MANAVA BHARATI
INDIA INTERNATIONAL SCHOOL

MID-TERM EXAMINATION (2017-18)
SUBJECT: MATHEMATICS
CLASS: XII Ratika Maan

TIME - 3 HOURS

MAXIMUM MARKS - 100

General Instructions

(i) Q.No. 1 to 4 carry 1 mark each.
(ii) Q.No. 5 to 12 carry 2 marks each.
(iii) Q.No 13 to 23 carry 4 marks each.
(iv) Q.No 24 to 29 carry 6 marks each.

1. Solve $\cos(\tan^{-1} x) = \sin(\cot^{-1}\frac{3}{4})$

2. If $A = \begin{bmatrix} 3 & -4 \\ 7 & 8 \end{bmatrix}$ Show that $A - A'$ is a skew symmetric matrix. Where A' is the transpose of Matrix A.

3. Evaluate $\begin{vmatrix} cos15^0 & sin15^0 \\ sin75^0 & cos75^0 \end{vmatrix}$

4. Evaluate $\int sec^2(7 - 4x)\, dx$

5. For what value of k the following function is continuous at $x = 0$?

$$f(x) = \begin{cases} \frac{1-cos4x}{8x^2} & , x \neq 0 \\ k & , x = 0 \end{cases}$$

6. Write in simplest form $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$

7. Differentiate $\sin\sqrt{x} + \cos^2\sqrt{x}$

8. Evaluate $\int \frac{cosx}{\sqrt{1+sinx}}\, dx$

9. Evaluate $\int_0^{\pi/2} \cos^2 x \, dx$

10. Evaluate $\int_0^1 \frac{2x+3}{5x^2+1} \, dx$

11. Find the interval in which $y = x^2e^{-x}$ is increasing

12. Find a point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining the Points $(2, 0)$ and $(4, 4)$.

13. For well being of an orphanage, three trust A, B and C has donated 10 %, 15 % and 20 % of Their total fund 2,00,000, 3,00,000 and 5,00,000 respectively. Using matrix multiplication. Find the total amount of money received by orphanage by three trusts. By such donations, which values are generated?

14. If $x^y = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{\log x}{(\log xe)^2}$

Or

If $y = \text{Sin}^{-1}\left\{\frac{5x+12\sqrt{1-x^2}}{13}\right\}$, then find $\frac{dy}{dx}$

15. Evaluate $\int \frac{4x+5}{\sqrt{2x^2+x-3}} \, dx$

16. If $y = e^x(\sin x + \text{Cos } x)$, then show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

Or

If $y = \log\left(x + \sqrt{x^2 + a^2}\right)$, Show that $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

7. By using properties of determinants, prove that

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

8. Show that the function $f(x) = |x-3|$, $x \in R$, is continuous but not differentiable at $x = 3$

Or

If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$

9. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$

10. Two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per Second. How fast is the area decreasing when the two equal sides are equal to the base?

11. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b], where a =1 and b =3. Find all $c \in (1, 3)$ for which $f'(c) = 0$.

12. Prove that $\sin^{-1}(\frac{8}{17}) + \sin^{-1}(\frac{3}{5}) = \cos^{-1}(\frac{36}{85})$

13. If a, b, c are in A.P, then evaluate the determinant

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

14. Evaluate $\int_0^4 (x + e^{2x})dx$ as the limit of a sum.

Or

Evaluate $\int_1^4 (x^2 - x)dx$ as the limit of a sum.

25. Solve the system of following equations, using matrix method

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4 \; ; \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1 \; ; \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2 \; ;$$

26. Show that height of the cylinder of greatest volume which can be inscribed in a right circular Cone of height h and semi vertical angle $\propto$ is one third that of the cone and the greatest volume of the cylinder is $\frac{4}{27}\pi h^3 \tan^2 \propto$.

Or

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

27. (a) Evaluate $\int x \sin^{-1} x \; dx$ (b) Evaluate $\int \frac{Cos\, x}{(1-Sin\, x)(2-Sin\, x)} dx$

28. (a) Evaluate $\int \frac{Cos\, x - Sin\, x}{1+Sin\, 2x} dx$ (b) Differentiate with respect to x, $y = \log_7(\log x)$

29. (a) Find the interval in which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly increasing or Decreasing

(b) Evaluate $\int \sqrt{x^2 + 4x - 5} \; dx$

Or

(a) Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at $x = e$

(b) Evaluate $\int \sqrt{1 - 4x - x^2} \; dx$