

GENERAL INSTRUCTIONS:-

1. All The questions are compulsory
2. Questions 1-4 in Section A carrying one mark each
3. Questions 5- 12 in Section B carrying two marks each
4. Questions 13- 23 in Section C carrying 4 marks each
5. Questions 24-29 in Section D carrying 6 marks each.

Section - A

1. Let  $N$  be the set of natural numbers and a relation  $R$  is defined over  $N$  as:  
 $R = \{(2,4), (4,3), (6,2), (8,1)\}$  check for transitivity.
2. Evaluate :  $\int \frac{1}{e^{x+1}} dx$
3. If  $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$ . Find  $x$  and  $y$ .
4. If  $A$  and  $B$  are square matrices of the same order such that  $|A| = 3$  and  $AB=I$ , then write the value of  $|B|$ .

Section - B

5. Let  $*$  be a binary operation on the set  $Q$  of rational numbers as follows:  $a*b = ab^2$   
 Find the binary operation is commutative and associative.
6. If  $f : R \rightarrow R$  and  $g : R \rightarrow R$  be functions defined by  $f(x) = [x]$  and  $g(x) = |x|$  then evaluate the following :  $(f+2g)(-2)$  and  $gof\left(\frac{2}{3}\right) - fog\left(\frac{2}{3}\right)$
7. Find the value of  $\cot\left(\frac{\pi}{2} - 2 \cot^{-1} \sqrt{3}\right)$
8. Express the following matrix as the sum of a symmetric and a skew- symmetric matrices  
 $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$
9. If  $\sec^{-1}\left(\frac{x+y}{x-y}\right) = a$ , prove that  $\frac{dy}{dx} = \frac{y}{x}$
10. The radius of cylinder is increasing at the rate of 2cm/sec. and its altitude is decreasing at the rate of 3cm/sec. Find the rate of change of volume when the radius is 3 cm and altitude is 5cm.
11. A wholesaler starts his business with 24000 rupees. He purchase the wheat and the rice at rate of 400 rupees and 600 rupees per quintal respectively. The capacity of his store is 200 quantity. He earns a profit of 25 rupees on wheat and 40 rupees on rice per quintal. Represent it in mathematical form if he stocks the wheat and rice  $x$  and  $y$  quintals respectively.

*If  $y = \cos t x$ , find  $\frac{d^2y}{dx^2}$  in terms of  $y$  alone*

12. Integrate  $\frac{1}{\sin x \cos^2 x}$

Section - C

13. Let  $f : N \rightarrow R$  be a function defined by  $f(x) = 4x^2 + 12x + 15$ . Show that  $f : N \rightarrow \text{Range}(f)$  is invertible. Find the inverse of  $F$ .

OR

Discuss the commutativity and associativity of binary operation  $*$  defined on  $Q$  by the rule  $a*b = a-b+ab$  for all  $a, b \in Q$ .

14. Prove that  $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$ .
15. Solve the equation for  $x$ :  $\cos^{-1} x + \sin^{-1} \frac{x}{2} = \frac{\pi}{6}$ .
16. Show that the function  $f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$  is continuous but not differentiable at  $x=0$ .
17. Differentiate with respect to  $x$ :  $y = \left(x + \frac{1}{x}\right)^x + x^{\left(x + \frac{1}{x}\right)}$

OR

- Verify Lagrange's mean value theorem for the function  $f(x) = x^2 - 2x + 4$  in  $[1,5]$
18. Find the intervals in which the following functions are strictly increasing or decreasing  
 $f(x) = -2x^3 - 9x^2 - 12x + 1$
19. Find the equation of the normals to the curve  $y = x^3 + 2x + 6$  which are parallel to the line  $x + 14y + 4 = 0$ .

OR

- Find the points on the curve  $x^2 + y^2 - 2x - 3 = 0$  at which the tangents are parallel to the  $x$ -axis.
20. Find the approximate value of  $f(5.001)$  where  $f(x) = x^3 - 7x^2 + 15$

21. Evaluate  $\int \frac{d\theta}{\sin(\theta - \alpha) \sin(\theta - \beta)}$

22. Evaluate  $\int \frac{(x^2 + 1)e^x}{(x + 1)^2} dx$

23. Evaluate  $\int_0^\pi x \sin^2 x dx$

*Handwritten notes:*  
 $x^2 - 1 = (x-1)(x+1)$   
 $\frac{2x^2 + 11x}{(x-1)(x+1)}$   
 $\frac{-2x^2 - 9x - 11}{(x-1)(x+1)}$   
 $\frac{-2x^2 + 2x + 11}{(x-1)(x+1)}$   
 $\frac{-15x - 11}{(x-1)(x+1)}$   
 $\frac{-15x - 11}{(x-1)(x+1)}$

Section - D

24. The monthly incomes of Aryan and Babban are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. If each saves Rs. 15,000 per month, find their monthly incomes using matrix method. This problem reflects which value?

OR

Using elementary row operations find the inverse of matrix  $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$  and hence solve the following system of equations  $3x - 3y + 4z = 21$ ,  $2x - 3y + 4z = 20$ ,  $-y + z = 5$ .

25. Using properties of determinants, prove that  $\begin{vmatrix} 2y & y - z - x & 2y \\ 2z & 2z & z - x - y \\ x - y - z & 2x & 2x \end{vmatrix} = (x + y + z)^3$ .

26. Find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{2}$  when  $x = a(\theta + \sin \theta)$  and  $y = a(1 - \cos \theta)$

27. In the printed page of a book, the combined width of the margin at the top and the bottom is 3 inches and that on the sides 2 inches. Find the dimensions of the page, so that the area of the printed matter may be maximum, if the total area of the page is 150 sq. Inches.

OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius  $R$  is  $\frac{8}{27}$  of the volume of the sphere.

28. A factory uses three different resources for the manufacture of two different products, 20 units of the resource, 12 units of B and 16 units of C being available. One unit of the first product requires 2, 2 and 4 units of the respective resources and one unit of the second product requires 4, 2 and 0 units of resources respectively. It is known that the first product gives a profit of 2 monetary units per unit and the second 3. Formulate the linear programming problem. How many units of each product should be manufactured for maximizing the profit? Solve it graphically?

29. Evaluate:  $\int \sqrt{a^2 - x^2} dx$

*Handwritten notes:*  
 (28) using properties of Determinant  
 $\begin{vmatrix} x & 2x & 4z \\ 4 & 4z & 2x \\ 2 & 2z & 2y \end{vmatrix} = (x-y)(y-z)(z-x)$   
 $(xy + yz + zx)$