FIRST TERM (2015-2016)

BB

CLASS XII

SUBJECT : MATHS

Time : 3 Hours **M.M. : 100**

General Instructions:

(i) All questions are compulsory.

(ii) This question paper contains 29 questions.

(iii) Questions 1-4 in section A are very short answer type questions carrying one mark each.

(iv) Questions 5-12 in Section B are short answer type questions carrying two marks each.

(v) Questions 13-23 in Section C are Long Answer Type-I questions carrying four marks each.

(vi) Questions 24-29 in Section-D are Long Answer Type-II questions carrying six marks each.

SECTION-A

1. If the binary operation * on the set of integers Z is defined by $a*b = a + 3b^2$, then find the value of 2*4.

2. Write the principle values:

$\cos^{-1}(\frac{1}{2}) - 2\sin^{-1}(-\frac{1}{2})$

3. Write the value of x – y + z from the following equations:

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

4. A is a non-singular matrix of order 3 and $|A| = -4$, find $|Adj\, A|$.

SECTION-B

5. Simplify: $\tan^{-1}\left[\frac{a\cos x - b\sin x}{a\cos x + b\sin x}\right]$, if $\frac{a}{b}\tan x > -1$

6. Solve: $\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$

7. Using elementry transformations find the inverse of the matrix: $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$

8. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find values of a and b.

9. Find matrix A, if: $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} A \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 3 & -2 \end{bmatrix}$

10. Verify mean value theorem for the function $f(x) = \frac{1}{4x-1}$ in $[1, 4]$

11. Using differentials find the approximate value of $(1.999)^5$.

12. Evaluate: $\int \frac{\sqrt{1+x^2}}{x^4} dx$

SECTION-C

13. Let A = R – {2} and B = R – {1}. If f : A → B is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find f^{-1}.

OR

Let A = {1, 2, 3, ..., 9} and R be the relation in A × A defind by (a, b) R (c, d) if a + d = b + c, for (a, b), (c, d) ∈ A × A. Prove that R is an equivalence relation, also obtain the equivalence class [(2, 5)].

14. A trust caring for handicapped children gets ₹30,000 every month from its donors. The trust spends half of the funds recieved for medical and educational care of the children and for that it charges 2% of the spent amount from them and deposits the balance amount in a private bank to get the money multiplied so that in future the trust goes on functioning regularly. What percent of interest should the trust get from the bank to get a total of ₹1800 every month? Use matrix method, to find the rate of interest. Do you think people should donate in such trusts?

15. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + x I = yA$. Hence find A^{-1}.

16. If $a + b + c \neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then using properties of determinants. Prove taht $a = b = c$.

17. For what value of k, is the following function continuous at $x = 0$

$$f(x) = \begin{cases} \dfrac{1-\cos 4x}{8x^2} & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$

18. Find $\dfrac{dy}{dx}$, if $y = \sin^{-1}\left(\dfrac{5x + 12\sqrt{1-x^2}}{13}\right)$

OR

If $x = a(\cos t + \log \tan t/2)$, $y = a(1 + \sin t)$. Find $\dfrac{dy}{dx}$.

19. Find the equations of the normal lines to the curve $y = 4x^3 - 3x + 5$ which are parallel to the line $9y + x + 3 = 0$.

20. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of an edge is 12 cm?

21. Integrate $\dfrac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$

22. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x} dx$

OR

$\int_0^{\frac{\pi}{2}} \sqrt{\sin x} \cos^5 x \, dx$

23. Evaluate $\int_1^3 2x^2 + x + 9 \, dx$ as limit of the sum.

SECTION-D

24. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinants, find the value of $f(2) - f(x)$.

25. Using matrices solve the following system of equations $x - y + z = 4$, $2x + y - 3z = 0$, $x + y + z = 2$.

26. Differentiate $(x)^{\sin x} + (\sin x)^{\cos x}$ w.r.t. x.

OR

If $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log x}{(\log xe)^2}$

27. Find the intervals in which the function $f(x) = \frac{3}{4}x^4 - 4x^3 - 45x^2 + 51$ is:

(a) strictly increasing

(b) strictly decreasing

28. Prove that the volume of the largest cone, that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

OR

Prove that the radius of the right circular cylinder of greatest ~~greatest~~ curved surface area which can be inscribed in a given cone is half of that of the cone.

29. Evaluate: $\int \sqrt{\tan x} + \sqrt{\cot x}\ dx$

OR

$$\int \frac{x^2+1}{(x-1)^2(x+3)} dx$$