Sahodaya

FIRST TERM EXAMINATION 2016

MATHEMATICS CLASS 12

Q1-6 CARRIES 1 MARK EACH

Q7-19 CARRIES 4 MARKS EACH

Q20-26 CARRIES 6 MARKS EACH

1. If $R = \{(x, y): x + 2y = 8,\ x, y \in N\}$ is a relation on N . Find the range of R

2. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, $xy < 1$, then write the value of x +y + xy

3. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of x + y

4. Evaluate $\int_2^4 \frac{x}{x^2+1} dx$

5. Differentiate $\log_x 4$ with respect to x

6. If $y = e^{\tan 5x}$ find $\frac{dy}{dx}$

7. Using properties of determinants ,prove that $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

8. If $x = \sin t$, $y = \sin k\,t$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2 y = 0$

9. It is given that for the function $f(x) = x^3 + bx^2 + ax + 5$ $on [1,3]$, Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$, find the values of a and b

10. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$, $-\frac{1}{\sqrt{x}} \le x \le 1$

OR

If $\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{x+4}\right) = \frac{\pi}{4}$ find the value of x

11. Prove that : $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$

OR

Solve for x : $\sin^{-1} 6\sqrt{3}x + \sin^{-1} 6x = -\frac{\pi}{2}$

12. Prove that the relation R in the set A ={5,6,7,8,9} given by

R ={$(a, b): |a - b|$ *is divisible by* $2; a, b \in A$} is an equivalence relation . Find all elements related to the element 6

13. If the function $f: R \to R$ *defined by* $f(x) = x^2 + 2$

and $g: R \to R$ *defined by* $g(x) = \frac{x}{x-1}$, x≠ 1 find f o g and g o f and hence find $fog(2)$ *and* $gof(-3)$

14. To raise the money for an orphanage , students of three schools A,B,C organized an exhibition in their locality , where they sold paper bags, scrap books and pastel sheets made by them using recycled paper , at the rate of Rs 20, Rs15, and Rs 5 per unit respectively . School A sold 25 paper bags , 12 scrap books ,and 34 pastel sheets School B sold 22 paper bags , 15 scrap books ,and 28 pastel sheets while School C sold 26 paper bags , 18 scrap books ,and 36 pastel sheets .Using matrices find the total amount raised by each school . By such exhibition ,which values are inculcated in the students ?

15. Find the values of x for which y = $[x(x-2)]^2$ is an increasing function .

16. Find the equations of the tangent and normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(\sqrt{2}a, b)$

17. Evaluate $\int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x}$ dx

OR

Evaluate $\int \frac{x+2}{\sqrt{x^2+5x+6}}$ dx

18. Using differentials , find the approximate value of $(0.999)^{\frac{1}{10}}$

19. Show that the function $f(x) = \begin{cases} \frac{1-\cos 2x}{2x^2} & if\ x \neq 0 \\ 1+3x & if\ x = 0 \end{cases}$, *is continuous at* $x = 0$

20. Use the product $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ to solve the system of equations;

$$x - y - 3 = 0, 2x + 3y + 4z = 17, y + 2z = 7$$

21. Prove , using properties of determinants ;

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & x+z & x+y \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

22. Evaluate $\int \frac{1}{\cos^4 x + \sin^4 x}\, dx$

OR

Evaluate $\int (3x-2)\sqrt{x^2+x+1}\, dx$

23. Evaluate $\int_0^4 (|x| + |x-2| + |x-4|) dx$

24. Show that the height of the cylinder of maximum volume , that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume .

25. If $y = \left(x + \sqrt{x^2+1}\right)^m$, then prove that $(1+x^2)\, y_2 + x\, y_1 - m^2 y = 0$

26. If $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\operatorname{cosec}\theta)$, $(\theta \neq 0)$, then find the value of θ