

1. Samyukt Behavior

Academic Session: 2016-17  
First Term Examination  
Subject: Mathematics  
M/2/2

Time : 3 hrs.

Max Mks : 100

General Instructions:

1. All questions are compulsory
2. This question paper consists of 29 questions divided into four sections A, B, C and D. Section A contains 4 questions of 1 mark each. Section B contains 8 questions of 2 marks each. Section C contains 11 questions of 4 marks each. Section D contains 6 questions of 6 marks each.
3. Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.
4. If you wish to attempt any question again, then cancel out the previous answer.
5. This paper has 4 printed sides.

SECTION A

Q1. Find the value of  $\sin\left(2\sin^{-1}\frac{3}{5}\right)$

Q2. Let  $A = \{1, 2, 3, \dots, 16\}$ . Let  $R$  be the equivalence relation on  $A \times A$  defined by

$(a, b)R(c, d)$  iff  $ad = bc$ . Find the equivalence class  $[(1, 3)]$

Q3. Evaluate:  $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

Q4. If  $y = \operatorname{cosec}^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$ , then find  $\frac{dy}{dx}$

SECTION B

Q5. If  $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$  then find the value of  $a$ .

Q6. Find the point(s) on the curve  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  where tangent is parallel to the y-axis.

Q7. Write the function  $\tan^{-1}(\sqrt{1+x^2} + x)$  in the simplest form

Q8. If  $x^x = y^y$ , then find  $\frac{dy}{dx}$

Q9. If A and B are two events such that  $P(A) = 0.4$ ,  $P(B) = 0.8$  and  $P(B|A) = 0.6$  then find  $P(A|B)$

Q10. Show that the function  $f$  given by  $f(x) = \cot^{-1}(\sin x + \cos x)$ ,  $x > 0$  is strictly decreasing in  $\left(0, \frac{\pi}{4}\right)$

Q11. Evaluate:  $\int \frac{x-3}{(x-1)^3} e^x dx$

Q12. Solve for  $x$ :  $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2} \tan^{-1} x$ ,  $x > 0$

### SECTION C

Q13. Differentiate  $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$  w.r.t.  $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Q14. The two equal sides of an isosceles triangle with fixed base  $b$  are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base?

OR

Using differentials, find the approximate value of  $\frac{1}{(2.002)^2}$ .

Q15. Evaluate:  $\int \frac{x^2+1}{(x-1)^2(x+3)} dx$

Q16. If  $y^x = e^{x^y}$ , prove that  $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$

Q17. Evaluate:  $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$

Q18. Evaluate:  $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9+16\sin 2x} dx$

Q19. Evaluate:  $\int \frac{\sin x - x \cos x}{x(x + \sin x)} dx$  OR  $\int \frac{1}{\sin(x-\alpha)\cos(x-\beta)} dx$

Q20. If  $x = a \sin pt$  and  $y = b \cos pt$ , then find  $\frac{d^2y}{dx^2}$  at  $t = 0$

Q21. Let  $A = Q \times Q$ , where  $Q$  is the set of all rational numbers and  $*$  be a binary operation on  $A$  defined by  $(a, b) * (c, d) = (ac, b + ad)$  for  $(a, b), (c, d) \in A$ . Then find

- (i) the identity element of  $*$  in  $A$
- (ii) invertible elements of  $A$

Q22. Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spades. Hence find the mean of the distribution.

OR

For 6 trials of an experiment, Let  $X$  be a binomial variable which satisfies the relation  $9P(X = 4) = P(X = 2)$ . Find the probability of success in a single trial.

Q23. Show that  $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

Section D

Q24. Evaluate:  $\int \frac{(\sin x + 2)\cos x}{\sqrt{(\sin x - 2)(\sin x - 3)}} dx$

OR

Evaluate  $\int_0^3 (2x^2 + 3x + 5) dx$  as the limit of sum.

Q25. Find the area of the region  $\{(x, y) : x^2 + y^2 \leq 4, x + y \geq 2\}$ , using the method of integration.

OR

Prove that the curves  $y^2 = 4x$  and  $x^2 = 4y$  divide the area of the square bounded by  $x = 0$ ,  $x = 4$ ,  $y = 4$  and  $y = 0$  into three equal parts.

Q26. A dealer wishes to purchase some sewing machines. He has only Rs 57600 to invest and has space for at most 20 items. An electronic machine costs him Rs 3600 and a manually operated machine costs Rs 2400. He can sell an electronic machine at a profit of Rs 220 and a manually operated machine at a profit of Rs 180. Assuming that he can sell all the machines that he buys, how should he invest his money in order to maximize his profit? Make it as a L.P.P. and solve it graphically.

Q27. Consider  $f : R_+ \rightarrow (-9, \infty)$  given by  $f(x) = 5x^2 + 6x - 9$  Show that  $f$  is invertible and find the inverse of  $f$ .

Q28. Find the coordinates of a point on the parabola  $y = x^2 + 7x + 2$  which is closest to the straight line  $y = 3x - 3$

OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius  $R$  is  $\frac{8}{27}$  of the volume of the sphere.

Q29. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.