^2+y^2}{x^2+xy}$ (4)

15. Solve the differenial equation :

$\left(x^2+xy\right)dy=\left(x^2+y^2\right)dx$, when x = 1, y = 0. (4)

16. Using principle of mathematical induction, prove that $(aI+bA)^n=a^nI+na^{n-1}bA$

where $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ (4)

17. Using properties of determinants prove that :

$$\begin{vmatrix}a^2+2a & 2a+1 & 1\\ 2a+1 & a+2 & 1\\ 3 & 3 & 1\end{vmatrix}=(a-1)^3$$

(4)

18. If $2\tan^{-1}(\cos\theta)=\tan^{-1}(2\operatorname{cosec}\theta)$: solve for θ. $\pi/4$ (4)

19. Find the value of k if

$$f(x)=\begin{cases}kx+1 & x\le\pi \\ \cos x & x>\pi\end{cases} \quad at\ x=\pi$$

cont. (2)

20. If $x = e^{x/y}$, prove : $\frac{dy}{dx} = \frac{x-y}{x\log x}$. (4)

21. For the curve $y = 4x^3 - 2x^5$, find all points at which tangent passes through origin. (3)

22. A kite is 120 m vertically high and 130m of string is out. If the kite is moving away horizontally at a rate of 52 m/s, find the rate at which the string is being paid out. (4)

OR

Show that the function :

$f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing in R. ; $a \geq 1$

23. Using mean value theorem, prove that there is a point on the curve $y = 2x^2 - 5x + 3$ between the points A(1, 0) and B(2, 1) where tangent is parallel to the chord AB. Also find the point. (4)

OR

Find the absolute maximum and minimum values for

$f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 1$ in $[1, 4]$

SECTION - D

24. Of all the closed right circular cylindrical cans of volume $128\pi\ cm^3$, find the dimensions of the can which has minimum surface area. (6)

OR

Of all the lines tangent to the curve $y = \frac{6}{x^2+3}$ find the tangent lines of minimum and maximum slope.

25. Evaluate using properties of integrals :

$$\int_0^\pi \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

(4)

OR

Evaluate using properties of integrals : $\int_0^\pi x\log\sin x\ dx$

26. Solve the differential equation :

$$\operatorname{cosec} x \log y \frac{dy}{dx} + x^2y^2 = 0 \text{ when } x = 0, y = 1$$

27. A dietician has to develop a special diet using two food P and Q. Each packet (containing 30g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholestrol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholestrol and 3 units of

vitamin A. The diet requires atleast 240 units of calcium atleast 460 units of iron and atmost 300 units of cholestrol. Using LPP find how many packets of each food should be used to minimise the amount of Vitamin A in the diet? What is the minimum value of Vitamin A? What is the importance of a balanced diet?

28. Evaluate :

✓ $\int_0^4 \left(x + e^{2x}\right) dx$ as the limit of sum. (5)

29. Draw a rough sketch of the region :

$\left\{(x, y) : y^2 \leq 6ax \ and \ x^2 + y^2 \leq 16a^2\right\}$. Also find the area of the region sketched using integration.

OR

Using integration compute the area bounded by the lines

✓ x + 2y = 2, y − x = 1 and 2x + y = 7 (6)