A/c Chaubey

APEEJAY SCHOOL SHEIKH SARAI-I

FIRST TERMINAL EXAMINATION, 2016-17

SS-27

CLASS-XII

MATHEMATICS

Time allowed : 3 Hrs. **M.M. : 100**

General Instructions :

(i) All questions are compulsory.
(ii) Questions 1-4 in Section—'A' carry 1 mark each.
(iii) Questions 5-12 in Section—'B' carry 2 marks each.
(iv) Questions 13-23 in Section—'C' carry 4 marks each.
(v) Questions 24-29 in Section—'D' carry 6 marks each.

(SECTION—A)

1. For what value of x, is the following matrix singular ?

$$\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$$

2. Evaluate :

$$\tan^{-1}(1)+\sin^{-1}\left(\frac{-1}{2}\right)$$

3. Is the binary operation defined by $a * b=\frac{a+b}{2}, \forall \ a, b \in Q$ associative. Justify your answer.

4. Discuss the applicability of Rolle's theorem for $x^{2/3}$ in $[-1,1]$.

5. Let $A=\begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B=\begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ find $(AB)'$.

6. A football is hit by a player, the ball travels along the path $y=2+x-\frac{x^2}{60}$, where y is the height attained by the ball when Horizontal distance is x metres. Find the turning point of the ball.

P.T.O.

7. If $x = a(\theta + \sin\theta)$ and $y = a(1+\cos\theta)$

Find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$

8. Let f be a function defined by $f: N \to N$, $f(x) = 9x^2 + 6x - 5$. Show that $f: N \to S$, where S is the range of f, is one-one onto function.

9. Prove that $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$.

10. If $|A| = 5$ and A is a 3×3 Matrix, find $|2\,adj\,A|$.

11. Evaluate :

$$\int \tan^4 x\,dx$$

12. Find the area bounded by the curve $x^2 = 4y$ and the lines $x = 0, y = 4$ in first quadrant.

13. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2},\ x \in \left(0, \frac{\pi}{4}\right)$.

14. If a, b, c are all positive and are the p^{th}, q^{th} and r^{th} terms of a G.P., then show that

$$\Delta = \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

OR

By using elementary row transformations find

$$A^{-1} \text{ if } A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}.$$

15. Show that of all the rectangles in a given circle the square has the maximum area.

16. Evaluate :

$$\int \frac{x^2}{(x-1)(x-2)}\,dx$$

17. Evaluate :

$$\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}}\,dx$$

18. Evaluate :

$$\int_{-1}^{2} |x^3 - x|\,dx$$

19. Evaluate :

$$\int_0^{\pi} e^{2x}\sin\left(\frac{\pi}{4}+x\right)dx \text{ OR } \int\frac{(3\sin\theta-2)\cos\theta}{5-\cos^2\theta-4\sin\theta}\,d\theta.$$

20. Find the area of the region enclosed between the two curves $x^2+y^2=4$ and $(x-2)^2+y^2=4$.

21. Show that the relation R in the set $A=\{x\in Z: 0\le x\le 12\}$, given by $R=\{(a,b):|a-b|$ is a multiple of 4, is an equivalence relation.

OR

Show that the relation R in the set $\{1, 2, 3\}$ given by $R=\{(1,2),(2,1)\}$ is symmetric but neither reflexive nor transitive. Write the required ordered pairs which can make the relation equivalence.

22. Define a binary operation * on the set $\{0, 1, 2, 3, 4, 5\}$ as $a*b=\begin{cases} a+b, & \text{if } a+b<6 \\ a+b-6, & \text{if } a+b\ge 6.\end{cases}$

Show that '0' is the Identity for this operation and each element $a\ne 0$ of the set is invertible with $6-a$ being the inverse of a.

23. Discuss the continuity for the function

$$f(x)=\begin{cases} x\sin\frac{1}{x}, & x\ne 0 \\ 0, & x=0\end{cases}$$

at $x=0$.

24. An open box with square base is to be made out of a given quantity of metal sheet of area c^2, show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$.

25. Using integration find the area of the region $\{(x,y): x^2+y^2\le 2ax,\ y^2\ge ax,\ x,y\ge 0\}$.

OR

Find the area of the region enclosed by the parabola $x^2=y$, the line $y=x+2$.

26. If $A=\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & +1 & 1\end{bmatrix}$, find A^{-1}, and hence solve the system of linear equations

$$x+2y+z=4$$
$$-x+y+z=0$$
$$x-3y+z=2.$$

OR

Using elementary row transformations find the inverse of $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$.

27. Prove that :

$\begin{vmatrix} yZ - x^2 & Zx - y^2 & XY - Z^2 \\ zX - Y^2 & XY - Z^2 & YZ - x^2 \\ xy - Z^2 & YZ - x^2 & Zx - y^2 \end{vmatrix}$ is divisible by $(x + y + z)$ and hence find the quotient.

28. For the curves $y = 4x^3 - 2x^5$, find all points at which the tangent passes through the origin.

29. It $x = \sqrt{a^{\sin^{-1} t}}$

and $y = \sqrt{a^{\cos^{-1} t}}$

Show that $\frac{dy}{dx} = -\frac{y}{x}$.

OR

If $y = x^x$ prove that

$$\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$