BLUEBELLS SCHOOL INTERNATIONAL
KAILASH, NEW DELHI
FIRST TERMINAL EXAMINATION (2016 – 17)
CLASS : XII

SET : B **M.M 100**
SUBJECT : MATHEMATICS **Time: 3 hours**

Syllabus: **Relations and functions, Inverse trigonometric Functions, Matrices& Determinants, Continuity& differentiability, AOD & Indefinite Integrals.**

Instructions:

- *All questions are compulsory; however certain question(s) contain internal choices.*
- *The question paper consists of 29 questions divided into four sections A, B, C and D.*
  Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
  Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
  Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
  Question 24-29 in Section D are long-answer type questions carrying 6 marks each.
- *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
- *Use of calculators is not permitted.*
- *Write neatly and legibly. Marks will be deducted for untidy work.*

SECTION A

Q1: Find the principal value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$

Q2: Evaluate: $\int \frac{(x^4 - x)^{1/4}}{x^5} dx$

Q3: If $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$, show that $A^{-1} = \frac{1}{19} A$

Q4: If the mappings f and g are given by $f = \{(1,2),(3,5),(4,1)\}$ and $g = \{(2,3),(5,1),(1,3)\}$, write gof.

SECTION B

Q5: Use differentials to approximate the value of $(1.999)^5$

Q6: Prove that the function f given by $f(x) = |x-5|, x \in R$ is not differentiable at $x = 5$.

Q7. If $f : R - \{-1\} \to R - \{-1\}$ be defined as $f(x) = \frac{x}{x+1}$, find $f^{-1}(x)$.

Q8. If $f(x)=\begin{cases}\dfrac{1-\cos 4x}{x^2}, \text{if } x\le 0\\ a, \text{if } x=0\\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, \text{if } x>0\end{cases}$, determine the value of a so that $f(x)$ is continuous at $x=0$.

Q9. Evaluate: $\int \dfrac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}dx$

Q10. If $A^{-1}=\begin{pmatrix}5 & -2\\ -7 & 3\end{pmatrix}$ and $B^{-1}=\dfrac{1}{2}\begin{pmatrix}9 & -7\\ -8 & 6\end{pmatrix}$, find $(AB)^{-1}$

Q11. If matrix $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ and $A^2=kA$. Then write the value of k.

Q12. If $x=e^{x/y}$, then prove that $\dfrac{dy}{dx}=\dfrac{x-y}{x\log x}$

SECTION C

Q13: Find the condition that the curves $2x=y^2$ and $2xy=k$ intersect orthogonally.

Or,

Show that the line $\dfrac{x}{a}+\dfrac{y}{b}=1$ touches the curve $y=be^{-x/a}$ at the point where the curve intersects the y axis.

Q14: If $x=\sec\theta-\cos\theta$, $y=\sec^n\theta-\cos^n\theta$, show that $(x^2+4)\left(\dfrac{dy}{dx}\right)^2=n^2(y^2+4)$.

Q15: Evaluate: $\int \dfrac{\sin x+\cos x}{9+16\sin 2x}dx$

Or,

Evaluate: $\int \sqrt{\operatorname{cosec} x-1}\,dx$

Q16: Water is running into a conical vessel. 15 cm deep and 5 cm in radius at the rate of 0.1 cm^3/sec. When the water is 6 cm deep, find at what rate is

i) the wetted surface of the vessel increasing?

ii) the water surface area increasing?

Q17. On the set $R-\{-1\}$, a binary operation * is defined by $a*b=a+b+ab$ for all $a,b\in R-\{-1\}$.

a) Prove that * is commutative as well as associative on $R-\{-1\}$.

b) Find the identity element with respect to * and prove that every element of $R-\{-1\}$ is invertible.

10000
5000
Hard 5000

Q18: If $A=\begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$, then find A^{-1} using elementary row operation.

Q19. If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$

Q20. If $\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\alpha$, then prove that $9x^2-12xy\cos\alpha+4y^2=36\sin^2\alpha$

Q21 Find the intervals in which the function f given by $f(x)=\sin x+\cos x$, $0\le x\le 2\pi$ is strictly increasing or strictly decreasing.

Q22. If $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ find x and y such that $A^2+xA+yI=0$. Hence find A^{-1}.

Q23 Show that $\sin^p\theta\cos^q\theta$ attains a maximum value when $\theta=\tan^{-1}\sqrt{\frac{p}{q}}$

SECTION D

Q24: In a survey of 20 richest persons of three residential society A,B, C it is found that in society A , 5 believe in honesty, 10 in hard work and 5 in unfair means while in B, 5 believe in honesty, 8 in hard work and 7 in unfair means and in C, 6 believe in honesty , 8 in hard work and 6 in unfair means. If the per day income of 20 richest persons of society A, B, C are Rs. 32500, Rs. 30500, Rs. 31000 respectively, then find the per day income of each type of people by matrix method.

a) Which type of person has more per day income?

b) According to you, which type of person is better for country?

Q25: A point on the hypotenuse of a right triangle is at a distance a and b from the sides of the triangle. Show that the maximum length of the hypotenuse is $\left(a^{2/3}+b^{2/3}\right)^{3/2}$

Or,

Prove that the volume of the largest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\pi h^3\tan^2\alpha$.

Q26: Using properties of determinants to prove that: $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}=1+a^2+b^2+c^2$

Q27: Evaluate 1) $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

2) $\int \frac{8\cot x+1}{3\cot x+2}\,dx$

Q28: If $y = Ae^{-kt}\cos(pt+c)$, prove that $\frac{d^2y}{dt^2}+2k\frac{dy}{dt}+n^2y=0$, where $n^2=p^2+k^2$

Q29. If $x=\frac{\sin^3 t}{\sqrt{\cos 2t}}$, and $y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$, prove that $\frac{dy}{dx}=-\cot 3t$

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