

Name Ananya Class & Section XII-B Roll No. 3

## FIRST TERM EXAMINATION—2017-18

### CLASS—XII

### SUBJECT—MATHEMATICS

Time : 3 Hours

M.M. : 100

#### General Instructions :

This question paper contains 29 questions.

Questions 1–4 in Section A carry 1 mark each.

Questions 5–12 in Section B carry 2 marks each.

Questions 13–23 in Section C carry 4 marks each.

Questions 24–29 in Section D carry 6 marks each.

All the questions are compulsory. However, internal choice has been given in a few questions.

#### Section-A

- Find the equation of line joining A (3, 1) and B (9, 3) using determinants.
- Evaluate :  $\sin [2 \tan^{-1} (0.75)]$
- Find an angle,  $0 < \theta < \frac{\pi}{2}$ , which increases twice as fast as its sine.
- Evaluate :  $\int \frac{\sin x}{\sin(x-a)} dx$

#### Section-B

- Check whether the relation R in the set of real numbers, defined by  $R = \{(a, b) : a \leq b^3\}$  is symmetric or transitive.
- If  $a_1, a_2, \dots, a_n$  is an AP with common difference  $d$ , then evaluate

$$\tan \left[ \tan^{-1} \left( \frac{d}{1 + a_1 a_2} \right) + \tan^{-1} \left( \frac{d}{1 + a_2 a_3} \right) + \dots + \tan^{-1} \left( \frac{d}{1 + a_{n-1} a_n} \right) \right]$$

7. Examine the consistency of the system of equations

$$x + 3y = 5; 2x + 6y = 8$$

8. If  $\log(x^2 + y^2) = 2 \tan^{-1} \frac{y}{x}$ , show that  $\frac{dy}{dx} = \frac{x+y}{x-y}$

9. Find the approximate change in the volume of a cube of side  $x$  metres caused by increasing the side by 3%.

10. Find the points of maxima or minima for the function,

$$f(x) = 4x^3 - 18x^2 + 27x - 7$$

11. Evaluate :  $\int e^{\tan^{-1}x} \left( \frac{1+x+x^2}{1+x^2} \right) dx$

12. Evaluate :  $\int \frac{8^{1+x} + 4^{1-x}}{2^x} dx$

### Section-C

13. Given a non empty set  $X$ , let  $*$  :  $P(X) \times P(X) \rightarrow P(X)$  be defined as  $A * B = (A - B) \cup (B - A)$ ,  $\forall A, B \in P(X)$ . Show that the empty set  $\phi$  is the identity for the operation  $*$  and all the elements of  $A$  of  $P(X)$  are invertible with  $A^{-1} = A$ .

14. Simplify :  $\tan^{-1} \left\{ \frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}} \right\}$ ,  $\pi < x < \frac{3\pi}{2}$

OR

If  $x = \operatorname{cosec} \left[ \tan^{-1} \left\{ \cos(\cot^{-1}(\sec(\sin^{-1} a))) \right\} \right]$ , express  $x$  in terms of  $a$ .

15. Check the differentiability of the function  $f$  given by  $f(x) = |2x - 1| \sin x$  on  $\mathbb{R}$ .

16. Two men  $M_1$  and  $M_2$  start with velocities 'v' at the same time from the junction of two roads inclined at  $45^\circ$  to each other. If they travel by different roads, find the rate at which they are separated.

'Speeding and driving from the wrong side contribute to major accidents in Delhi.' Comment.

17. Given  $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ , find BA and use this to solve the

system of equations  $y + 2z = 7$ ,  $x - y = 3$ ,  $2x + 3y + 4z = 17$

18. If  $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ , then find  $A^2 - 5A - 14I$ . Hence, find  $A^3$ .

19. Find the equation of tangent to the curve  $y = \sqrt{3x - 2}$  which is parallel to the line  $4x - 2y + 5 = 0$

20. Show that the function

$$f(x) = 2x + \cot^{-1} x + \log \left( \sqrt{1+x^2} - x \right) \text{ is increasing in } \mathbb{R}.$$

**OR**

Find the intervals in which the function given by

$$f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi \text{ is increasing or decreasing.}$$

21. Evaluate the integral :

$$\int \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

22. Evaluate : 
$$\int \frac{\sqrt{x^2 + 1} \{ \log(x^2 + 1) - 2 \log x \}}{x^4} dx$$

**OR**

$$\int \frac{1}{2 - 3 \cos 2x} dx$$

23. Evaluate :

$$\int_0^1 e^{2-3x} dx \text{ as a limit of sum.}$$

**Section-D**

24. Prove that 
$$\begin{vmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ac - b^2 \end{vmatrix}$$
 is divisible by  $a + b + c$  and find the quotient.

25. Evaluate the integral  $\int e^{-3x} \cos^3 x dx$

**OR**

Evaluate  $\int \frac{\tan \theta + \tan^3 \theta}{1 + \tan^3 \theta} dx$

26. Evaluate :  $\int_{-\pi/4}^{\pi/4} \log |\sin x + \cos x| dx$

27. Find the area bounded by the lines  $y = 4x + 5$ ,  $y = 5 - x$  and  $4y = x + 5$

28. Prove that the volume of the largest cone that can be inscribed in a sphere of radius  $R$  is  $\frac{8}{27}$  of the volume of the sphere.

29. If  $x = \sin t$  and  $y = \sin pt$ , prove that  $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0$

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