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DAV Public School, Kailash Hills
First Terminal Examination, 2017
CLASS-XII, MATHEMATICS

Time Allowed: 3 Hrs **Maximum Marks: 100**

General Instructions:

(i) **All questions are compulsory.**
(ii) **This question paper contains 29 questions.**
(iii) **Questions 1-4 in Section A carries 1 mark each.**
(iv) **Questions 5-12 in Section B carry 2 marks each**
(v) **Questions 13-23 in Section C are long-answer-I type questions carry 4 marks each.**
(vi) **Questions 24-29 in Section D are long-answer-II type questions carry 6 marks each.**

Section – A

1. Write the value of $\tan\left(2\tan^{-1}\frac{1}{5}\right)$.

2. If A is a non-singular square matrix of order 3 such that $|adjA| = 225$, *then find* $|A'|$.

3. Write the derivative of x^x.

4. The radius of a circle is increasing at the rate of $3cm/sec$. Find the rate at which the area of the circle is increasing when the radius is $10cm$.

Section – B

5. If $\begin{bmatrix}2x & 3\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}\begin{bmatrix}x\\ 8\end{bmatrix} = 0$, find the value of x.

6. Evaluate $\int \frac{\cos 2x}{\sin^2 x \cos^2 x}\,dx$.

7. If the matrix $\begin{bmatrix}5-x & x+1\\ 2 & 4\end{bmatrix}$ is singular, find x.

8. Find $\frac{dy}{dx}$ when $(x^2+y^2)^2 = xy$.

9. Evaluate $\int \frac{\sin 2x}{a^2+b^2\sin^2 x}\,dx.$

10. If $x = a\sec\theta,\ y = b\tan\theta$, find $\frac{d^2y}{dx^2}$

11. Find the slope of the normal to the curve $x = 1 - a\sin\theta,\ y = b\cos^2\theta$ at $\theta = \frac{\pi}{2}$

12. Using differentials find approximate value of $\sqrt{25.3}$

Section – C

13. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$

14. Using Roll's theorem, find the point on the curve $y = x(x-4),\ x \in [0,4]$, where the tangent is parallel to $x-$ axis.

15. Solve for x; $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$.

16. Using properties of determinants prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

17. Show that the function $f(x) = 2x - |x|$ is continuous but not differentiable at $x = 0$.

18. Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut orthogonally If $c^4 = 32a^4$.

19. Evaluate: $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\,dx$

20. If $x^py^q = (x+y)^{p+q}$ then prove that $\frac{dy}{dx} = \frac{y}{x}$

21. Evaluate: $\int \frac{e^x(x-4)}{(x-2)^3}\,dx$

22. Evaluate $\int \frac{x^2}{(x^2+4)(x^2+9)}\,dx$

23. Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 15$ is increasing and decreasing.

Section – D

24. Solve by matrices:

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

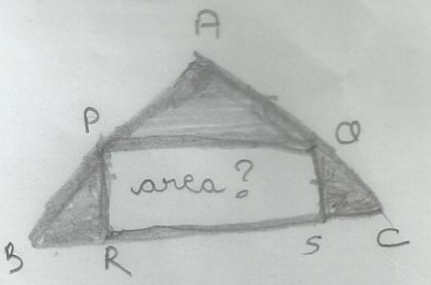
25. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is $12\ m$, find the dimensions of the rectangle so that it may produce the largest area of the window.

26. An open box with a square base is to be made out of a given quantity of sheet of area a^2. Show that the maximum volume of the box is $\frac{a^3}{6\sqrt{3}}$.

27. A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of semicircle. Find the dimensions of the rectangle so that its area is maximum. Also find the maximum area.

28. Evaluate: $\int (x-3)\sqrt{x^2+3x-18}\,dx$.

29. Evaluate: $\int \{\sqrt{\tan x} + \sqrt{\cot x}\}dx$.



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