

APS

Time: 3 hours
M.M: 60

FIRST TERM EXAMINATION 2017-18

CLASS : XII MATHS

INSTRUCTIONS : Question 1 to 4 carry 1 marks each, question 5 to 12 carry 2 marks each , question 13 to 24 carry 4 marks each and question 25 to 29 carry 6 marks each .

- 1) The relation R in the set A of all the lines in a plane defined as $R = \{ (l_1, l_2) \in A \times A ; l_1 \text{ is parallel to } l_2 \}$ is an equivalence relation . Write the equivalence class related to the line $2y = 5x + 7$
- 2) If for matrix A $|A| = 5$, find $|4A|$ where matrix A is of order 2×2
- 3) $*$ is a binary operation on the set N , natural numbers, defined by $a * b = \text{HCF} (a , b)$. Evaluate $3 * (2 * 5)$
- 4) If I is an identity matrix and A is a square matrix of the same order such that $A^2 = A$, then what is the value of $(I + A)^2 - 3A$?
- 5) I is multiplicative identity for matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 2 & 6 \end{bmatrix}$. State true or false with a reason.
- 6) If $y = \left(\frac{\sin x}{2} + \frac{\cos x}{2} \right)^4$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$
- 7) Find the points on the curve $y = \frac{1}{x-1}$, where tangent has slope equal to 2.
- 8) Evaluate $\int e^{\sqrt{x}} dx$
- 9) Evaluate $\tan^{-1} \left(-\tan \frac{3\pi}{4} \right)$
- 10) Without using derivatives find the maximum and minimum values of $(3 \sin x + 2)$, $x \in R$
- 11) If $y = \cos^{-1} \left(\frac{x^2 - 1}{x^2 + 1} \right)$ find $\frac{dy}{dx}$
- 12) Evaluate $\int \frac{\sqrt{x} dx}{x+1}$
- 13) If $A = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find k so that $A^2 = kA - 2I$
- 14) For what values of p , the function $f(x) = \begin{cases} \frac{\sqrt{1+px} - \sqrt{1-px}}{x} & -1 \leq x < 0 \\ \frac{2x+1}{x-2} & 0 \leq x \leq 1 \end{cases}$ is continuous at $x = 0$
- 15) If $y = x + \tan x$, prove that $\cos^2 x \cdot \frac{d^2 y}{dx^2} - 2y + 2x = 0$
- 16) Verify mean value theorem for the function $f(x) = x^2 + 2x + 3$ in $[4, 6]$

17) Find the values of a and $f(0)$, if f is continuous at $x = 0$ where $f(x) = \frac{\sin 2x + a \sin x}{x^3}$

18) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ show that $(1-x^2) \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} - y = 0$

19) Find the approximate value of $f(2.01)$, where $f(x) = 4x^3 + 5x^2 + 2$

20) A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. Find the maximum volume of the box. Write the importance of calculating the sides of a square to be cut off to get box of maximum value.

21) Evaluate $\int \frac{\sin a}{\sin 3a} da$

22) If $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$, find AB and hence solve the equations

$$2x - y + z = -1$$

$$-x + 2y - z = 4$$

$$x - y + 2z = -3$$

23) By using properties of determinant prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

24) Show that the relation S in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ given by $S = \{(a,b) : a, b \in \mathbb{Z}, |a-b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all the elements related to 1.

25) Prove that $\tan(\pi/4 + \frac{1}{2} \cos^{-1} a/b) + \tan(\pi/4 - \frac{1}{2} \cos^{-1} a/b) = 2b/a$.

26) By using elementary transformation, find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

27) If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinants find the value of $f(2x) - f(x)$.

28) Find the intervals in which the function f given by $f(x) = x^3 - 12x^2 + 36x + 17$ is increasing or decreasing.

29) Evaluate $\int \frac{(3 \sin x - 2) \cos x}{5 - \cos^2 x - 4 \sin x} dx$